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NUMERICAL INVESTIGATIONS OF STRESS CONCENTRATION IN REINFORCEMENT STEEL STRUCTURE BY COMPOSITE OVERLAYS

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Abstract

The stress concentration observed in the vicinity of cut-outs and holes in structural elements significantly influences the fatigue endurance of machines subjected to cyclic loads. Numerous studies have been made so far to improve this situation and increase the structure lifetime. Several design recommendations have also been worked out to avoid the problem of premature failure. The proposed article illustrates the influence of the composite overlays applied around the cut-outs made in flat steel constructional elements subjected to axial tension. The detailed study concerns the reinforcement made from the FRP (fibre reinforcement polymer) composite applied around the notches. Two types of composite materials were used, namely: TVR 380 M12/R-glass (glass fibres embedded in epoxy resin matrix) and AS4D/9310 (carbon fibres embedded in epoxy resin matrix). In the first step, the detailed numerical studies (finite element analysis) were performed for the steel samples (with no overlays added) with cut-outs made in the form of circle, square and triangle hole (the last two with rounded corners). The results of these studies were compared with the existing analytical solutions with respect to the stress concentration factors (SCF) estimation. The relatively good conformity was observed when using dense meshes of finite elements placed around the void vicinity. In the next step, the composite overlays were applied around cut-outs and their influence on the stress concentration was investigated. The influence of the fibre orientation, numbers of layers, sizes of the composite overlay used were considered. It was proved that the application of composite overlays evidently decreases the stress concentration around the notches.

Keywords: *plate, hole, cut-out, stress concentration, FRP composite, reinforcement, FEM*

1. Introduction

The necessity of overall structure mass reduction, the providing of communications between the segments of structures or presence of inspection holes and channels, and others is the usual cause of the various cut-outs application. So that the presence of different geometric discontinuities in constructional elements it is sometimes unavoidable and is accompanied by the local elevation of stress. The typical discontinuities are holes, grooves, keyways, fillets, cross-bores, etc. and these are usually called stress raisers. The notion of the notch is introduced for such raisers and these structural elements are called notched members. These notched members demand special attention in the designing process, as the presence of notches may reduce significantly the fatigue endurance and shorten the element lifetime. The stress concentration factor α_k (K_t) is employed to characterize the importance of an introduced notch, which is defined as below [1-3]:

$$\alpha_k = \frac{\sigma_{\max}}{\sigma_{nom}}. \tag{1}$$

This coefficient is defined as the ratio of the maximum stress occurring in the notch area to the nominal stress determined in the weakened part of the cross-section. The formula given above is valid in the field of elastic deformations, which is crucial in case of high-cyclic fatigue endurance.

Depending on the shape and type of the notch, the value of maximum stresses in notch may exceed even several times the nominal values of stresses determined in a certain distance from the notch [1, 2]. The investigations concerning stress concentration in selected types of notches have been conducted for many years. One of the first works providing solutions for structural elements with notches are works by A. Kirsch [4] and C. E. Inglis [5], where the Authors provide analytical solutions for plates of infinite width with circular and elliptical holes. In the following years, numerous theoretical considerations were conducted – among the most important works on the mathematical modelling of notches in the isotropic elements; one can find the works of N. I. Muschelishvili [6] or H. Neuber [2]. The fundamental work concerning the anisotropic elements is the monograph of S. G. Lekhnickij [7]. At the same time, intensive experimental studies of notches were carried out. Here, a special role-played the investigations with the application of photo-elastic properties of certain materials, and in this subject, the monography of M. M Frocht [8] is considered to be the classical one. The results of numerous studies concerning the stress concentrations were gathered in a pioneering monograph by R. E. Peterson entitled as Stress Concentration Factors, which first issue took place in 1953. It is worth mentioning that the chapter of this monograph dealing with the stress concentration around the holes made in structural elements contains more than 100 diagrams for various combinations of shapes of holes and applied loads. Another effective experimental tool used in stress concentration studies was the use of strain gauges [9, 10].

The fundamental change in the assessment of the stress concentration was the introduction and application of Finite Element Method (FEM) [11], which provides a relatively precise and quick prediction of stress concentration factor in a sense of the applied structure and material model. At present, the researchers received the additional non-destructive method, which enables the efficient analysis of displacements, strains, and stresses – this is the Digital Image Correlation Method (DIC). The DIC method has become popular in the late 80-ties and 90-ties of the previous century [12]. Thanks to the existence of diagrams, charts, etc. given in guides and books, and FEM modelling or application of DIC the designing of even complex in shape structural elements has become relatively comfortable and well-controlled [13].

Following the theoretical and experimental results, it can be stated that the stress concentration factor is the function of the notch shape, its size, the relation of the respective dimensions, type of applied loads and others. Figure 1 shows various shapes of notches in the form of the hole applied in flat, thin-walled structural element subjected to tension. In all these cases, the value of the stress concentration factor strongly depends on the notch shape.

For certain notches shown in Figure 1, the analytical formulas for stress concentration factor can be given [3-7]. In case of the elliptic hole cut in the flat, isotropic plate (Fig. 1c) with finite width, the α_k is as follow [2, 4-6]:

$$\alpha_k = C_1 - C_2 \cdot \frac{2a}{D} + C_3 \cdot \left(\frac{2a}{D}\right)^2 - C_4 \cdot \left(\frac{2a}{D}\right)^3, \tag{2}$$

where: $1.0 \leq a/b \leq 8.0$, the detailed formulas for constants C_1-C_4 are easily accessible in [3].

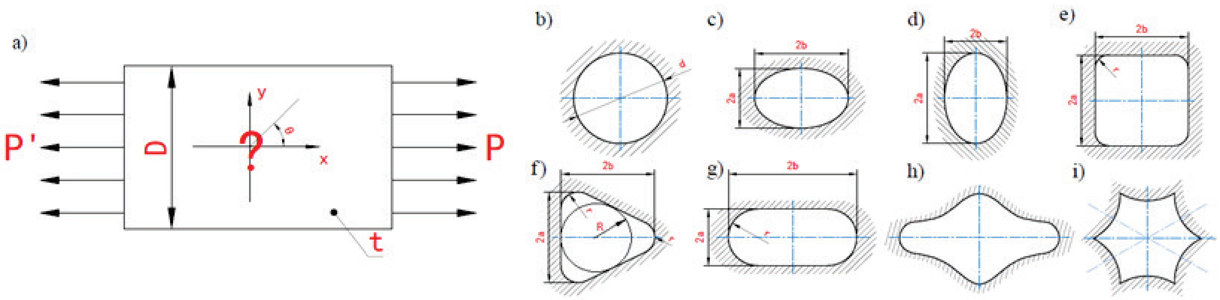

Fig. 1. a) Flat, thin-walled element under tension with notch placed in the centre of the plate b) circular hole, c) horizontal ellipsis, d) vertical ellipsis e) rectangle hole with rounded corners f)triangular hole, g) oval or slot type hole, h) so-called almost elliptic, i) hole with a special case

For other geometric shapes of applied notches (Fig. 1 d-i), the analytical formulas for plates with infinite width are accessible. Their values depend on different roundness radii, proportions of dimensions and notch orientation with respect to the load direction or panel thickness. In case of the rectangular hole (Figure 1e) with rounded corners cut in an infinitely wide thin plate subjected to uniaxial, tension the formulae takes the simplified form [3]:

$$\alpha_k^{\infty} = C_1 + C_2 \cdot \frac{b}{a} + C_3 \cdot \left(\frac{b}{a}\right)^2 + C_4 \cdot \left(\frac{b}{a}\right)^3, \tag{3}$$

valid when: $0.06 \leq r/b \leq 1.0$ and $0.2 \leq b/a \leq 1.0$,
constants C_1-C_4 are calculated as following [3]:

$$\begin{aligned} C_1 &= 14.815 - 22.308 \cdot \sqrt{\frac{r}{2b}} + 16.298 \cdot \frac{r}{2b}, \quad C_2 = -11.201 - 13.789 \cdot \sqrt{\frac{r}{2b}} + 19.200 \cdot \frac{r}{2b}, \\ C_3 &= 0.202 + 54.620 \cdot \sqrt{\frac{r}{2b}} - 54.748 \cdot \frac{r}{2b}, \quad C_4 = 3.232 - 32.530 \cdot \sqrt{\frac{r}{2b}} + 30.964 \cdot \frac{r}{2b}. \end{aligned} \tag{4}$$

In several works the correction factors, which take into account the finite width of the investigated sample can be found [14]. In case of investigations of the orthotropic materials, the stress concentration factors also can be found. For example for elliptic cut-out:

$$\alpha_k^{\infty} = 1 + \frac{b}{a}\sqrt{2\left(\sqrt{E_y/E_x} - \nu_{yx} + E_y/2G_{xy}\right)}. \tag{5}$$

where a and b are the semi-axes of the ellipsis and orientation angle is equal to 0º [14, 15]. More complex shapes of notches in the form of holes are seldom used and the assessment of the respective stress concentration factors can be found in papers [2, 3, 6, 7]. For engineering purposes, stress concentration can be assessed by means of charts or diagrams [3].

2. Numerical model

The stress concentration factor for simple and complex shape notches can be effectively estimated by means of numerical calculations, namely FEM. The detailed study was performed for the flat plate with a square hole having rounded corners subjected to unidirectional axial tension. The comparative analysis was done with the use of the ANSYS code [16] for three types of elements, namely plane elements (PLANE182), shell elements (SHELL181) and solid elements (SOLID185). Due to the symmetry of geometry and boundary conditions and type of applied load, it was enough to analyse only symmetric quadrant of the investigated plate. The spotted differences of results (maximum stress value) for the respective, comparable meshes were negligible so that finally the plane elements were chosen for the detailed *h*-convergence study. The idea of the mesh pattern, proposed line divisions and results obtained are presented in Fig. 2. Here, in Figure 2a the exemplary mesh for the symmetric quadrant is shown. In this approach the regular mesh in the neighbourhood of the stress concentration zone is applied In Fig. 2b the number of the line divisions along the ¼ hole edges is shown. Fig. 2c presents the dependency between the stress concentration factor defined in (1) and the number of so-called active degrees of freedom, which in fact defines the number of equations to solve in the analysed problem. As it was expected, the best results were obtained for the densest mesh, corresponding to the sequence of divisions: 96-192-96 along the quarter of the cut hole. Here the energy error estimator was used ('smxb' value) for estimation of the maximum stress, and for the densest applied mesh, the discrepancy between the 'smxb' (estimated) and 'smx' (current) values are accepted from the engineering point of view. The proposed mesh choice takes into account also the calculation time aspect, which rapidly increases with the greater number of applied finite elements.

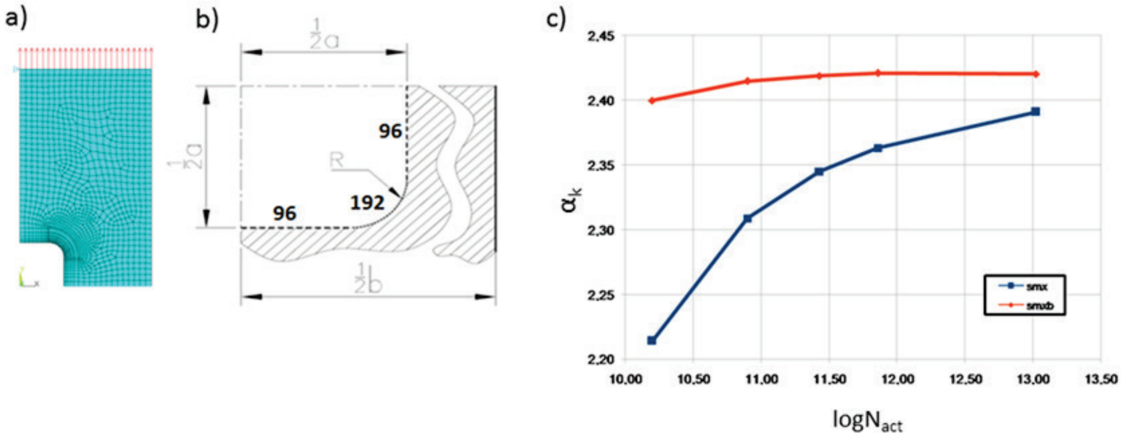

Fig. 2. a) symmetric quadrant of analysed model, b) number of divisions for straight (96) and rounded corner (196) lines, c) stress concentration factor value versus log of number of active degrees of freedom in investigated model (N_{act}, here 'smx' – current solution attributed to chosen mesh 'smxb' – estimated value of stress concentration factor with energy error estimator)

It is worth noting that the numerically estimated value of the stress concentration factor (α_k) is about 2.41 (for the plate of the same geometrical parameters but with the infinite width α_k raises to 3.28). The observed stress concentration is particularly dangerous for structures subjected to loads changing during the exploitation time usually resulting in the premature destruction of machine components. In order to minimize the harmful influence of the notch presence, certain steps can be made. One of them is the well-known local increase of the thickness around the cut hole. The respective results can be found for various shapes of notches in the monography [2-7]. Here, in the presented article the application of properly chosen composite overlays is suggested. The idea of such applications follows among others the known solutions used for concrete, bridge structures [17]. There the composite bands are attached where certain defects or damages were spotted and glued for example at the bottom of the bridge spans subjected to tension or along the compressed pillars supporting bridges. Such applications are definitely cheaper than full structure reconstruction and let for further structure safe exploitation. This idea was also exploited in the papers of Z. Y. Wang and Y. Wang and others [18, 19] with respect to metallic components with circular holes. There the Authors investigated the influence of the applied overlays on the fatigue endurance in the high cyclic fatigue range area. The general idea of the performed in the article numerical studies illustrates the Fig. 3.

As it can be seen, such overlays can have the shape similar to the cut hole, which seems to be recommended. However, overlays with other shapes can also be applied. These above given can also be spread out over the respective cut-hole. The outer size of the respectively applied overlay can be the subject of the separate study. In the above-presented examples, the description of fatigue damage becomes much more complex than in the simple isotropic plate. In such a case, except the typical rupture modes for steel plates, additional failures characteristic for fibre-reinforced composite materials (FRCM) can be observed. These are as follow: interface debonding between the plate and composite, fibre failure, the resin matrix degradation and so on. This opens the possibility for wide studies concerning the number of layers applied, fibres orientation, the types of composite overlay, its size and shape used. In the article, the Authors concentrated on the analysis of the plate with a rectangular hole with rounded corners subjected to unidirectional axial tension and the results of that study are presented in the next paragraph.

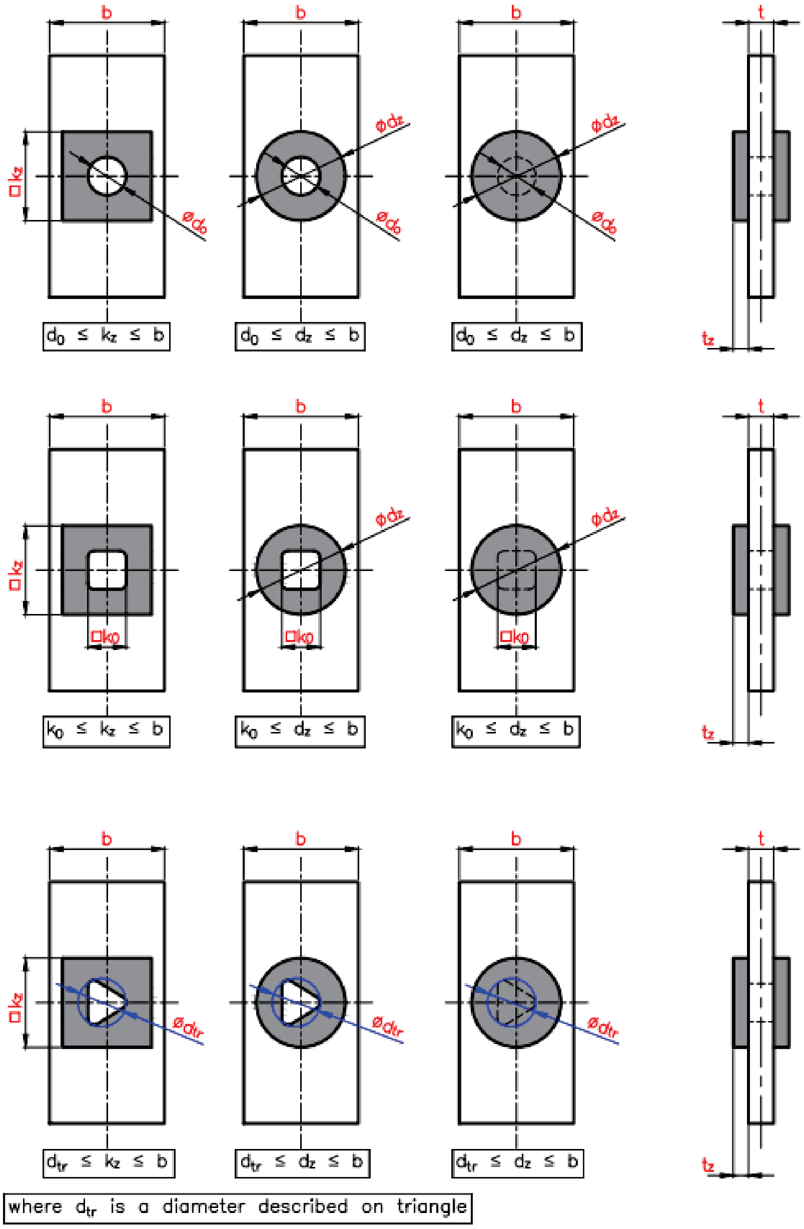

Fig. 3. Various concepts of two-sided composite overlays applied in stress concentration zone

3. Numerical results

The numerical studies were performed for bare steel plate with square opening and for plates with two types of FRCM overlays applied. These were composites with glass fibres: TVR 380 M12/R-glass and composite with carbon fibres: AS4D/9310, which are easily accessible on the market with various angle orientations of fibres in stacking sequence. The basic, analysed model with no overlays is shown in Fig. 4. The bare plate is subjected to unidirectional tension (axial force $Fy = 1000$ N, Fig. 4a) and due to the symmetry, the only ¼ of the structure is modelled for future comparison of the results certain parts around the cut hole were distinguished (zones *A1* and *A2*, Fig 4b). In Figs. 4c, d the distributions of σ_y stress are presented. In Fig. 4 d, the peak of the stress rises to 44.9 MPa on the boundary in the rounded corner area.

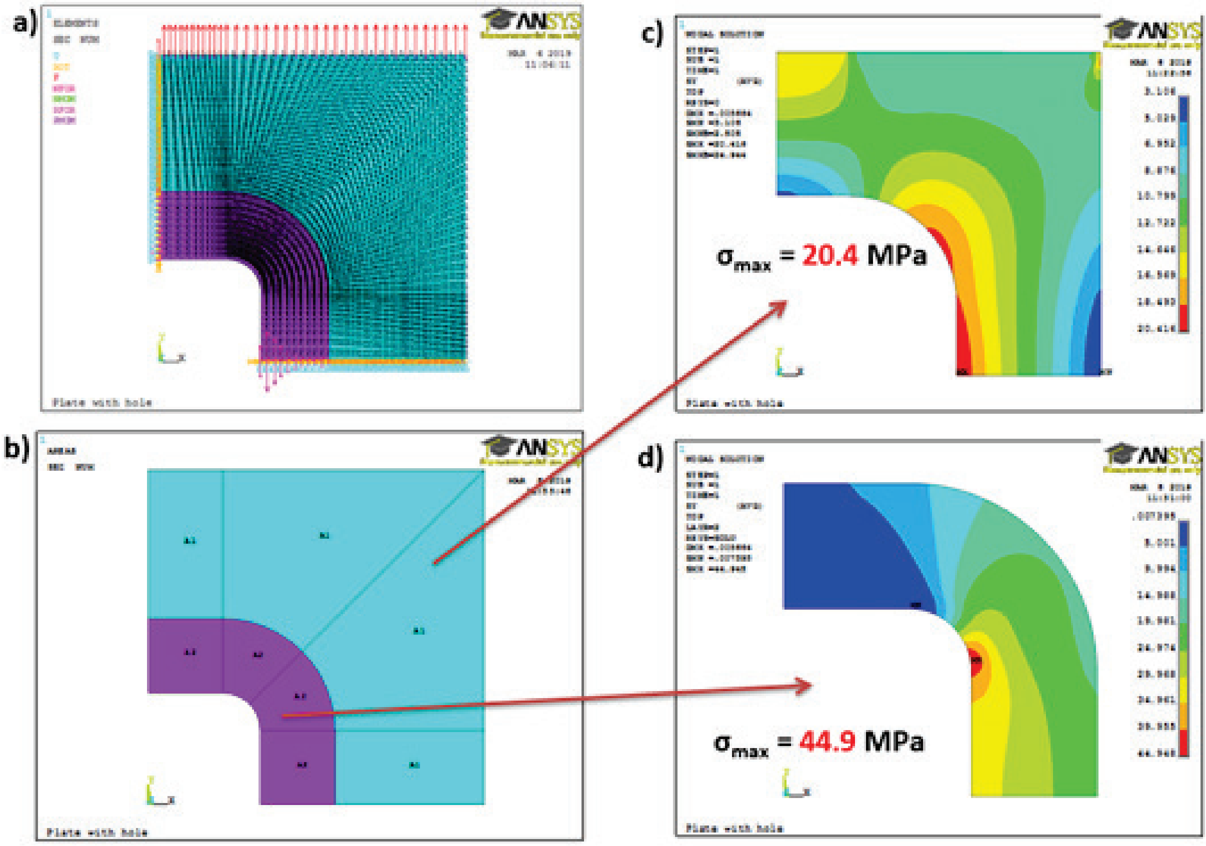

Fig. 4. Model – a) loads, boundary conditions; b) distinguished areas A1 – outer part and A2 – inner part with stress concentration; c) distribution of σ_y stress in outer part of model (A1); d) distribution of σ_y stress in A2 areas

In order to reduce the stress concentration, the composite overlays were introduced in the area *A2* over the top and bottom of the plate. The outer boundary of applied overlays is the homothetic contour of the cut hole, but it can be also of the other form presented in Fig. 3. The idea of the application of the two outer overlays is given in Fig. 5 below. Layer 2 is the steel part, which is still isotropic and exhibits typical mechanical properties, namely Young's modulus $E = 210$ GPa and Poisson's coefficients $v = 0.3$. Layer L1 and L3 – shown in Fig. 5 – are the composite overlays with the material data specified in Tab. 1. The thickness of both applied overlays was the same and equal to 0.5 mm while the basic plate had a thickness of 2 mm.

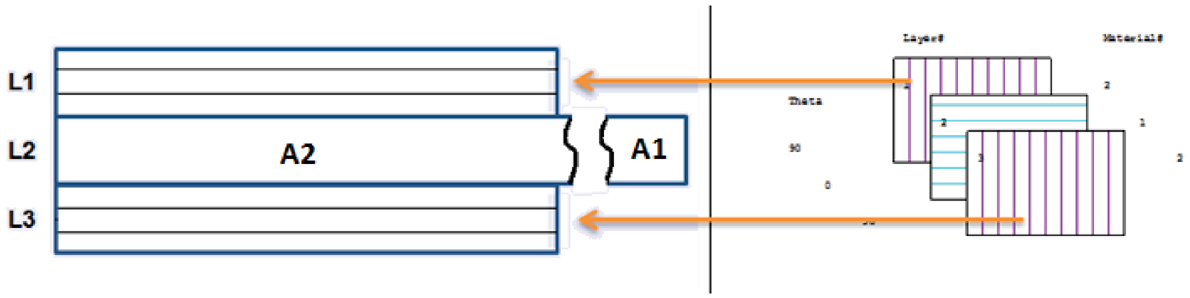

Fig. 5. Stacking sequence of model part with overlays, orientation angle for composite overlays α = 90° with respect to x axis

Tab. 1. Material data for investigated FRCM overlays

Modulus or coefficients	TVR 380 M12/R-glass	AS4D/9310
Young	$E_x = 46.43$ GPa, $E_y = E_z = 14.92$ GPa	$E_x = 133.86$ GPa, $E_y = E_z = 7.71$ GPa
Kirchhoff	$\mathrm{G}_{xy} = \mathrm{G}_{xz} = 5.23$ GPa, $\mathrm{G}_{yz} = 9.15$ GPa	$G_{xy} = G_{xz} = 4.31$ GPa, $G_{yz} = 2.76$ GPa
Poisson	$\nu_{xy} = \nu_{xz} = 0.269$, $\nu_{yz} = 0.3$	$\nu_{xy} = \nu_{xz} = 0.301$, $\nu_{yz} = 0.396$

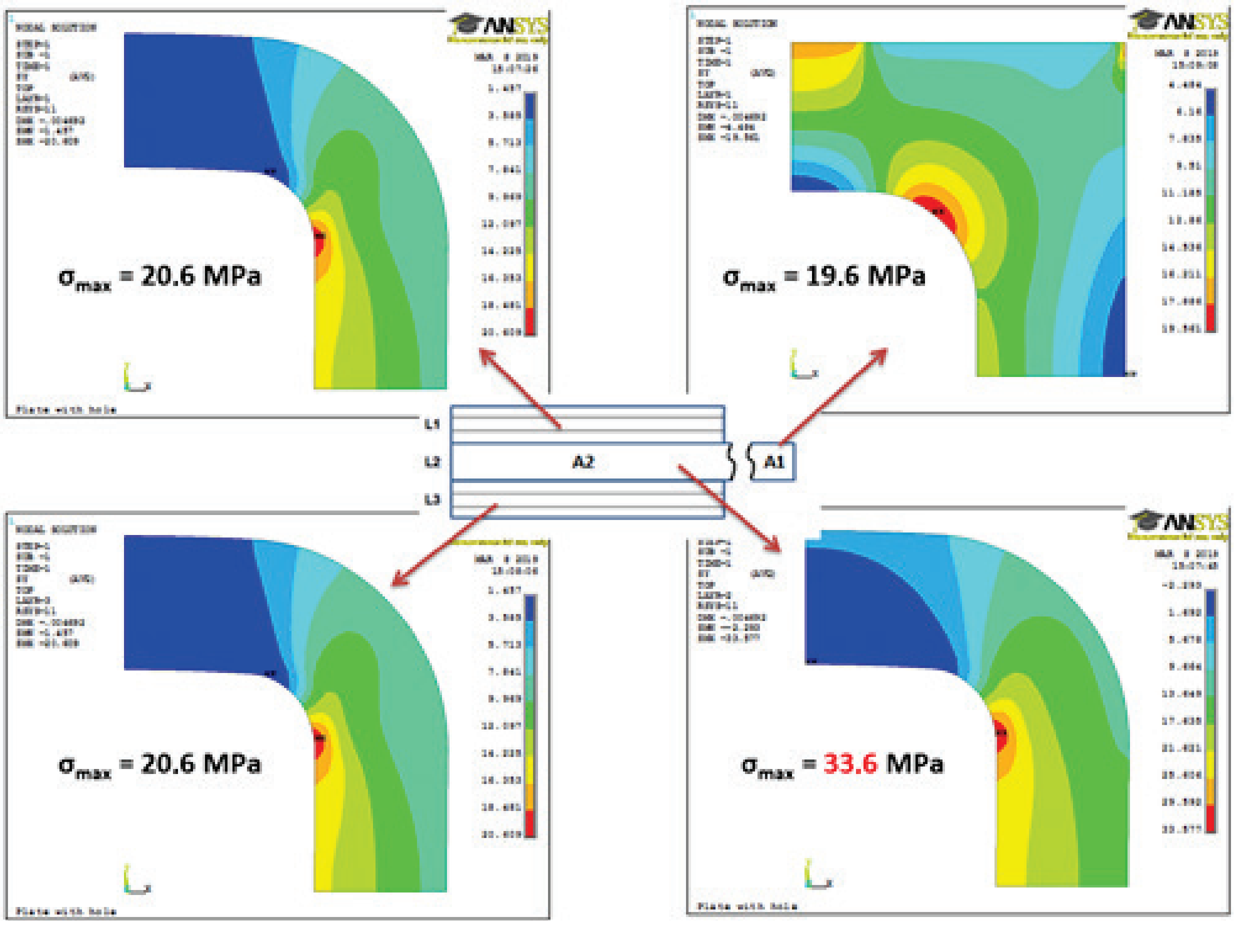

Fig. 6 Distributions of σ_y stress in structure with AS4D/9310 composite overlays

The obtained numerical results registered for both types of applied overlays reduction of the stress, concentration. The application of AS4D/9310 composite appeared to be more effective in the peak stress reduction (about 25% reduction). The slightly smaller reduction – by 11% – was observed in case of use of the glass – fibre reinforced composite.

4. Concluding remarks

The numerical analysis of the stress concentration in a flat plate with a square hole cut was performed for the chosen value of the corner radius. The stress concentration was observed with the $\alpha_k = 2.41$. The bare plate structure was modified with the application of two composite overlays applied on both sides of the bare plate around the hole cut. In such a modified plate the peak stress was reduced by 11% (glass fibre reinforced composite) and 25% (carbon fibre reinforced composite) respectively. The obtained results are very promising so that further investigations will be made. The numerical tests are planned for multilayer overlays with other fibre orientation angles. Another question, which remains to solve, is the outer size and shape of the applied overlays. All these above-given aspects open the optimal choice of the overlays configurations. In the next step, the Authors provide the experimental tests and verification for both types of loads - static and fatigue.

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