THE MAGNETIC FIELD CURVATURE CORRECTION ALGORITHM DEDICATED FOR HELMET MOUNTED CUEING SYSTEMS

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Abstract
The article presents the new built in Poland helmet mounted cueing system NSC-1 Orion with a magnetic method, dedicated to the multi-purpose helicopters W-3PL Gluszec. This system uses the magnetic field generated by the system of three mutually orthogonal electric coils. The principle of operation of this system is described and the method of determining the angular position of the pilot's helmet relative to the helicopter's cabin using the reference magnetic field and the directional cosines matrix are discussed. Electrical flat coils, constructed in the Polish Air Force Institute of Technology (AFIT), generating a magnetic field with curved symmetry axis characteristics, causing errors in determining the angular position of the pilot's helmet are shown. As a way to minimize these errors, an original proprietary algorithm for correcting the negative impact of the magnetic field curvature generated by the on-board system has been presented. Mathematical relations describing the presented correction process as well as selected results of simulation and experimental investigations in the area of inaccuracy of the "before" and "after" systems of applying the developed algorithm were given. Inaccuracies determined from computer simulations of the developed mathematical relations were compared with experimental data from magnetic field measurements using the integrated three-axis sensor ADIS-16405, used in the laboratory of the AFIT Avionics Division for helmet control of the angular position of the moving observation and sighting head and reflector-search light.

Keywords: transport, avionics, helmet-mounted cueing systems, errors of the angular positioning, magnetic field measurement, flat coil profile modelling

1. Introduction

One of the most modern and most important on-board systems of modern multi-purpose military aircrafts and helicopters is the helmet mounted cueing system. The task of such a system is the nether pointing, tracking of the target and the helmet-based display of pilot-navigational, and observation-target parameters [3, 5, 9, 10].

Such a system makes it possible to indicate the target for on-board weapon systems (including mobile fire stations and infrared-directed missile warheads) by means of the pilot's head movement. Particularly in the field of helmet mounted cueing systems, currently being the basic „work tool“ for pilots of military aircraft in obtaining and maintaining so-called advantages in the air using the „first look – first shot“ principle [10, 16].

One of the main methods used in aerial helmet mounted cueing systems is the magnetic (also electromagnetic) method [1, 8, 10, 15]. It involves the generation in the cabin of an aircraft of a reference magnetic field with known characteristics, enabling the determination of the angular position of the pilot's helmet relative to the reference system associated with the cabin [11-13].
An example of an aeronautical aiming system with a magnetic method is the JHMCS system [10] installed on F-16 aircraft, operated, among others in the Armed Forces of the Republic of Poland. It uses an on-board magnetic field transmitter (Fig. 1), built-in in the aircraft cabin, with a characteristic enabling determination of the angular position of the pilot's helmet in the coordinate system associated with the cabin [10, 15, 16].

The JHMCS system uses a magnetic helmet tracking system with active noise reduction. In the available advertising materials, however, there is no information about the algorithm for determining the position of the helmet [10, 15].

The analysis of the available literature shows that one of the main problems related to the combat use of the JHMCS systems is the achievement of insufficient accuracy of precisely determines the pilot's visa line, currently required for new types of on-board combat agents, including rockets aimed at targets indicated by laser [10, 16].
To meet these needs, as part of the implementation of a research project carried out with the funds of the Ministry of Science and Higher Education of the Republic of Poland for R&D (Research and Development), a demonstration of technologies in the NSC-1 helmet mounted cueing system was built at the AFIT (Fig. 2), intended for the multi-purpose helicopters W-3PL Głuszec [15].

This system, as one of the tested methods, uses a magnetic method to determine the momentary angular position of the pilot's helmet relative to the aircraft cabin [15-17]. As a magnetic field generator, a system of three flat coils and a specialized control system for supplying and measuring the magnetic field components necessary for determining the coordinate system associated with the aircraft cabin were used.

The developed algorithm for determining the angular position of the helmet in relation to the aircraft cabin is based on a matrix (using the directional cosines) and magnetic field components measured by a sensor mounted in the overlay mounted on the pilot's helmet.

2. The magnetic system used in the NSC-1 helmet mounted cueing system

The magnetic system applied in the NSC-1, presented in the work in the form of a physical model (Fig. 3), consists of three orthogonally built flat coils, generating an artificial magnetic field on the aircraft.

This system includes a longitudinal coil (denoted as A) with an axis of symmetry consistent with the longitudinal axis of the aircraft, a transverse coil (labelled B) with an axis of symmetry consistent with the transverse axis of the aircraft and a vertical coil (denoted as C) with an axis of symmetry consistent with the axis vertical aircraft. Each of the coils has the same geometric dimensions and contains 26 turns (wound in the form of a spiral with an outer radius of 6 cm).

Under the AFIT laboratory conditions, a DC power supply with a digital regulation and stabilization of the current flowing in the winding of each coil was used to supply the coils. To control the operation of the power supply system and the measuring system with the magnetic sensor ADIS-16405 [2, 15], a specialized KG-1HC computer was used, which is an integral part of the NSC-1 helmet mounted cueing system [15].

Fig. 3. A view of the model of a system of three orthogonal coils intended for the generation of a magnetic field at the point of finding the helmeted measuring system ADIS-16405 [own study]
The measuring cycle consists of four successive stages of feeding individual flat coils (longitudinal, transverse and vertical as described):

- Stage 1: Supplying the longitudinal coil (denoted as coil A) and measuring the field by the dial sensor in the steady state for the given angular position of the pilot’s helmet;
- Step 2: Supplying the transverse coil (denoted as coil B) and measuring the field by the dial sensor in the steady state for the given angular position of the pilot's helmet;
- Stage 3: Supplying of the vertical coil (denoted as coil C) and measurement of the field by the dial sensor in the steady state for the given angular position of the pilot's helmet;
- Stage 4: Measurement of the ambient field (without coil supply) by the helmet detector in steady state for the given angular position of the pilot's helmet.

The magnetic field generated by the flat coil is treated as a reference field, forming a coordinate system associated with the cabin of the aircraft. By taking into account the measurement of the environmental magnetic field (carried out in step 4) three measurement vectors (from stages 1, 2 and 3) are obtained, containing differential signals without interfering with the external magnetic field (natural magnetic field of the Earth and magnetic field generated by on-board electrical circuits) structural ferromagnetic elements of the aircraft, including pilot's seat, engine, girders).

3. Mathematical description of the magnetic field generated in the NSC-1 system

The mathematical description of the individual components of the magnetic field generated in the area of the pilot's helmet should take into account the geometry of the flat coil, the distance of the helmet from the coil and the deviation of the helmet from the calibration point of the helmet relative to the reference system associated with the aircraft cabin [4, 6, 11, 15, 17].

The magnetic field, produced in individual stages of the measurement cycle, contains components oriented in the reference system associated with the cabin of the aircraft, which are described as follows:

\[
[B_{sp}(A)] = [B_{sp}^x(A), B_{sp}^y(A), B_{sp}^z(A)],
\]

(1)

where:

\( B_{sp}(A) \) – the vector of the magnetic field generated by the longitudinal coil (coil A),

\( B_{sp}^x(A), B_{sp}^y(A), B_{sp}^z(A) \) – components of the magnetic field generated by the coil A.

\[
[B_{sp}(B)] = [B_{sp}^x(B), B_{sp}^y(B), B_{sp}^z(B)],
\]

(2)

where:

\( B_{sp}(B) \) – the vector of the magnetic field generated by the transverse coil (coil B),

\( B_{sp}^x(B), B_{sp}^y(B), B_{sp}^z(B) \) – components of the magnetic field generated by the coil B.

\[
[B_{sp}(C)] = [B_{sp}^x(C), B_{sp}^y(C), B_{sp}^z(C)],
\]

(3)

where:

\( B_{sp}(C) \) – the vector of the magnetic field generated by the vertical coil (coil C);

\( B_{sp}^x(C), B_{sp}^y(C), B_{sp}^z(C) \) – components of the magnetic field generated by the coil C.

The components of the generated magnetic field were determined at the measuring point \( P(x_0, y_0, z_0) \), lying on the axis of symmetry of each flat coil and spaced from the coil by the distance \( D \), characterizing the so-called the neutral position of the pilot's helmet in the adjustment process (Fig. 4).

The initial analysis of the magnetic field components generated by individual flat coils (orthogonally built in the aircraft cabin constituting the reference system) can be carried out assuming uniformity of the field with respect to the symmetry axis of the given coil.
For the longitudinal coil (coil A) supplied with current $I(A)$ with a symmetry axis oriented along the longitudinal axis of the aircraft, the magnetic field components are defined as:

$$B^x_{SP}(A, x_0, y_0, z_0) = \mu \cdot I(A) \cdot A_y(A, x_0, y_0, z_0),$$

$$B^y_{SP}(A, x_0, y_0, z_0) = 0,$$

$$B^z_{SP}(A, x_0, y_0, z_0) = 0,$$  \hspace{1cm} (4) \hspace{1cm} (5) \hspace{1cm} (6)

where:

$$A_y(A, x_0, y_0, z_0) = \frac{1}{2} \cdot \frac{R_{max}(A)-6 \text{ cm}}{R_{min}(A)-0 \text{ cm}} \sum \frac{R^2_y(A)}{[(D(A)^2 + R^2_y(A))]^{3/2}}.$$  \hspace{1cm} (7)

For a transverse coil (coil B) supplied with current $I(B)$ with a symmetry axis oriented along the longitudinal axis of the aircraft, the magnetic field components are defined as:

$$B^x_{SP}(B, x_0, y_0, z_0) = 0,$$

$$B^y_{SP}(B, x_0, y_0, z_0) = \mu \cdot I(B) \cdot A_y(B, x_0, y_0, z_0),$$

$$B^z_{SP}(B, x_0, y_0, z_0) = 0,$$  \hspace{1cm} (8) \hspace{1cm} (9) \hspace{1cm} (10)

where:

$$A_y(B, x_0, y_0, z_0) = \frac{1}{2} \cdot \frac{R_{max}(B)-6 \text{ cm}}{R_{min}(B)-0 \text{ cm}} \sum \frac{R^2(B)}{[(D(B)^2 + R^2_y(B))]^{3/2}}.$$  \hspace{1cm} (11)

For a vertical coil (coil C) supplied with current $I(C)$ with a symmetry axis oriented along the longitudinal axis of the aircraft, the magnetic field components are defined as:

$$B^x_{SP}(C, x_0, y_0, z_0) = 0,$$  \hspace{1cm} (12)
where:

$$A_1(C, x_0, y_0, z_0) = \frac{1}{2} \sum_{R_{\min}(C) = 0 \, \text{cm}}^{R_{\max}(C) = 6 \, \text{cm}} \frac{R^2_i(C)}{[(D(C)^2 + R^2_i(C))]^{3/2}}.$$  

In order to determine the inaccuracy in determining the angular position of the pilot's helmet relative to the aircraft cabin, more complex magnetic field models should be adopted, taking into account its curvature resulting from the spatial characteristics of the field generated by the electrical flat coils [4, 6, 11, 14].

4. The algorithm of generating a magnetic field generated in the NSC-1 system

The developed, proprietary algorithm for generating and analysing the magnetic field generated by the flat coils consists in consecutively powering the individual coils within the adopted measurement cycle [1, 15]. In order to compensate for the impact of the Earth's magnetic field and aircraft equipment, an additional field measurement is used in a state in which the magnetic coil is not powered, and for further calculations, differential signals are used (obtained for both powered and non-energized coils). To determine the angular position of the pilot's helmet relative to the aircraft cabin, a matrix account describing relations between the magnetic field components generated by individual flat coils (orthogonal built-in) and magnetic field components measured by the sensor placed on the pilot's helmet can be used (Fig. 5).

Fig. 5. A view of the integrated ADIS-16405 three-axis sensor built into the pilot's helmet during the experimental researches with magnetic field generated by the electrical flat coils [own study]

Between the components of the magnetic field produced by the flat coils (coils A, B and C) and the magnetic field components measured by the sensor placed on the pilot's helmet in a given angular position (represented by elevation angle and azimuth angle) occur:
where:  
\[ B_{PO}(A,B,C) \] – a square matrix composed of magnetic field vector components generated by coils A, B and C;  
\[ M^P_{SP}(A,B,C) \] – a square matrix composed of magnetic field vector components measured by the ADIS-16405 sensor;  
\[ M^M_{SP}(A,B,C) \] – a square matrix defining the angular position of the pilot's helmet relative to the aircraft cabin (directional cosine matrix).

The matrix of the magnetic field measured by the integrated three-axis sensor ADIS-16405 with measuring axes oriented in the helmet's pilot system is determined in the form:

\[
[B_{PO}(A,B,C)] = 
\begin{bmatrix}
B_{PO}^X(A) & B_{PO}^Y(B) & B_{PO}^Z(C) \\
B_{PO}^X(A) & B_{PO}^Y(B) & B_{PO}^Z(C) \\
B_{PO}^X(A) & B_{PO}^Y(B) & B_{PO}^Z(C)
\end{bmatrix}
\]

(18)

Matrix of a magnetic field generated by individual flat coils A, B and C with axes of symmetry oriented in the aircraft cabin system is determined in the form of:

\[
[B_{SP}(A,B,C)] = 
\begin{bmatrix}
B_{SP}^X(A) & B_{SP}^Y(B) & B_{SP}^Z(C) \\
B_{SP}^X(A) & B_{SP}^Y(B) & B_{SP}^Z(C) \\
B_{SP}^X(A) & B_{SP}^Y(B) & B_{SP}^Z(C)
\end{bmatrix}
\]

(19)

The directional cosine matrix defining the angular position of the pilot's helmet relative to the aircraft cabin is defined in the form:

\[
[M^P_{SP}(A,B,C)] = 
\begin{bmatrix}
M_{11}(A,B,C) & M_{12}(A,B,C) & M_{13}(A,B,C) \\
M_{21}(A,B,C) & M_{22}(A,B,C) & M_{23}(A,B,C) \\
M_{31}(A,B,C) & M_{32}(A,B,C) & M_{33}(A,B,C)
\end{bmatrix}
\]

(20)

On the basis of the determined quaternion components, it is possible to determine the angles of the spatial orientation of the pilot's helmet, relative to the aircraft cabin in the form of:

– the angle of the helmet’s elevation relative to the cabin reference system:

\[
\Theta_{SP}^P(A,B,C) = \text{ARCTAN} \left( \frac{-M_{13}(A,B,C)}{\sqrt{1 - [M_{13}(A,B,C)]^2}} \right)
\]

(21)

– the azimuth of the pilot's helmet relative to the cabin reference system:

\[
\Psi_{SP}^P(A,B,C) = 2 \text{ARCTAN} \left( \frac{M_{12}(A,B,C)}{\sqrt{1 - [M_{13}(A,B,C)]^2 + M_{11}(A,B,C)}} \right)
\]

(22)

– the angle of tilting the pilot’s helmet relative to the cabin reference system (not used in work):

\[
\Phi_{SP}^P(A,B,C) = 2 \text{ARCTAN} \left( \frac{M_{23}(A,B,C)}{\sqrt{1 - [M_{13}(A,B,C)]^2 + M_{33}(A,B,C)}} \right)
\]

(23)

The errors of determination of spatial angles of the pilot’s helmet relative to the aircraft cabin for the assumed inaccuracy of the magnetic field measurement by the ADIS-16405 sensor (Fig. 6) can be determined using the above dependences, taking into account sensor errors in equations (4)-(6), (8)-(10), (12)-(14) for the field components generated by the individual flat coils, respectively.
Error values for selected angular positions of the pilot's helmet relative to the aircraft cabin for the assumed inaccuracy of the magnetic field measurement by the ADIS-16405 sensor at the level of ±0.5 [mGs] in the individual measuring axes are shown in Tab. 1 (for elevation channel) and in Tab. 2 (for the azimuth channel).

**Tab. 1. Errors in determining the elevation angle of the helmet's relative to the reference system associated with the cabin resulting from the measuring error of the magnetic field generated by the coils [own study]**

<table>
<thead>
<tr>
<th>ELEVATION</th>
<th>AZIMUTH</th>
<th>-45°</th>
<th>-30°</th>
<th>-15°</th>
<th>0°</th>
<th>+15°</th>
<th>+30°</th>
<th>+45°</th>
</tr>
</thead>
<tbody>
<tr>
<td>+45°</td>
<td>0.068</td>
<td>0.067</td>
<td>0.066</td>
<td>0.065</td>
<td>0.066</td>
<td>0.067</td>
<td>0.068</td>
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<tr>
<td>+30°</td>
<td>0.064</td>
<td>0.063</td>
<td>0.062</td>
<td>0.062</td>
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<td>0.063</td>
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<tr>
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<td>0°</td>
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<td>-15°</td>
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<td>-30°</td>
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<td>-45°</td>
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<td>0.067</td>
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</tbody>
</table>

**Tab. 2. Errors in determining the azimuth angle of the helmet's relative to the reference system associated with the cabin resulting from the measuring error of the magnetic field generated by the coils [own study]**

<table>
<thead>
<tr>
<th>ELEVATION</th>
<th>AZIMUTH</th>
<th>-45°</th>
<th>-30°</th>
<th>-15°</th>
<th>0°</th>
<th>+15°</th>
<th>+30°</th>
<th>+45°</th>
</tr>
</thead>
<tbody>
<tr>
<td>+45°</td>
<td>0.085</td>
<td>0.081</td>
<td>0.078</td>
<td>0.076</td>
<td>0.078</td>
<td>0.081</td>
<td>0.085</td>
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</tr>
<tr>
<td>+30°</td>
<td>0.075</td>
<td>0.073</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
<td>0.073</td>
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<tr>
<td>+15°</td>
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<tr>
<td>-15°</td>
<td>0.067</td>
<td>0.066</td>
<td>0.065</td>
<td>0.064</td>
<td>0.065</td>
<td>0.066</td>
<td>0.067</td>
<td></td>
</tr>
<tr>
<td>-30°</td>
<td>0.075</td>
<td>0.073</td>
<td>0.071</td>
<td>0.071</td>
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<tr>
<td>-45°</td>
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<td>0.081</td>
<td>0.078</td>
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<td>0.078</td>
<td>0.081</td>
<td>0.085</td>
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</tbody>
</table>
The error values obtained do not exceed the level of 0.1° and may be reduced in the future by using a more accurate magnetic field sensor [2, 7, 16, 17].

5. The algorithm for correction of magnetic field curvature generated in the NSC-1 system

One of the main reasons for errors in determining the angular position of the pilot's helmet relative to the aircraft cabin is, next to the measuring errors of the sensor and errors in calculations of the on-board computer, the heterogeneity of the magnetic field produced by the actual coils manifesting as its curvature. The components of the curvature were determined at the measuring point \(P(x_0 + \Delta x, y_0 + \Delta y, z_0 + \Delta z)\), lying outside the axis of symmetry of the coil and away from the so-called position of the pilot's neutral helmet (at \(P(x_0, y_0, z_0)\)) deviations \(\Delta x\), \(\Delta y\) and \(\Delta z\) (Fig. 7).

The original correction algorithm for this curvature is based on the determination of the momentary position of the linear pilot helmet relative to the cabin based on the analysis of signals received from the magnetic field sensors. The generation and measurement of the magnetic field takes place analogically to the above-described case with a homogeneous field, for the pilot's helmet placed in the neutral position. The measuring cycle consists of four successive stages of feeding individual coils (A, B and C). In order to reduce interference (originating from the Earth's magnetic field and the magnetic field generated by the aircraft and its equipment), the magnetic field of the environment is measured with the power coil not being supplied [1, 16, 17].

For the longitudinal coil (coil A) supplied with current \(I(A)\) with a symmetry axis oriented along the longitudinal axis of the aircraft, the magnetic field components are defined as:

\[
B_{SP}^X(A, \Delta x, \Delta y, \Delta z) = B_{SP}^X(A, x_0, y_0, z_0) + \mu \cdot I(A) \cdot \left[ 2A_1 (A, x_0, y_0, z_0) \cdot \Delta x \right] + \\
+ \mu \cdot I(A) \cdot \left[ A_2 (A, x_0, y_0, z_0) \cdot \left( \Delta x^2 - \frac{1}{2} \Delta y^2 - \frac{1}{2} \Delta z^2 \right) \right] + \\
+ \mu \cdot I(A) \cdot \left[ \frac{1}{6} A_4 (A, x_0, y_0, z_0) \cdot \left( 4\Delta x^3 - 6\Delta y^2 \Delta x^2 - 6\Delta z^2 \Delta x^2 \right) \right],
\]

(24)
\[ B_{S0}^I(A, \Delta x, \Delta y, \Delta z) = B_{S0}^I(A, x_0, y_0, z_0) + \mu \cdot I(A) \cdot \left[ A_1(A, x_0, y_0, z_0) \cdot \Delta y \right] + \mu \cdot I(A) \cdot \left[ A_2(A, x_0, y_0, z_0) \cdot \Delta x \cdot \Delta y \right] + \mu \cdot I(A) \cdot \left[ \frac{1}{4} A_4(A, x_0, y_0, z_0) \cdot (\Delta y^2 + \Delta z^2 - 4 \Delta x^2) \cdot \Delta y \right], \] (25)

\[ B_{S0}^Z(A, \Delta x, \Delta y, \Delta z) = B_{S0}^Z(A, x_0, y_0, z_0) + \mu \cdot I(A) \cdot \left[ A_1(A, x_0, y_0, z_0) \cdot \Delta z \right] + \mu \cdot I(A) \cdot \left[ A_2(A, x_0, y_0, z_0) \cdot \Delta x \cdot \Delta z \right] + \mu \cdot I(A) \cdot \left[ \frac{1}{4} A_4(A, x_0, y_0, z_0) \cdot (\Delta y^2 + \Delta z^2 - 4 \Delta x^2) \cdot \Delta z \right], \] (26)

where:

\[ A_1(A, x_0, y_0, z_0) = \frac{3}{4} \sum_{R(A)=0 \text{cm}}^{R(A)=6 \text{cm}} \frac{D(A) \cdot R_z^2(A)}{[(D(A)^2 + R_z^2(A))]^{5/2}}, \] (27)

\[ A_2(A, x_0, y_0, z_0) = \frac{3}{4} \sum_{R(A)=0 \text{cm}}^{R(A)=6 \text{cm}} \frac{R_y^2(A) \cdot [4 \cdot D^2(A) - 3 \cdot R_z^2(A)]}{[(D(A)^2 + R_z^2(A))]^{7/2}}, \] (28)

\[ A_4(A, x_0, y_0, z_0) = \frac{15}{8} \sum_{R(A)=0 \text{cm}}^{R(A)=6 \text{cm}} \frac{R_y^2(A) \cdot [4 \cdot D^2(A) - 3 \cdot R_z^2(A)]}{[(D(A)^2 + R_z^2(A))]^{9/2}}. \] (29)

For a transverse coil (coil B) supplied with current \( I(B) \) with a symmetry axis oriented along the transverse axis of the aircraft, the magnetic field components are defined as:

\[ B_{S0}^X(B, \Delta x, \Delta y, \Delta z) = B_{S0}^X(B, x_0, y_0, z_0) + \mu \cdot I(B) \cdot \left[ A_1(B, x_0, y_0, z_0) \cdot \Delta x \right] + \mu \cdot I(B) \cdot \left[ A_2(B, x_0, y_0, z_0) \cdot \Delta y \right] + \mu \cdot I(B) \cdot \left[ \frac{1}{4} A_4(B, x_0, y_0, z_0) \cdot (\Delta x^2 + \Delta z^2 - 4 \Delta y^2) \cdot \Delta x \right], \] (30)

\[ B_{S0}^Y(B, \Delta x, \Delta y, \Delta z) = B_{S0}^Y(B, x_0, y_0, z_0) + \mu \cdot I(B) \cdot \left[ 2A_1(B, x_0, y_0, z_0) \cdot \Delta y \right] + \mu \cdot I(B) \cdot \left[ A_2(B, x_0, y_0, z_0) \cdot (\Delta y^2 - \frac{1}{2} \Delta x^2 - \frac{1}{2} \Delta z^2) \right] + \mu \cdot I(B) \cdot \left[ \frac{1}{6} A_4(B, x_0, y_0, z_0) \cdot (4 \Delta y^3 - 6 \Delta x^2 \Delta y^2 - 6 \Delta z^2 \Delta y^2) \right], \] (31)

\[ B_{S0}^Z(B, \Delta x, \Delta y, \Delta z) = B_{S0}^Z(B, x_0, y_0, z_0) + \mu \cdot I(B) \cdot \left[ A_1(B, x_0, y_0, z_0) \cdot \Delta z \right] + \mu \cdot I(B) \cdot \left[ A_2(B, x_0, y_0, z_0) \cdot \Delta y \cdot \Delta z \right] + \mu \cdot I(B) \cdot \left[ \frac{1}{4} A_4(B, x_0, y_0, z_0) \cdot (\Delta x^2 + \Delta z^2 - 4 \Delta y^2) \cdot \Delta z \right], \] (32)

where:

\[ A_1(B, x_0, y_0, z_0) = \frac{3}{4} \sum_{R \text{max}(B)=0 \text{cm}}^{R \text{max}(B)=6 \text{cm}} \frac{D(B) \cdot R_z^2(B)}{[(D(B)^2 + R_z^2(B))]^{5/2}}, \] (33)

\[ A_2(B, x_0, y_0, z_0) = \frac{3}{4} \sum_{R \text{min}(B)=0 \text{cm}}^{R \text{max}(B)=6 \text{cm}} \frac{R_y^2(B) \cdot [4 \cdot D^2(B) - R_z^2(B)]}{[(D(B)^2 + R_z^2(B))]^{7/2}}, \] (34)

24
For a vertical coil (coil C) supplied with current $I(C)$ with a symmetry axis oriented along the vertical axis of the aircraft, the magnetic field components are defined as:

$$B_{sp}^x(C,\Delta x,\Delta y,\Delta z) = B_{sp}^x(C, x_0, y_0, z_0) + \mu \cdot I(C) \cdot [A_1(C, x_0, y_0, z_0) \cdot \Delta x] +$$

$$+ \mu \cdot I(C) \cdot \left[ \frac{1}{4} A_4(C, x_0, y_0, z_0) \cdot (\Delta x^2 + \Delta y^2 - 4\Delta z^2) \cdot \Delta x \right],$$  \hspace{1cm} (36)

$$B_{sp}^y(C,\Delta x,\Delta y,\Delta z) = B_{sp}^y(C, x_0, y_0, z_0) + \mu \cdot I(C) \cdot [A_1(C, x_0, y_0, z_0) \cdot \Delta y] +$$

$$+ \mu \cdot I(C) \cdot \left[ \frac{1}{4} A_4(C, x_0, y_0, z_0) \cdot (\Delta x^2 + \Delta y^2 - 4\Delta z^2) \cdot \Delta y \right],$$  \hspace{1cm} (37)

$$B_{sp}^z(C,\Delta x,\Delta y,\Delta z) = B_{sp}^z(C, x_0, y_0, z_0) + \mu \cdot I(C) \cdot [2A_1(C, x_0, y_0, z_0) \cdot \Delta z] +$$

$$+ \mu \cdot I(C) \cdot \left[ \frac{1}{6} A_4(C, x_0, y_0, z_0) \cdot (4\Delta x^3 - 6\Delta x^2\Delta z^2 - 6\Delta y^2\Delta z^2) \right],$$  \hspace{1cm} (38)

where:

$$A_1(C, x_0, y_0, z_0) = \frac{3}{4} \cdot \sum_{R_{\max(C)}=0 \text{cm}}^{R_{\max(C)}=6 \text{cm}} \frac{D(C) \cdot R^2(C)}{[(D(C)^2 + R^2(C))]^{3/2}},$$  \hspace{1cm} (39)

$$A_2(C, x_0, y_0, z_0) = \frac{3}{4} \cdot \sum_{R_{\min(C)}=0 \text{cm}}^{R_{\max(C)}=6 \text{cm}} \frac{R^2(C) \cdot [4 \cdot D^2(C) - R^2(C)]}{[(D(C)^2 + R^2(C))]^{1/2}},$$  \hspace{1cm} (40)

$$A_4(C, x_0, y_0, z_0) = \frac{15}{8} \cdot \sum_{R_{\min(C)}=0 \text{cm}}^{R_{\max(C)}=6 \text{cm}} \frac{R^2(C) \cdot [4 \cdot D^2(C) - 3 \cdot R^2(C)]}{[(D(C)^2 + R^2(C))]^{3/2}}.$$  \hspace{1cm} (41)

Knowledge of mathematical relations describing the magnetic field components generated by individual flat coils at a given measuring point $P'(x_0+\Delta x,y_0+\Delta y,z_0+\Delta z)$, away from the neutral position $P(x_0,y_0,z_0)$ allows to determine the angles of curvature magnetic field and is used in the developed algorithm for error correction of the position of the pilot's helmet.

Error values before making corrections for selected angular positions of the pilot's helmet relative to the cabin and the deviations accepted: $\Delta x = 1 \text{ cm}$, $\Delta y = 1 \text{ cm}$, $\Delta z = 1 \text{ cm}$, are shown in Tab. 3 (for elevation channel) and in Tab. 4 (for azimuth channel).

The errors obtained in determining the angular position of the pilot's helmet reach maximum values of $6^\circ$ and considerably exceed the acceptable level adopted for the NSC-1 Orion helmet mounted cueing system [15-17].

In order to reduce these errors, a correction method was developed using the same measurement data, without the additional method of determining the position of the linear pilot helmet relative to the aircraft cabin (Fig. 8). The author's original correction algorithm is based on the determination of deviations $\Delta x$, $\Delta y$ and $\Delta z$ with respect to the position of the pilot's neutral helmet using the dependencies defining the module (the so-called norm) of the magnetic field vector generated by the individual coils A, B and C.
Tab. 3. Errors in determining the elevation angle of the helmet’s relative to the reference system associated with the cabin resulting from the error of the curvature of the magnetic field generated by the coils [own study]

<table>
<thead>
<tr>
<th>ELEVATION</th>
<th>-45°</th>
<th>-30°</th>
<th>-15°</th>
<th>0°</th>
<th>+15°</th>
<th>+30°</th>
<th>+45°</th>
</tr>
</thead>
<tbody>
<tr>
<td>+45°</td>
<td>5.235</td>
<td>5.023</td>
<td>4.825</td>
<td>4.735</td>
<td>4.825</td>
<td>5.023</td>
<td>5.235</td>
</tr>
<tr>
<td>0°</td>
<td>3.275</td>
<td>3.185</td>
<td>3.125</td>
<td>3.075</td>
<td>3.125</td>
<td>3.185</td>
<td>3.275</td>
</tr>
<tr>
<td>-45°</td>
<td>5.235</td>
<td>5.023</td>
<td>4.825</td>
<td>4.735</td>
<td>4.825</td>
<td>5.023</td>
<td>5.235</td>
</tr>
</tbody>
</table>

Tab. 4. Errors in determining the azimuth angle of the helmet’s relative to the reference system associated with the cabin resulting from the error of the curvature of the magnetic field generated by the coils [own study]

<table>
<thead>
<tr>
<th>ELEVATION</th>
<th>-45°</th>
<th>-30°</th>
<th>-15°</th>
<th>0°</th>
<th>+15°</th>
<th>+30°</th>
<th>+45°</th>
</tr>
</thead>
<tbody>
<tr>
<td>+45°</td>
<td>6.306</td>
<td>6.006</td>
<td>5.830</td>
<td>5.706</td>
<td>5.830</td>
<td>6.006</td>
<td>6.306</td>
</tr>
<tr>
<td>0°</td>
<td>3.745</td>
<td>3.414</td>
<td>3.228</td>
<td>3.145</td>
<td>3.228</td>
<td>3.414</td>
<td>3.745</td>
</tr>
</tbody>
</table>

Fig. 8. A view of the pilot’s helmet with integrated ADIS-16405 three-axis sensor during the experimental researches with magnetic field system in the W-3PL helicopter cabin [own study]

For a longitudinal coil (coil A) with an axis of symmetry oriented along the longitudinal axis of the aircraft, the vector module of the generated magnetic field is described in the form:

$$
\left[ (B_{sp}(A, \Delta x, \Delta y, \Delta z)) \right] = \sqrt{\left( B_{sp}^x(A, \Delta x, \Delta y, \Delta z) \right)^2 + \left( B_{sp}^y(A, \Delta x, \Delta y, \Delta z) \right)^2 + \left( B_{sp}^z(A, \Delta x, \Delta y, \Delta z) \right)^2}.
$$

(42)
which is at the same time equal to the magnetic field vector module measured by the sensor at the set linear position of the pilot's helmet relative to the aircraft cabin:

\[
\|B_{Pr0}(A,\Delta x,\Delta y,\Delta z)\| = \sqrt{(B_{Pr0}^x(A,\Delta x,\Delta y,\Delta z))^2 + (B_{Pr0}^y(A,\Delta x,\Delta y,\Delta z))^2 + (B_{Pr0}^z(A,\Delta x,\Delta y,\Delta z))^2}.
\] (43)

For a transverse coil (coil B) with an axis of symmetry oriented along the transverse axis of the aircraft, the vector module of the generated magnetic field is described in the form:

\[
\|B_{Pr0}(B,\Delta x,\Delta y,\Delta z)\| = \sqrt{(B_{Pr0}^x(B,\Delta x,\Delta y,\Delta z))^2 + (B_{Pr0}^y(B,\Delta x,\Delta y,\Delta z))^2 + (B_{Pr0}^z(B,\Delta x,\Delta y,\Delta z))^2},
\] (44)

which is at the same time equal to the magnetic field vector module measured by the sensor at the set linear position of the pilot's helmet relative to the aircraft cabin:

\[
\|B_{Pr0}(B,\Delta x,\Delta y,\Delta z)\| = \sqrt{(B_{Pr0}^x(B,\Delta x,\Delta y,\Delta z))^2 + (B_{Pr0}^y(B,\Delta x,\Delta y,\Delta z))^2 + (B_{Pr0}^z(B,\Delta x,\Delta y,\Delta z))^2}.
\] (45)

For a vertical coil (coil C) with an axis of symmetry oriented along the vertical axis of the aircraft, the vector module of the generated magnetic field is described in the form:

\[
\|B_{Pr0}(C,\Delta x,\Delta y,\Delta z)\| = \sqrt{(B_{Pr0}^x(C,\Delta x,\Delta y,\Delta z))^2 + (B_{Pr0}^y(C,\Delta x,\Delta y,\Delta z))^2 + (B_{Pr0}^z(C,\Delta x,\Delta y,\Delta z))^2},
\] (46)

which is at the same time equal to the magnetic field vector module measured by the sensor at the set linear position of the pilot's helmet relative to the aircraft cabin:

\[
\|B_{Pr0}(C,\Delta x,\Delta y,\Delta z)\| = \sqrt{(B_{Pr0}^x(C,\Delta x,\Delta y,\Delta z))^2 + (B_{Pr0}^y(C,\Delta x,\Delta y,\Delta z))^2 + (B_{Pr0}^z(C,\Delta x,\Delta y,\Delta z))^2}.
\] (47)

The above relationships allow to determine approximate values of deviations \(\Delta x, \Delta y\) and \(\Delta z\) (marked as \(\Delta x^*, \Delta y^*, \Delta z^*\)) relative to the position of the pilot's neutral helmet by solving the system of equations (42)-(47), after substituting variables from dependencies (24)-(26), (30)-(32) and (36)-(38).

For example, taking into account only the linear parts from the development of individual magnetic field components into the Taylor series, deviations \(\Delta x^*, \Delta y^*, \Delta z^*\) characterize the linear position of the pilot's helmet along the longitudinal, transverse and vertical axis of the aircraft in the curved magnetic field generated by individual coils A, B and C can be determined using the following relations:

\[
\Delta x^* = -\frac{\sqrt{[B_{Pr0}^x(A,\Delta x,\Delta y,\Delta z)]^2 - [A_1(A,x_0,y_0,z_0)]^2}}{4 \cdot A_1(A,x_0,y_0,z_0) \cdot A_1(A,x_0,y_0,z_0)},
\] (48)

\[
\Delta y^* = -\frac{\sqrt{[B_{Pr0}^y(B,\Delta x,\Delta y,\Delta z)]^2 - [A_1(B,x_0,y_0,z_0)]^2}}{4 \cdot A_1(B,x_0,y_0,z_0) \cdot A_1(B,x_0,y_0,z_0)},
\] (49)

\[
\Delta z^* = -\frac{\sqrt{[B_{Pr0}^z(C,\Delta x,\Delta y,\Delta z)]^2 - [A_1(C,x_0,y_0,z_0)]^2}}{4 \cdot A_1(C,x_0,y_0,z_0) \cdot A_1(C,x_0,y_0,z_0)}.
\] (50)

The determined deviation values \(\Delta x^*, \Delta y^*, \Delta z^*\) make it possible to determine the angles of curvature of the magnetic field at a given measurement point, in which there is a magnetic field sensor installed on the pilot's helmet, and thus apply corrections to the determined elevation angles and the azimuth of the pilot's helmet relative to the cabin reference system.

Knowledge of the angles of the curvature of the magnetic field is used in the developed algorithm for correcting errors in determining the angular position of the pilot's helmet.

The values of these errors after the correction for selected angular positions of the pilot's helmet relative to the cabin and the deviations accepted: \(\Delta x = 1\ \text{cm}, \Delta y = 1\ \text{cm}, \Delta z = 1\ \text{cm}\), are shown in Tab. 5 (for the elevation channel) and in Tab. 6 (for the azimuth channel).
The errors obtained in determining the angular position of the pilot's helmet reach the maximum values of 0.8° and do not exceed the permissible level adopted for the NSC-1 Orion cueing system [15-17]. The values of these errors can be reduced by using more developed models of the generated magnetic field (including further nonlinear components from the development into the Taylor series).

6. Summary

The guiding principle of the presented material is to discuss the latest technologies used in onboard equipment of modern military aircraft and helicopters. Particularly in the field of helmet mounted cueing systems, currently being the basic „work tool“ for pilots of military aircraft in obtaining and maintaining so-called advantages in the air using the „first look – first shot“ principle.

The scientific aspect of the presented subject is the presentation of the method of determining the angular position of the pilot's helmet in the NSC-1 Orion cueing system, developed in the AFIT for the multi-purpose helicopter W-3PL Głuszec. The simulation and verification tests carried out are aimed at determining the inaccuracy of determining the angular position of the pilot's helmet relative to the reference system associated with the aircraft cabin, using a magnetic method (with a magnetic system containing flat coils generating a magnetic field with curved characteristics from the symmetry axis).

One of the main substantive values in the presented material is the developed original algorithm for curving the curvature of the generated magnetic field, which consists in determining the instantaneous position of the linear helmet of the pilot relative to the cabin based on the analysis of signals received from magnetic field sensors and does not require additional measurement methods, e.g. optical or mechanical.

Tab. 5. Errors in determining the elevation angle of the helmet's relative to the reference system associated with the cabin resulting after correction of the curvature of the magnetic field generated by the coils [own study]

<table>
<thead>
<tr>
<th>ELEVATION</th>
<th>AZIMUTH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-45°</td>
</tr>
<tr>
<td>+45°</td>
<td>0.804</td>
</tr>
<tr>
<td>+30°</td>
<td>0.721</td>
</tr>
<tr>
<td>+15°</td>
<td>0.664</td>
</tr>
<tr>
<td>0°</td>
<td>0.636</td>
</tr>
<tr>
<td>-15°</td>
<td>0.664</td>
</tr>
<tr>
<td>-30°</td>
<td>0.721</td>
</tr>
<tr>
<td>-45°</td>
<td>0.804</td>
</tr>
</tbody>
</table>

Tab. 6. Errors in determining the azimuth angle of the helmet's relative to the reference system associated with the cabin resulting after correction of the curvature of the magnetic field generated by the coils [own study]

<table>
<thead>
<tr>
<th>ELEVATION</th>
<th>AZIMUTH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-45°</td>
</tr>
<tr>
<td>+45°</td>
<td>0.834</td>
</tr>
<tr>
<td>+30°</td>
<td>0.764</td>
</tr>
<tr>
<td>+15°</td>
<td>0.722</td>
</tr>
<tr>
<td>0°</td>
<td>0.688</td>
</tr>
<tr>
<td>-15°</td>
<td>0.722</td>
</tr>
<tr>
<td>-30°</td>
<td>0.764</td>
</tr>
<tr>
<td>-45°</td>
<td>0.834</td>
</tr>
</tbody>
</table>
The performed estimations of errors in the area of inaccuracy in determining the angular position of the pilot's helmet using the magnetic method have shown that for a system of three flat coils (orthogonal built-in) it is possible to correct errors by determining the linear deviations of the helmet from the neutral position (determined during adjustment of the helmet system). The advantage of the developed correction algorithm is the use of the same measurement data that is used to determine the angular position of the pilot helmet for the curved field. It was shown that the components of the magnetic field generated by the individual flat coils allow to determine the linear deviation of the helmet from the neutral position, and thus to determine the curvature of the magnetic field and make its correction based on the magnetic field models developed in the AFIT.

The developed computational algorithms, as implemented software applications, were used in the laboratory of the AFIT Avionics Division for helmet control of the angular position of the moving observation and sighting head and reflector-search light. It is anticipated that the positive results of these works will be applied on board the W-3PL helicopter Gluszec with the NSC-1 Orion helmet sighting system, among others for the implementation of SAR/CSAR search and rescue missions.

References


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