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THE ERRORS OF THE NUMERICAL CALCULATIONS OF DYNAMIC CHARACTERISTICS OF CANTILEVER BEAM MOUNTED ON THE PLATE

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Abstract

The safety, comfort of the crews, stability, economics of the equipment when ship operating is the leading requirement in the field of designing and manufacturing marine structure and machinery. As a result, all parts of the ships must be tested and inspected to meet the basic safety requirements of the shipping association. The design, manufacture, testing in the maritime field in general and shipbuilding sector in particular are expensive, time consuming: such as aerodynamic experiments of the engine, collision test, ship manoeuvring, vibration test and balance of deck beams, hull beams, hatch covers, shafts ...thus experimental works are sometimes impossible. Along with the development of computer science, many numerical models and software programs have been developed to solve these difficult problems. There are many numerical modelling methods, starting with the finite difference method, the boundary element method, the finite element method, the no mesh method, the weight residue or the energy method. The Work will be limited to the analysis of the most popular numerical modelling method - finite element method using Patran and Nastran software. In the first step of our research, T-beam was analysed as a part of ship hull structure (thin-walled structure). The article goes into the analysis of the accuracy of selected numerical models for the natural vibration frequency of the T-beams mounted on the plate. After modelling, calculating the natural frequency of the T-beam using the Patran - Nastran software, the results were compared with the theoretical values. From that, we evaluate the dispersion and error of different numerical models and select the optimal numerical model. Optimal model will be used for modelling full ship hull with superstructure.

Keywords*: T-Beam, vibration, ship structure, modal analysis, FEM errors*

1. Introduction

Vibration has always been a very important subject for naval architects and structural engineers as its presence can seriously affect the comfort of passengers on ships and the integrity of structures like ships, bridges, offshore structures, airplanes, cars etc. It is well known that structures can resonate, that is, relatively small forces can result in significant deformation, and possibly, damage can be induced in the structure. “Ship hull vibration is an old but a new problem” is often said by many naval architects and marine engineers. That is because the vibration of the hull structures caused serious problems in old times and that it still now brings new kinds of problems.

Determining the natural vibration frequencies are the first and most important problem of dynamic analysis. Determining the natural vibration frequency of the devices will help marine engineers solve the vibration resonance problem of equipments. This helps to increase the life span, reliability of the equipment, the safety, and comfort of the crew as well as passengers on board. In engineering field, vibration behaviour of an element plays a key role without which it is incomplete. Resonance is a key aspect in dynamic analysis, which is the frequency of any system matches with the natural frequency of the system, which may lead to catastrophes or system failure. Modal analysis has become a major alternative to provide a helpful contribution in understanding control of many vibration phenomena, which encountered in practice.

Ship hull and superstructure is typical thin-walled structure: plates with stiffeners – beams. Beams are basic structural members of a ship structure and their vibration analysis is thus very important. Vibration of deck beams of a ship takes place when the forcing disturbances come from the shafting or propellers [11]. Okumoto et al. [6] mentioned that the unbalanced forces of engines are so large that they can produce hull girder vibration. They also showed that the deck of a pure car carrier and a hatch cover of a bulk carrier could be effectively modelled as a beam to calculate their natural frequencies as these members can be subjected to vibrations.

Many researchers have worked on transverse vibration of beams and as this is a subject of practical engineering interest, has been the objective of many recent theoretical investigations. For instance, the bending linear vibration of an elastically restrained beam carrying concentrated masses located within the beam span was analysed by Hamdan and Jubran [1] and in the analysis, the base beam equation of motion is solved to obtain mode shape functions, which satisfy all the geometric and natural boundary conditions at the beam-ends. These functions are used in conjunction with Galerkin's method to obtain the free and the forced response. Rossit and Laura [8] presented the exact solution of free vibrations of a cantilever beam with a spring-mass system attached to the free end using the Bernoulli-Euler theory of beam vibrations. Natural frequencies are obtained for a wide range of the intervening physical parameters. The problem is of interest in naval and ocean engineering systems since in order to avoid dangerous resonance conditions the designer must be able to predict natural frequencies of the overall mechanical system: structure–motor and its elastic mounting. Many other researchers have made important contributions in the field of dynamic analysis of beams. They are Lau [3], Schafer and Holzach [10], Kojima et al [2], Liu and Huang [4], Nagaya and Ishikawa [5] etc. The target of these studies is to understand the behaviour of beams so that proper remedial measure can be taken to control the vibration that may result.

2. Analytical and FEM models of the beam

In the present case, we consider a T-beam is 1308 mm in length, of which 308 mm is mounted on the plate. The material, properties of the T-beam and the plate are presented in the Tab. 1. The analysed beam is presented in Fig. 1.

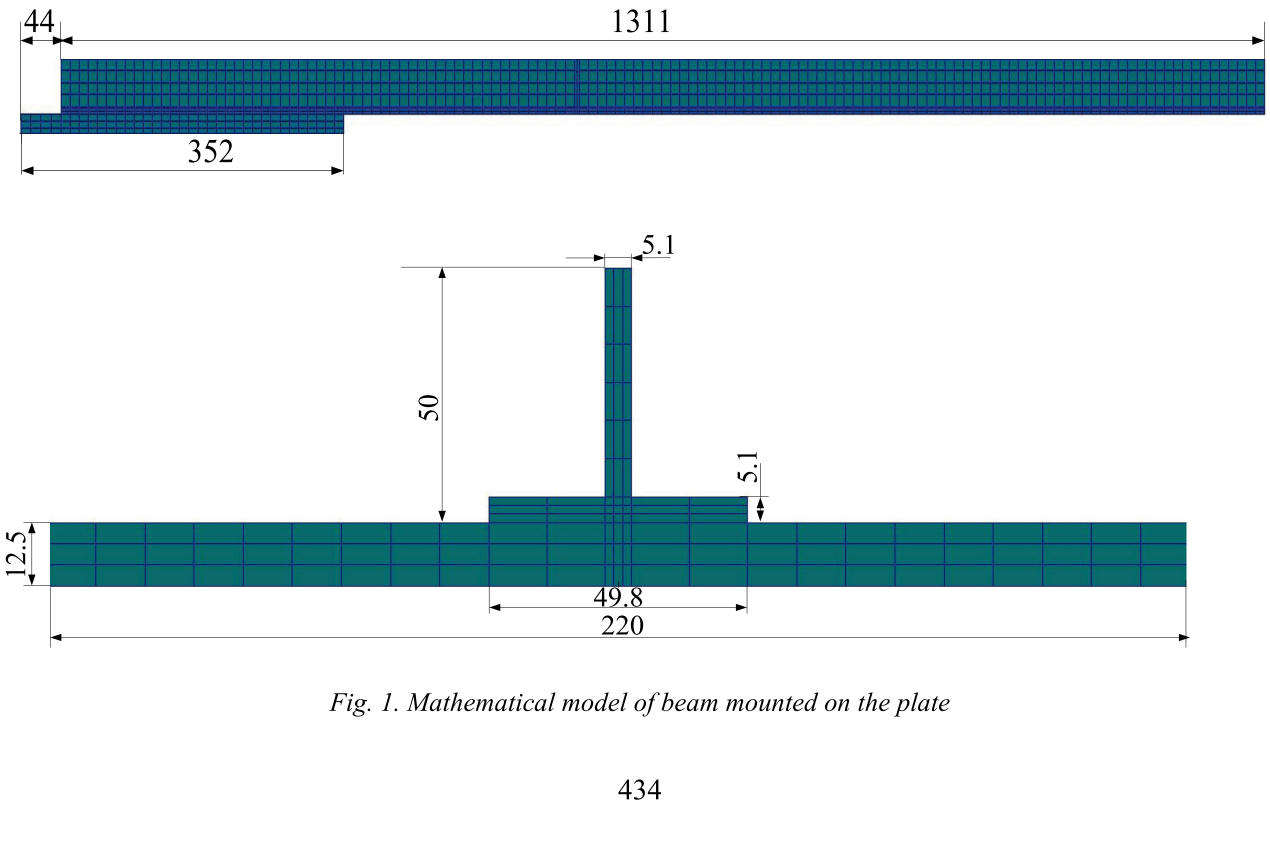


Fig. 1. Mathematical model of beam mounted on the plate

Tab. 1. Material properties of aluminum model used in the calculations

Dimension/Properties	T-beams made of aluminum	Plate made of aluminum
Length [mm]	1308	352
Width [mm]	49.8	220
Thickness [mm]	5.1	12.5
Height [mm]	50	-
Young's Modulus E [GPa]	70	70
Density of the material ρ [kg/m^3]	2800	2800
Poisson's ratio ν	0.33	0.33

First of all, we should calculate the natural vibration frequencies of the T-beam using the theory. Let us consider a T-beam mounted on the plate as shown in Fig. 1. According to documents of Rao [7], the natural frequency of transverse vibration and mode shapes are as follows:

– natural frequency $\omega_i = \dfrac{\lambda_i^2}{L^2}\left(\dfrac{EI}{\rho A}\right)^{1/2}$ in [rad/s] (i = 1, 2, 3, 4 and 5) and

– natural frequency $f_i = \dfrac{\lambda_i^2}{2\pi L^2}\left(\dfrac{EI}{\rho A}\right)^{1/2}$ in [Hz] (i = 1, 2, 3, 4 and 5), (1)

where $A = 0.00048297$ m^2 be the cross sectional area, $E = 7\cdot10^{10}$ Pa be the modulus of elasticity, $\rho = 2800$ kg/m^3 be the density, $L = 1$ m (free part of the T-beam from the fixed position to the free end) be the length of the beam, I be the moment of inertia, $\lambda_1 = 4.73004074$, $\lambda_2 = 7.85320462$, $\lambda_3 = 10.99560790$, $\lambda_4 = 14.13716550$ and $\lambda_5 = 17.27875970$.

Mode shapes are given by:

$$\cosh\frac{\lambda_i x}{L} - \cos\frac{\lambda_i x}{L} - \beta_i\left(\sinh\frac{\lambda_i x}{L} - \sin\frac{\lambda_i x}{L}\right), \tag{2}$$

where $\beta_i = \dfrac{\cosh\lambda_i - \cos\lambda_i}{\sinh\lambda_i - \sin\lambda_i}$, i = 1, 2, 3, 4 and 5.

One may find the detailed derivation of the above expressions in Rao [7].

Calculate the moment of inertia of the T-beam cross section I: according to the document by G. H. Ryder [9], the moment of inertia I in x-axis is calculated and presented in Fig. 2, where:

C_1 – centroid of rectangle number 1,
C_2 – centroid of rectangle number 2,
C – centroid of cross-section of the T-beam.

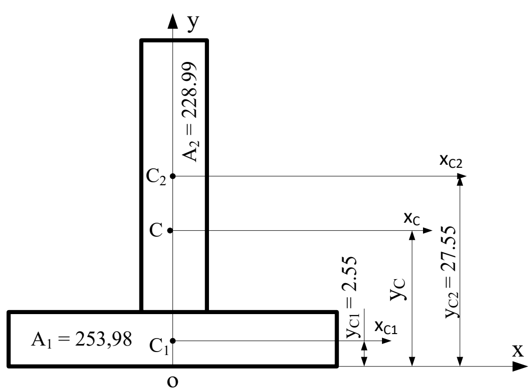


Fig. 2. Model for calculating moment of inertia of the T-beam cross section

Select the coordinate system as shown in Fig. 2, coordinate C is calculated as follows:

$$y_C = \frac{y_{C1}A_1 + y_{C2}A_2}{A_1 + A_2}, \tag{3}$$

$x_C = 0$, because y is the symmetry axis. So, the coordinate of C(0;14,403).

Moment of inertia of the T-beam cross section:

$$I_{xC} = I_{xC}^{(1)} + I_{xC}^{(2)}, \tag{4}$$

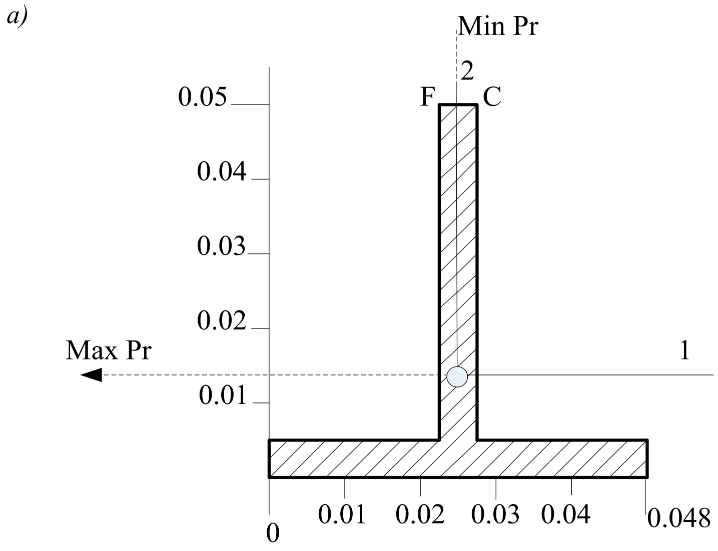
where:

$$I_{xC}^{(1)} = I_{xC1}^{(1)} + \overline{y_{C1}y_C}^2 \cdot A_1 = \frac{49.8 \cdot (5.1)^3}{12} + (11.854)^2 \cdot 253.98 = 36239.0896,$$

$$I_{xC}^{(2)} = I_{xC2}^{(2)} + \overline{y_{C2}y_C}^2 \cdot A_1 = \frac{5.1 \cdot (44.9)^3}{12} + (13.147)^2 \cdot 228.99 = 78049.9688.$$

So, $I = 114289.058\ \text{mm}^4 = 1.14289 \cdot 10^{-7}\ \text{m}^4$

In the next step, the beam was modelled with usage of Patran-Nastran FEM software. Beam property analysed by the program is presented in Fig. 3.



Area
A = 0.00048297
Moments of Inertia
I1 = 1.14283E-7
I2 = 5.298639E-8
I12 = 0.
Torsional Const. about Centroid
J = 4.300104E-9
Shear Stiff. Factors
K1 = 0.4741288
K2 = 0.5258712
Warping Coeff. about Shear Center
CWA = 5.074281E-13
Centroid Relative to Shear Center
N1A = 0.01185322
N2A = 0.

Centroid Relative to Origin
Hori. = 0.024900001
Vert. = 0.01440322
Shear Center Rel. to Origin
Hori. = 0.024900001
Vert. = 0.0025499994
Angle from Axis 1 to Max. Princ. Axis
Alpha = 180.
External Perimeter
Perim = 0.19960001

Dimensions
W = 0.0498
H = 0.05
t1 = 0.0051
t2 = 0.0051

Fig. 3. Beam property analysis by PATRAN software: shape of the cross-section (a) and properties (b)

Several, different FEM models can be used during structure characteristics analyses. The following models of the aluminium cantilever beam were considered (Fig. 4):

– Model T-beam one-dimensional (1D): T-beams are mounted on the plate with an offset equal to 0.014 m. The plate uses hexagonal three-dimensional finite elements (Hex8), beams use one-dimensional finite elements of bar form (Bar2), aluminium material: basic, diagonal mass matrix, number of nodes 606, number of elements 132 (Bar2) and 212 (Hex8),
– Model T-beam two-dimensional (2D): T-beams are mounted on the plate. The plate uses hexagonal three-dimensional finite elements (Hex8), beams use two-dimensional finite elements of quadrilateral form (Quad4), aluminum material: basic, diagonal mass matrix, number of nodes 13274, number of elements 5241 (Quad4) and 6011 (Hex8),
– Model T-beam three-dimensional (3D): T-beams are mounted on the plate. The plate and the beam use hexagonal three-dimensional finite elements (Hex8), aluminum material: basic, diagonal mass matrix, number of nodes 5114, number of elements 1950 (Hex8).

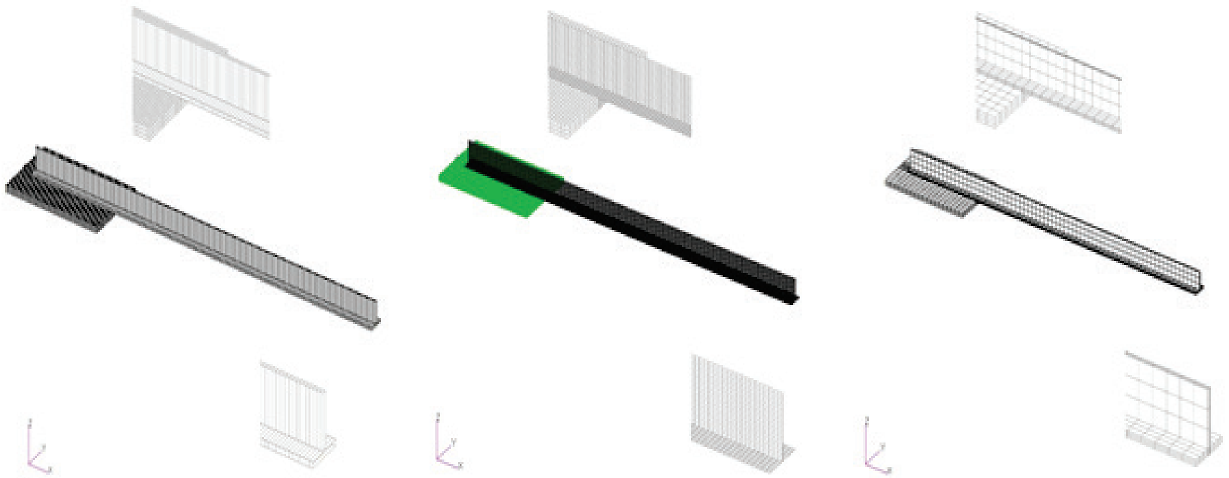


Fig. 4. FEM models of the analysed T-beam

3. Comparative results of natural frequencies

After replacing the parameters such as moment of inertia of the T-beam cross section I, cross-sectional area A, length of T-beam L, density and λ coefficient of the T-beam, into Equation 1, we will obtain the natural frequencies of the T-beam as theoretical. The models of the T-beam are simulated by determination of the geometry characteristics, material properties and density, boundary conditions and meshing in the Patran software. The data will be transferred to the Nastran software to calculate the natural frequencies of the T-beam. The final result processed and the graphics of the results can be presented again in the Patran software. Results of natural frequencies found out by hand calculation using equation (1) and by finite element modal analysis (free vibration without damping determined by professional software MSC NASTRAN) are presented in Tabs. 2 and 3. The comparative results was performed for evaluation the accuracy level and the errors of the selected model. Since then identifies the most optimal numerical model, which can be applied to calculate and evaluate the vibrations of other important marine structures. It is to be noted that finite element analysis produces flexural modes along with other modes like twisting, in plane and mixed. Within the framework of this article, we have only considered the flexural modes. Specifically, we will consider three bending modes of the T-beam. The comparison between one-, two-, and three-dimensional models is shown in Fig. 5.

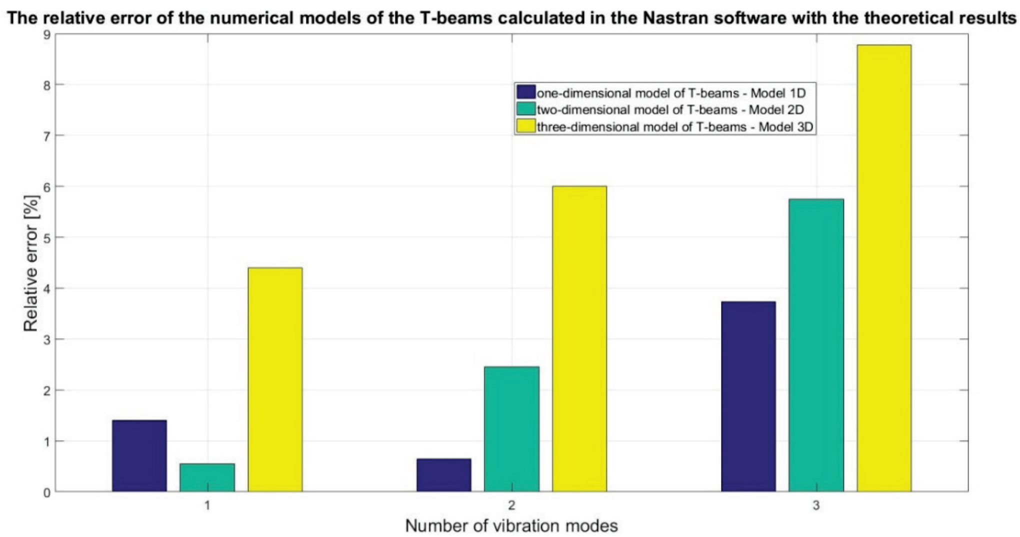


Fig. 5. Compare the relative error of the natural frequencies of 1D, 2D, 3D models with theoretical results

Tab. 2. Natural frequencies for each cantilever beam models

Shapes/Models	Natural frequencies [Hz]		
	T-beam one dimensional (1D)	T-beam two dimensional (2D)	T-beam three dimensional (3D)
1	29.799	28.698	29.563
2	43.674	42.829	41.173
3	185.11	115.13	121.66
4	268.11	175.31	181.55
5	511.17	263.24	253.66
6	727.45	339.18	359.09
7	982.29	447.99	469.44
8	1272.9	590.74	628.4
9	1365.9	712.2	689.3
10	1584.8	766.1	792.09

Tab. 3. Comparison of the natural frequencies of the beam calculated in theory and FEM software

Normal mode	Hand calculation		Model 1D f_1 [Hz]	$(\Delta f/f)$ [%]	Model 2D f_2 [Hz]	$(\Delta f/f)$ [%]	Model 3D f_3 [Hz]	$(\Delta f/f)$ [%]
	Frequency [rad/s]	Frequency f [Hz]						
1	270.467	43.068	43.674	1.407	42.829	0.0555	41.173	5.726
2	1694.739	269.863	268.11	0.637	263.24	2.442	253.66	6.004
3	4745.331	755.626	727.45	3,729	712.2	5,747	689.3	8.777

The results of natural vibration frequencies is better (smaller relative error) when increasing the number of finite elements (increasing the mesh density) – bigger meshing leads to errors and dispersion minimization of the calculated results. However, increasing the number of finite elements will increase the computational time, requiring higher computer configuration, making it difficult to calculate. It is important for such complicated structures as ship hull and superstructure. Therefore, it is important to choose the appropriate number of finite elements. In addition, the choice of one-dimensional, two-dimensional, three-dimensional models also influenced the results of natural vibration frequencies obtained by the software Nastran, one-dimensional model for the most accurate results - the maximum error does not exceed 4%, followed by two-dimensional numerical modelling – error of up to 6% and finally the three-dimensional model – the largest error of almost 9%.

With numerical models when the number of finite elements is too small, the resulting vibration curve is not smooth, Particularly prominent in the one-dimensional model (compare Fig. 6-8), so accuracy is not high, the dispersion of the results will be large, leading to large errors in the calculation. Determining the exact frequency of vibration alone will help designers, equipment manufacturers avoid the resonant vibration regions, endangering, destroying equipment.

Natural frequency for different mode shapes for T-beam mounted on the plate.

2nd mode *4th mode* *6th mode*

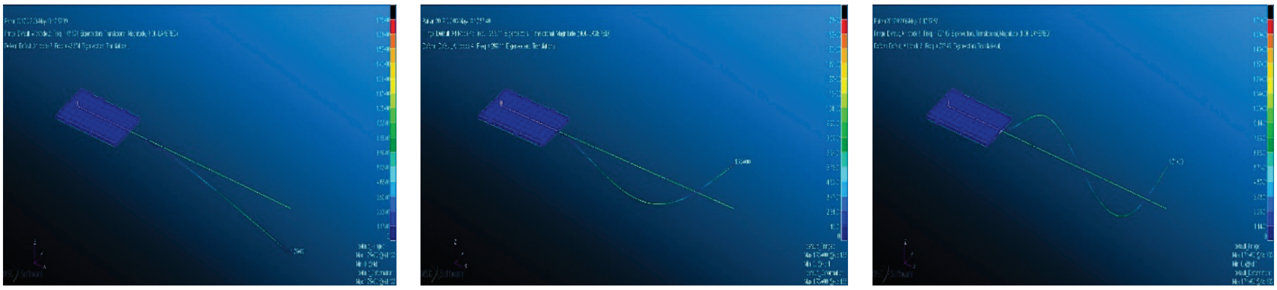


Fig. 6. Modes of vibration for T-beams of one-dimensional model with The T-beam: number of elements 132 (Bar2) and the plate: number of elements 212 (Hex8)

2nd mode *5th mode* *9th mode*

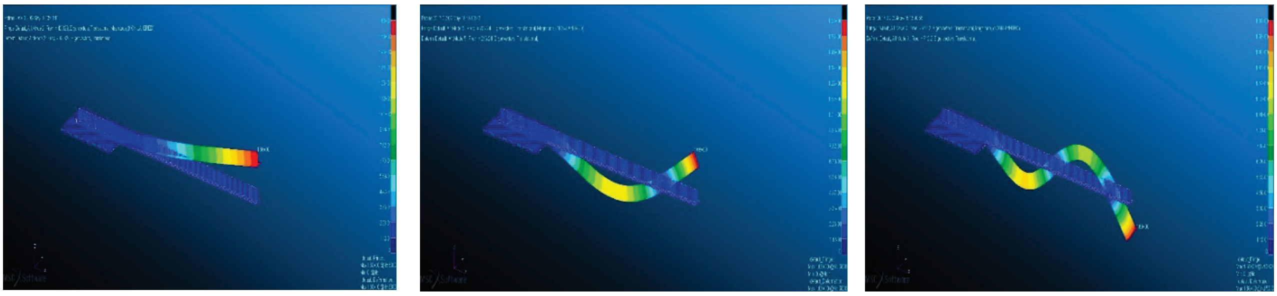


Fig. 7. Modes of vibration for T-beams of two-dimensional model with The T-beam: number of elements 5241 (Quad4) and the plate: number of elements 6011 (Hex8)

2nd mode *5th mode* *9th mode*

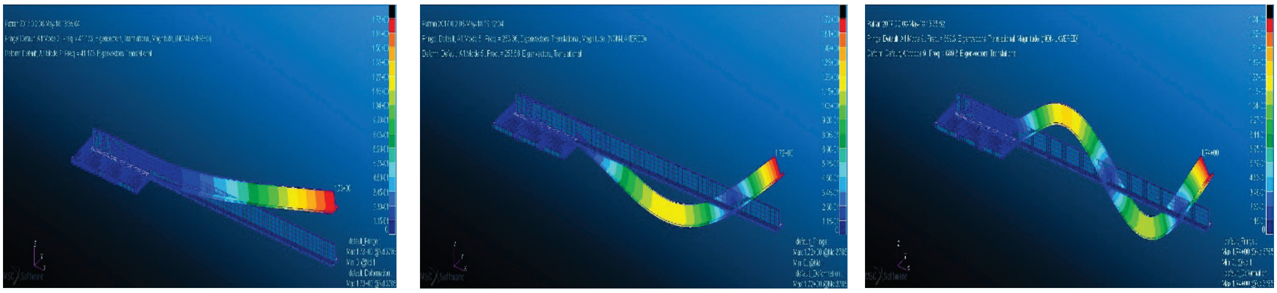


Fig. 8. Modes of vibration for T-beams of three-dimensional model with The T-beam and plate: number of elements 1950 (Hex8)

4. Conclusions

A detailed finite element analysis on the basic dynamic behaviour of the T-beam mounted on the plate is presented first. The results are then compared with hand calculation using theoretical equations. The closeness of comparison validates the choice of elements and procedure of analysis. The type of analysis was selected: modal analysis. By modal analysis, the natural frequencies and mode shapes are found.

The analysis also shows that the one-dimensional model for the most accurate results - the maximum error does not exceed 4%, followed by two-dimensional numerical modelling – error of up to 6% and finally the three-dimensional model – the largest error of almost 9%. However, the errors are different for different mode shape. Obviously, 3-D model should not be used during ship structure modelling. However, 1-D model gives better results for higher mode shape, when 2-D model is better for first natural mode. Therefore, during ship structure modelling, the best elements should be considered. Preliminary assumption, adopted by the authors of the work, is as follows: local stiffeners should be modelled as beams (1-D element) and global stiffeners (like main frames) should be modelled by 2-D elements. This assumption has to be checked.

Though the analysis was done for a beam, the same procedure can be applied to other structures like ship deck (stiffened panel), engine room of a ship etc. that may be subjected to resonant vibration. Those parts of the ship structure should also be analysed. The authors plan to perform comparative analyses with measurements with taking into account beams with failures.

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