

## MODELLING OF THE INFLUENCE OF TIRE CHARACTERISTICS ON STABILITY OF MOTION WITH USING VEHICLE AT A SCALE

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### Abstract

*The use of physical models of vehicles, at a scale allows them to be used when measuring the behavior of the full-size vehicles. The characteristics of the various systems and their impact on the dynamics of motion of the vehicle can be determined during stand tests or simulation and confirmed during testing of vehicles on test tracks. Testing vehicles built in small series, oversized or performed individually are conducted infrequently or not at all. In such cases, the alternative may be conducting tests of vehicles carried out at a scale. Research may be conducted in the boundary conditions (which could lead to loss of stability or overturning of the vehicle) impossible to achieve during testing of actual vehicles. They are particularly useful for assessing the stability of vehicles, the impact of the solutions or actions of a driver assistance system. Conducting research vehicles at a scale for the assessment of full-size vehicle must meet the criteria of similarity in the study of physical models of vehicles. One of the major issues is interaction of wheels and road. This article describes the results of wheels interaction with the road for tires with different characteristics used in the mobile model of the car and on the actual vehicle. The aim of the study was to prepare reference material to determine the correlation between the characteristics of the tire model and the full-scale tires. This will allow for made adjustment resulting from the impact of the characteristics of the tires on the motion dynamics of comparable cars especially in the curvilinear motion.*

**Keywords:** stability of vehicle motion, testing vehicles in scale, tires characteristics

### 1. Introduction

The tires and the condition of the road surface are responsible for the transmission of forces from the vehicle to the ground during the movement. Tires properties affect in a decisive manner on the transmission of forces and the behavior of the vehicle while driving. Terms of cooperation of tires with the road are so complex that it very difficult to define them using only theoretical models. The values of forces transmitted to the road, depends on the vehicle parameters and variables, which determine its movement. The longitudinal, lateral and vertical forces operate on tires during the curvilinear motion of the vehicle. Maintaining the balance of the vehicle motion requires a balance of the forces acting on the tire-road contact area. Longitudinal and lateral forces can be transmitted from the tire to the road using friction between the tread of the tire and the road surface. The characteristics of the tires depend largely on the construction and air pressure in the tires. The subject of the analysis was to evaluate the effect of the tire stiffness on the stability and steerability of the vehicle. Was used the physical model of the vehicle, made in scale, with the behavior of the similarity criteria to a full-size vehicle. Dimensionless parameters of vehicles motion were compared. For this purpose was used Buckingham's  $\pi$ -theorem [2]. Comparison of the parameters and characteristics of the tire for vehicles full dimensional and made in scale, required to carry out comparative studies under both static conditions and during movement of the vehicle.

## 2. Description of the model tests

In many cases, it is not possible to carry out tests on the real object, in particular at the reason of its large dimensions. These tests are difficult to perform, expensive and often dangerous. In such cases, you can use the research of physical models made on a smaller scale. Physical models of vehicles are built on the basis of dimensional analysis and the theory of similarity. The behavior of the scale of similarity allows for the transposition of the measured motion parameters of the tests of vehicle in scale on the parameters of the full dimensional object. The use of similarity theory requires behavior similarities: geometric, kinematic and dynamic similarities of the vehicle for real vehicle. This is achieved by extracting a number of parameters, which in dimensionless notation for the vehicles: in scale and the full size, should be the same. Fulfilling the criteria of similarity allows the interpretation of the test results gained from the vehicle in scale and relating to the full size vehicle. To determine the individual parameters in a dimensionless notation we used the Buckingham's  $\pi$ -theorem [1, 2].

There are a number of advantages of using a vehicle made to scale, to experimental testing of dynamics, rather than a full-size car:

- costs of vehicle tests performed on a scale are much smaller than the full-size vehicle, the same goes for supplies and spare parts.
- tests made in vehicle in scale require less space and are much safer to use.
- it is much easier to make changes to the vehicle in a smaller scale.
- scope of the research can look beyond the scope of the “safe”. Any overturning of the vehicle made in scale entails much lower repair costs.
- time testing is shorter.
- the vehicle is made smaller scale, otherwise the parameters analyzed by the similarity of its construction, is also characterized by the structural similarity of whole model and its assemblies.

Studies using vehicles in scale are being conducted in many scientific centers. Testing of vehicles at a scale are conducted in the Department of Internal Combustion Engines and Automotive Engineering in University of Bielsko-Biala. Currently our Department has a few of vehicles made in different scales, used for research on test tracks and a several test stands that allow for measurement of characteristics of assemblies, parts and tires [11, 15].

## 3. Impact of the tires characteristics on the behavior of the vehicles motion

The task of the tire is a smooth transfer of forces from the vehicle to the road in different traffic conditions, and ensures adequate insulation of vibrations generated by the unevenness of the road. The occurrence of lateral forces acting on the wheel axle to change the direction of their movement. Velocity vectors of wheels deviate from plane wheel about side slip angle. This affects among other things, to change of the turning radius and the vehicle path. The realization of driving around the specified radius requires increased steering angle by a value dependent on, among other things, of the tire parameters.

On the transfer of forces affect the elastic properties of the tire, inter alia, depend on the tire pressure, their dimensions and aspect ratio of the tire, its construction, normal load, camber, the side-slip angle and vehicle speed. The elastic properties of the tire are mainly determined by the static coefficients of rigidity of the tire  $C_x$ ,  $C_y$ ,  $C_z$  and cornering stiffness  $C_\alpha$ , determined from the relationship as follows: [4-8]

$$C_z = \frac{\Delta F_z}{\Delta s_z}, \quad C_y = \frac{\Delta F_y}{\Delta s_y}, \quad C_x = \frac{\Delta F_x}{\Delta s_x}, \quad C_\alpha = \frac{\Delta F_y}{\Delta \alpha}, \quad (1-4)$$

where:

- $\alpha$  – wheel side-slip angle, rad,
- $C_\alpha$  – tire cornering stiffness, N/rad,
- $C_x$  – tire longitudinal stiffness, N/m,
- $C_y$  – tire lateral stiffness, N/m,
- $C_z$  – tire radial stiffness, N/m,
- $s_x$  – longitudinal tire deflection, m,
- $s_y$  – lateral tire deflection, m,
- $s_z$  – radial tire deflection, m.

The cornering stiffness depends primarily on the pressure in the tire, its construction, the vertical load on the wheel and the camber. The determined parameters of the tire will change during motion of the vehicle and are described using stiffness coefficients (equations 1-4).

#### 4. The characteristics of the tires used in the test

Ensuring the similarity of the full-size vehicle and the physical model vehicle made in the scale was required designation of some parameters, which allow for comparison the tires. An additional problem is the fact that in vehicles at a scale with a length of 0.5 to 1 m is generally used tire with the polyurethane insert (Fig. 1). These inserts are divided into two groups of stiffness – soft and hard. In these dimensions of tires, we can also meet pneumatic tires for transport carriages, which are designed for use in low-speed of motion. These studies were designed to show the effect of the tires stiffness used on a vehicle in the scale as the results of road tests and compare them with the results of the full-size vehicle.



Fig. 1. The tire with a polyurethane insert used in a vehicle made in smaller scale

For comparison was selected tire car, 185R15C (used in the comparable vehicle) and tires used in the vehicle on a scale with inserts: hard and soft. Tab. 1 shows the compared tires.

Tab. 1. Tires selected for comparison

Parameter of tire	unit	185R15C SAVA	190/60 Desert Buster HD	190/60 Desert Buster HD
Aspect ratio	%	80	50	50
Diameter	in	15	4.5	4.5
Width	mm	185	60	60
Spring element	–	air	soft insert	hard insert
Air pressure	MPa	front 0.32 / rear 0.40	–	–
Type of tread	–	road	road	road

For comparison of tire, parameters were required to carry out a series of tests to identify the longitudinal, radial, lateral and angular stiffness of the tire, rolling resistance, tireprint, etc. In this study is assumed the constant coefficient of friction as a result of the research on the clean and dry road surface. Due to the inability to carry out the measurement of the angular stiffness of the tires of the truck, was used for this purpose elastic beam analogy suggested by Hewson [3]. Based on the presented dependence, we set the stiffness of the tire:

$$C_{\alpha} = \frac{2E \cdot b \cdot w^3}{(r + w \cdot a)^2 \cdot \sin\{\arccos[1 - (s \cdot w \cdot a)/(r + w \cdot a)]\} \cdot (\pi - \sin\{\arccos[1 - (s \cdot w \cdot a)/(r + w \cdot a)]\})}, \quad (5)$$

where:

$a$  – tires aspect ratio,

$b$  – thickness of the belt material, m,

$C_{\alpha}$  – tire cornering stiffness, N/rad,

$E$  – modulus of elasticity of the material of the tires belt, N/m<sup>2</sup>,

$r$  – rim radius, m,

$s$  – lateral deformation of the tire under a load, %,

$w$  – width of the belt, m.

These analyzes also uses the average steering angle  $\delta$  and the angle resulting from the Ackermann's relationship  $\delta_A$  – (equation 6 and 7) and the wheels side-slip angles for front and rear axle (equation 8 and 9) and understeer index (determined according equation 10). These relationships are shown below. Other motion parameters of the vehicle in a scale were measured during the tests. To determine the parameters of a moving vehicle was used motorcycle model, assuming that the lateral acceleration does not exceed 4 m/s<sup>2</sup> and suspension correspond to rigid axles and lateral inclination will have no significant impact on the values of the analyzed variables. In this case, it can be assumed that the cornering stiffness of the tire in the motorcycle model is the sum of this stiffness of individual wheels on one axle. The error resulting from this simplification begins to increase during increasing of lateral acceleration [12].

The steering angles determined from the Ackermann's dependence and taking into account of wheels side-slip angles can be presented as:

$$\delta_A = \frac{R}{L_w}, \quad \delta = \frac{R}{L_w} + \left( \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \right) \cdot \frac{V^2}{g \cdot R}, \quad (6, 7)$$

where:

$\delta$  – average steering angle of the front wheels, rad,

$\delta_A$  – average steering angle of the front wheels from the Ackermann's dependence, rad,

$C_{\alpha}$  – tire cornering stiffness of front/rear wheels, N/rad,

$g$  – acceleration of gravity, m/s<sup>2</sup>,

$L_w$  – wheelbase ( $L_w = a_1 + a_2$ ), m,

$R$  – radius of the track, m,

$V$  – vehicles speed, m/s,

$W_f, W_r$  – load on the front/rear axle, N.

The wheel side-slip angles can be determined from the relationship:

$$\alpha_r = \beta - \frac{a_2 \cdot \dot{\psi}}{V}, \quad \alpha_f = \beta + \frac{a_1 \cdot \dot{\psi}}{V} - \delta, \quad (8, 9)$$

where:

$\alpha$  – wheel side-slip angle for front/rear wheels, rad,

$\beta$  – vehicle side-slip angle, rad,

$\delta$  – average steering angle of the front wheels, rad,

$\dot{\psi}$  – yaw speed, rad/s,

$a_1, a_2$  – distance from the center of gravity to the front/rear axle, m.

The understeer coefficient  $K$  can be determined from the relationship:

$$K = \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}, \quad (10)$$

where:

$C_{\alpha}$  – tire cornering stiffness for front/rear wheels, N/rad,

$W_f, W_r$  – load on the front/rear axle, N.

Characteristic parameters of tire of vehicle made in the scale were established on the basis of measurements on test benches.

## 5. Characteristics of tires

The measurements of tires stiffness for use in stationary conditions was carried out in the Vehicle's Laboratory in University of Bielsko-Biala. For measurements were selected tires 185R15C. These tires were tested with nominal and reduced air pressure in the tires. During the test, the tire was placed on a sliding basis as shown in Fig. 2.



*Fig. 2. Measurement of tires stiffness – 185R15C*

The vertical force was applied to the wheel axis (in the case of measuring the rigidity of the normal force was varied in the range of 0 to a maximum ~50% above the static load), and in other cases (measurement of longitudinal and lateral stiffness) the tire was tested at two values of the loading force. Tangential forces was applied to the sliding plate, on which was based the wheel. Similar studies were carried out to tire of the vehicle in scale (Fig. 3).

In Tab. 2 are summarized the results of measurements of full-size tires and tires used in the vehicle on scale.

Testing of cornering stiffness of the tire was conducted using the platform with the movable track, on which were pending the wheel. The longitudinal axis of the wheel was deflected by a predetermined angle from the axis of the track, which made it possible to generate lateral forces and simulate phenomena occurring during curvilinear movement at the contact area of the tire and road. On the test bench was measured angle of deflection of the wheel from the axis of the track, radius of the wheel, vertical force and lateral force that was generated by the tire. Fig. 4 shows the test bench for measuring the angular of stiffness of tires.

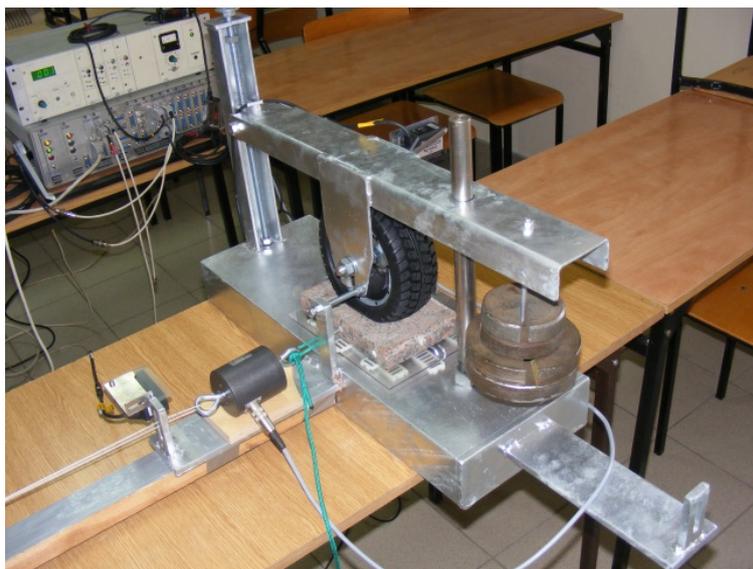


Fig. 3. The test bench to the measurement of the tire on a scale

Tab. 2. Results of the measurements of the tire stiffness used in full-size vehicle and vehicle in scale

Parameter	185R15C SAVA			190/60 Desert Buster HD		
	load [N]	0.22 MPa	0.33 MPa	load [N]	soft insert	hard insert
Radial stiffness, N/mm	–	625	923	–	14.09	17.09
Longitudinal stiffness, N/mm	4340	340	376	48	10.74	15.0
	8240	336	362	88	11.59	15.5
	12400	342	372	128	13.99	15.5
	–	–	–	168	14.32	16.3
Lateral stiffness, N/mm	4340	55.6	66.8	48	3.21	6.05
	8240	56.1	66.6	88	4.5	6.0
	12400	56.3	69.4	128	5.5	5.8
	–	–	–	168	5.69	8.02



Fig. 4. The test bench for measuring the tire cornering stiffness

Measurements of radial, longitudinal and lateral stiffness were taken under static conditions. Strain measurement of tires: the vertical longitudinal and transverse, was carried out for the full-size tire with air pressures of the tire: reduced 0.22 MPa and 0.33 MPa nominal, for the tire of vehicle in scale with the hard and soft inserts. The problems associated with the choice of tire for the vehicle scale have been described in the literature [9, 13]. Fig. 5 shows the change of cornering stiffness of tires depending on the side-slip angle of the tire. The cornering stiffness for full-size vehicle tire (was determined from the equation 5) and is 56630 N/rad for reduced air pressure in tire of 0.22 MPa and 68 805 N/rad for the nominal pressure of 0.33 MPa.



Fig. 5. The results of measurements of the tires cornering stiffness of the vehicle on a scale

On the basis of measurements, the values of the cornering stiffness were established for the tires on a scale with hard and soft insert. For further analysis assumed the values  $C_{\alpha\_hard} = 500$  N/rad and  $C_{\alpha\_soft} = 290$  N/rad.

## 6. Measurements of the vehicle dynamics

To evaluate the stability and maneuverability of vehicles are used standardized tests [12]: steady-state circular tests (ISO 14792), lateral transient response test methods (ISO 14793), dynamic vehicle stability (AVTP 03-160 – Test avoid an obstacle on the road) and others. The study selected two tests modeled on driving under specified conditions on a circular path and the lateral transient response test [16, 17]. Using the theory of similarity, it has been chosen to scale the vehicle trajectory and motion parameters. These tests have been programmed in the controller of the vehicle on the scale.

The vehicle on scale was equipped with measuring apparatus Racelogic Vbox based on GPS technology, allowing for the registration of the track and traffic parameters like: speed, acceleration, longitudinal and transverse angular velocity of the center of gravity, tilt angles. Fig. 6 shows the vehicle in a scale during testing.

The vehicle in scale was moving in a circular path at a given steering angle. Testing of vehicle dynamics allowed determining the impact of changes in the stiffness of the tire resulting from the reduced air pressure on the movement of the vehicle. Tires with hard insert reflect the situation in which a tire pressure are nominal. The vehicle with mounted hard inserts moves in a circular path with a radius of ~4.5 m. Mounting of tire with soft insert on the front axle (simulating of reducing the air pressure to 0.22 MPa in a real car) the effect of increasing the turning radius to ~5.7 m and when was mounted tires with soft insert on the rear axle the radius of the circular path was reduced to ~3.75 m. Vehicle speed resulted from the rolling resistance and cornering resistance in the circular path.



Fig. 6. The vehicle in scale  $\sim 1:5$  during road tests

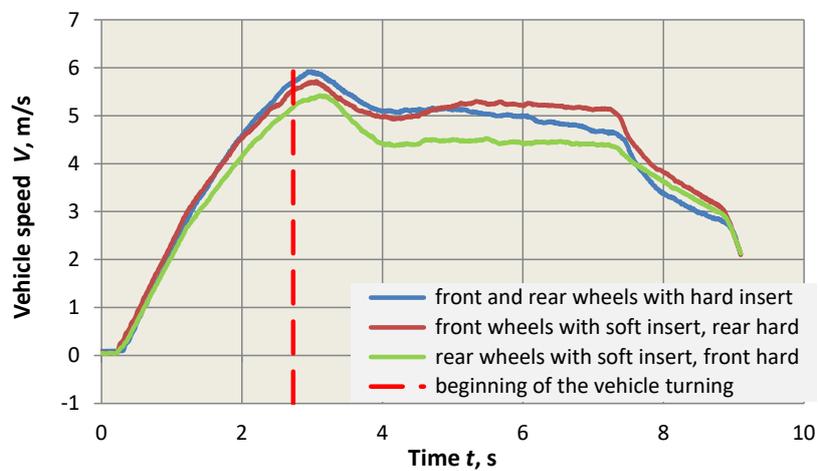


Fig. 7. The vehicle speed measured on the vehicle in scale for tires on axles with various characteristics

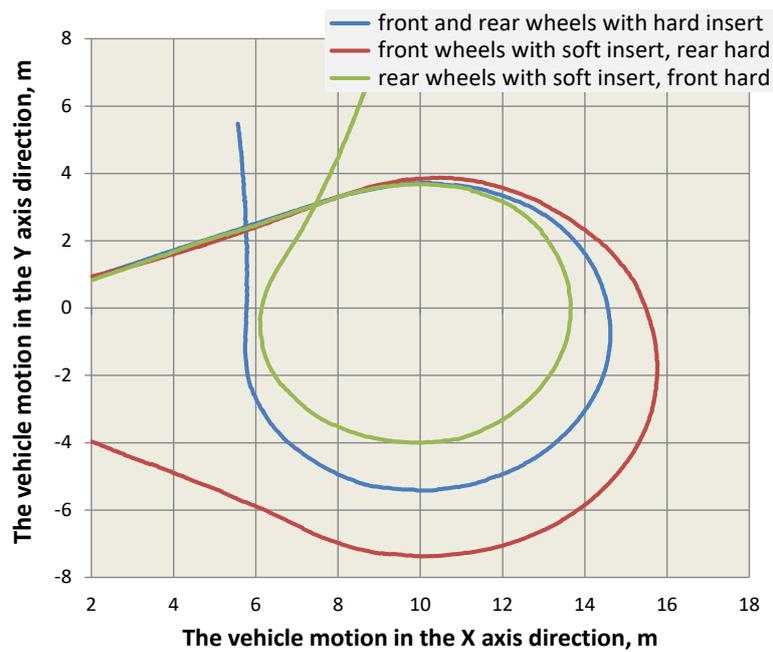


Fig. 8. Path of vehicle on scale with tires with different characteristics

In the case of tires with hard insert, during the turn, speed is the highest about 6.0 m/s and under stabilized conditions is lower about 5 m/s. For tires with a soft insert on the front axle speed during a turn is 5.7 m/s and the under stabilized conditions of 5.2 m/s. The lowest speed occurs when mounting the tire with a soft insert on the rear axle and speed assumed the values 5.3 and 4.4 m/s. The side-slip angle of center of gravity varied between  $7.1^\circ$  for the tire with the hard insert on all axles to  $6.5^\circ$  for the tire with a soft insert on the front and  $7.7^\circ$  from a soft insert on the rear axle. Fig. 8 shows the track of the vehicle in scale with tires with varying cornering stiffness.

The Fig. 9-11 was shown curves of lateral acceleration, side-slip angles of the center of gravity and difference of wheels side-slip angle on front and rear axles for various cornering stiffness of tires.

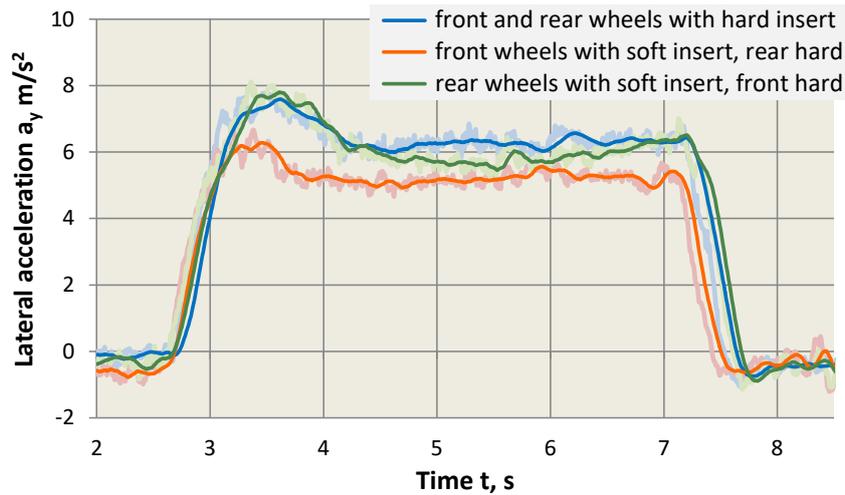


Fig. 9. The vehicle lateral acceleration measured on the vehicle in scale for tires on axles with various characteristics

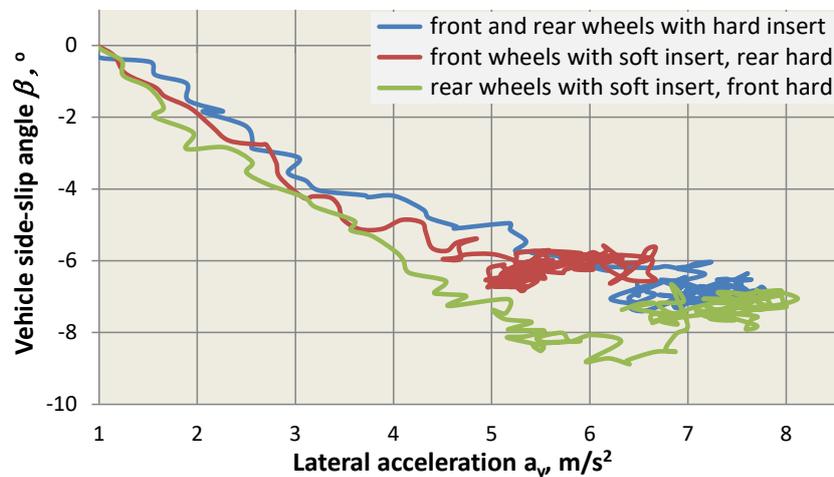


Fig. 10. The vehicle side-slip angle to lateral acceleration measured on the vehicle in scale for tires on axles with various characteristics

The ratio of yaw speed to the steering angle, well characterized motion of the vehicle with tires which have different characteristics - tires with the hard insert have this ratio 5.12 1/s and tires with soft insert were mounted on the front axle, 4.73 1/s, and 6.20 1/s and if tires with soft insert which were mounted on the rear axle 1/s.

In the Tab. 3 have been listed values, which were obtained after recalculating the results on the parameters of motion for the full-size vehicle.

Those set out above results of measurements and analysis are used to determine the effect of use of tires with reduced air pressure on the front or rear axle of the vehicle.

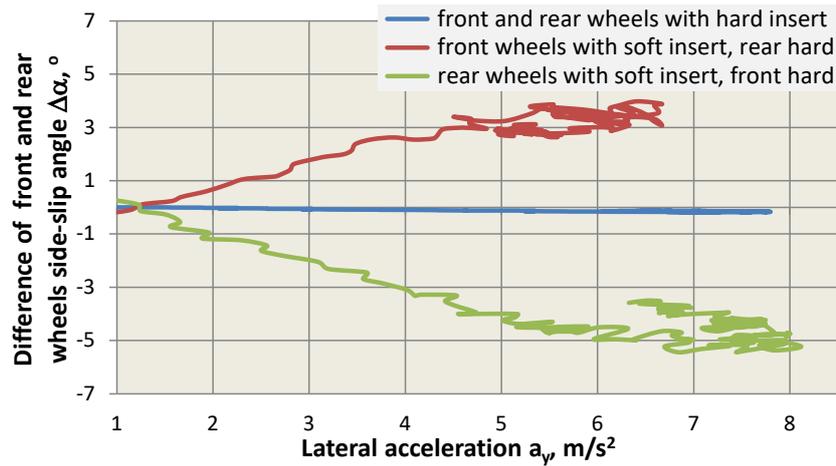


Fig. 11. The difference of wheels side-slip angle to lateral acceleration measured on the vehicle in scale for tires on axles with various characteristics

Tab. 3. The real vehicle motion parameters obtained by transferring the results of measurements of the physical model motion

Tire location	$v_x$ [m/s]	$\dot{\psi}$ [rad/s]	$\phi$ [°]	$\Delta\delta$ [°]	$a_y$ [m/s <sup>2</sup> ]	$K$ [°/g]
Tires with hard insert on all axles	9.85	0.438	4.7	0.01	5.07	-0.0244
Tires with soft insert on the front axle	10.23	0.541	4.7	3.20	4.81	0.6306
Tires with soft insert on the rear axle	8.67	0.433	4.9	-1.74	5.07	-0.6972
Tire location	$\beta$ [°]	$\alpha_r$ [°]	$\alpha_f$ [°]	$\Delta\alpha$ [°]	$a_y/\delta$ [m/s <sup>2</sup> /rad]	$\dot{\psi}/\delta$ [1/s]
Tires with hard insert on all axles	-6.95	5.29	5.44	-0.14	0.520	2.637
Tires with soft insert on the front axle	-6.36	6.90	4.11	2.98	0.379	2.435
Tires with soft insert on the rear axle	-7.07	4.57	8.09	-3.69	0.654	3.193

### 7. Conclusions

Comparison of the results of measurement for vehicle full-size and in scale requires the preservation criteria of similarity. Using the vehicle in scale, you can examine the actual parameters of vehicle motion (the assumed accuracy of the analysis).

The tests performed of dynamically change the direction of driving and driving in a circular path with a fixed speed has been used to compare vehicle motion with tires of different stiffness. These tests allow the assessment of the behavior of the vehicle during a motion with reduced tire pressure on front or rear axle. Transfer of the results from the vehicle in scale to the full-size vehicle requires the conversion by the relevant scale factors.

Decrease in air pressure in tires on front axle will change from almost neutral characteristic on a strongly understeering and in the event of a reduction of air pressure in the rear wheels clearly to oversteer. Information obtained during the investigation of the lateral acceleration  $a_y$ , yaw speed  $\dot{\psi}$  and differences of wheels side-slip angle  $\Delta\alpha$  are comparable with the results of measurements of the full-size vehicle. There is also a compliance of indicators: yaw speed  $\dot{\psi}/\delta$  and lateral acceleration  $a_y/\delta$  depending on the steering angle. This is also confirmed understeer index ( $K$ , which takes on the values respectively for the tire with hard insert -0.0244 °/g), and for tire with soft insert mounted on the front axle 0.6306 °/g, and on rear axle -0.6972 °/g. The values obtained were averaged for the front and rear axle.

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