

MODEL OF VEHICLE ELECTRIC DRIVE SYSTEM

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Abstract

Models of electrical energy storage devices – a battery and a supercapacitor – as well as models of motor and a control system, were used to feature the work of a hybrid electric drive system. The models were adapted to simulate driving with a given driving cycle, in a quasi-static method, with the use of MATLAB-Simulink software. The advantage of this method is that calculations can be done quickly and results can be easily compared.

A battery model enables determining voltage and charge level in relation to load current and temperature. The basic parameter of a battery is a nominal capacity. It is a measure of energy stored in the battery, which can be absorbed within discharge time, in conditions of temperature and current. The energy can be absorbed until the minimal voltage is reached in the clips. The article presents simulation of calculations lead that the described electric drive model. The model makes it possible to analyse different concepts of steering the whole system already in the initial phase. In particular, the article presents PMSM model and trajectories of selected parameters during a trapezoidal driving cycle.

Keywords: transport, electric vehicle, energy storage, electric drive, driving cycle, battery model

1. Battery model

A battery model enables determining voltage U_{aku} and charge level SOC in relation to load current I_{aku} and temperature T . A general scheme of the model is depicted in Fig. 1 [as cited in 1].

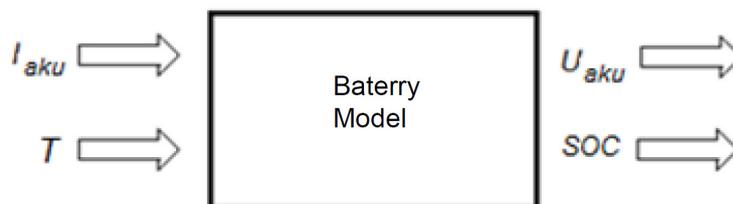


Fig. 1. Battery model

This proposal includes two functional sections:

- a section where the charge level SOC is determined,
- a section where voltage U_{aku} is calculated.

1.1. Charge level section

The basic parameter of a battery is a nominal capacity Q_n . It is a measure of energy stored in the battery, which can be absorbed within discharge time t , in conditions of certain temperature T and current I_n . The energy can be absorbed until the minimal voltage is reached in the clips.

$$Q_n = I_n t. \quad (1)$$

Different nominal capacities are defined for different discharge times. Usually they are determined for times of 10 or 5 hours, which correspond with capacities C_{10} and C_5 for discharge currents $0.1C_{10}$ and $0.2C_5$ accordingly.

The effective capacity of the battery Q_w is different from the nominal value Q_n and it depends on the type of load, as well as the temperature and the time of work. The correlation between the capacity and the load is called Peukert's law [1]:

$$Q_w = I_w(t)^{p(T)} t = const, \tag{2}$$

where:

Q_w – effective capacity of the battery,

t – discharge time,

$p(T)$ – Peukert's factor (which depends on the type of battery and the temperature T),

$I_w(t)$ – discharge current.

Tab. 1. Peukert's factor values [1]

Type of electric cell	p	Type of electric cell	p
Pb	1.27÷1.34	Ni-Zn-H	1.09
Ni-Cd	1.14	Ag-Zn	1.05
Ni-MH	1.1	Na-S	1.1

The constant for a charging process is $p=1$.

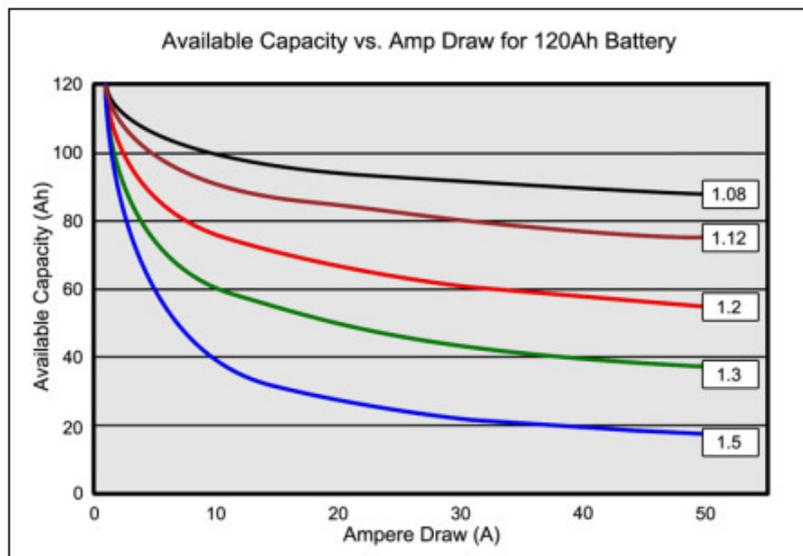


Fig. 2. Capacity changes in relations to discharge current and Peukert's factor varying between 1.08 and 1.5 [2]

Peukert's law includes an inside resistance and a speed of battery discharge. Peukert's factor close to 1 indicates that the battery works well, with high efficiency and minimal losses. The higher Peukert's constant value, the less efficient the battery is.

Previous equations indicate that there is a correlation between two different capacities Q_{w1} and Q_w , as well as between adequate discharge currents I_{w1} and I_w [1]:

$$Q_{w1} = Q_w \left(\frac{I_w}{I_{w1}} \right)^{p(T)-1} \tag{3}$$

Hence, it is possible to determine the quantity of the charge acquired from the battery loaded with given constant current, when the battery capacity and the battery rated current are known. Peukert's equation can be used to determine remaining capacity of the battery [1]:

$$Q(T, t) = Q(T, t_0) - \int_{t_0}^t \left(I(t) \left(\frac{I(t)}{I_{nom}} \right)^{p(T)-1} \right) dt, \quad (4)$$

where:

$I(t)$ – discharge current,

$Q(T, t_0)$ – initial capacity,

I_{nom} – rated current of the battery (usually C_{10} or C_5).

Capacity changes during the charging process can be likewise presented in an equation:

$$Q(T, t) = Q(T, t_0) + \int_{t_0}^t I(t) dt. \quad (5)$$

While considering the charging processes, following issues should be taken into account:

- a decrease of temperature stunts diffusion processes and chemical activity of electrode material, therefore the battery capacity is reduced,
- in a longer period, a self-discharge process can be observed. Its intensity increases with the growth of temperature – to different extent, depending on the type of electric cell,
- batteries age with the passage of time, which results in a decline of capacity.

If the battery charge level SOC , is defined as [1]:

$$SOC(T, t) = \frac{Q(T, t)}{Q_{max}(T)}, \quad (6)$$

where:

$Q_{max}(T)$ – maximal capacity at given temperature and load current,

$Q(T, t)$ – quantity of charge stored in the battery,

it is possible to define the actual battery charge level SOC by the following equation:

$$SOC(T, t) = SOC(T, t_0) - s(T) \frac{I_{nom}^{1-p(T)}}{Q_{nom}} \int_{t_0}^t I_{aku}(t)^{p(T)} dt = SOC_0 - C s(T) \int_{t_0}^t I_{aku}(t)^{p(T)} dt, \quad (7)$$

where:

C – constant value, which depends on the rated capacity Q_{nom} for assumed rated load current I_{nom} and Peukert's factor p ,

SOC_0 – initial battery charge level,

s – usage factor (e.g. a decline of capacity to 80% after 1000 cycles); for a new battery $s=1$.

When the battery is fully charged: $SOC=1$, whereas when the battery is fully discharged: $SOC=0$. In practise it is impossible to reach discharge level of $SOC=0$, because the maximal quantity of electric charge which can be acquired without a risk of damage is limited by electrochemical reactions.

1.2. Voltage level section

Fig. 3 depicts a battery equivalent circuit model.

According to Fig. 3 battery clips voltage can be calculated as follows:

$$U_{aku} = U_{aku-w}(SOC, t) \pm I_{aku}(t) * (R_{el}(SOC, t) + R_e(SOC, t)). \quad (8)$$

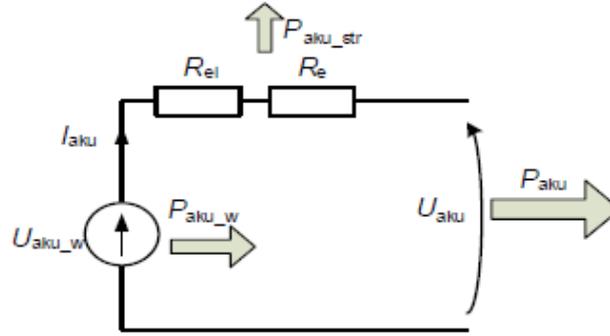


Fig. 3. Battery equivalent circuit model for energy-related calculations [4]: R_{el} – electrolyte resistance, R_e – electrode resistance, U_{aku_w} – electromotive force of the battery, U_{aku} – battery clips voltage, I_{aku} – battery current, P_{aku_w} – power of battery electromotive force, P_{aku_str} – power losses in internal resistances, P_{aku} – initial power of the battery

When the formula $P_{aku}=U_{aku}*I_{aku}$ is taken into account, it is possible to describe the instantaneous current during a charging process as [1, 4]:

$$I_{aku}(t) = \frac{-U_{aku_w}(SOC) + \sqrt{U_{aku_w}(SOC)^2 + 4R_{aku}(SOC) * P_{aku}(t)}}{2R_{aku}(t)} \quad (9)$$

and during a discharging process as:

$$I_{aku}(t) = \frac{U_{aku_w}(SOC) - \sqrt{U_{aku_w}(SOC)^2 - 4R_{aku}(SOC) * P_{aku}(t)}}{2R_{aku}(t)} \quad (10)$$

Because of high influence of time and temperature on the parameters, it is recommended to use results from experimental researches in the foregoing equations.

For instance, [4] demonstrates the results of researches on the electric cell Ni-MH Panasonic HHR-650D/FT with rated parameters: voltage $U_n = 1.2$ V and capacity $Q_n = 6.5$ Ah, which is used in i.e. hybrid Honda cars. Fig. 4 depicts charts of changes in electric cell voltage during discharging with the current of constant value.

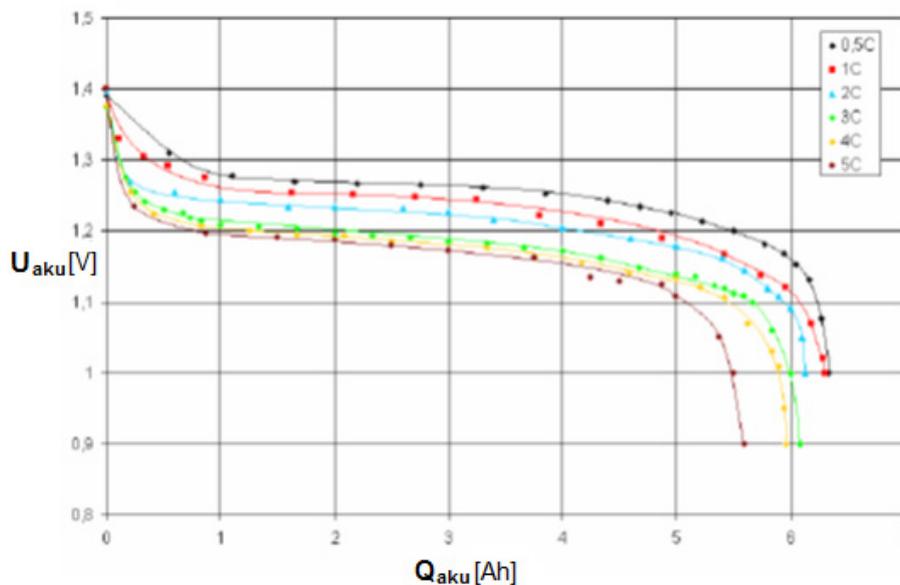


Fig. 4. Charts of changes in HHR-650D/FT battery voltage for discharge currents 0.5C, 1C, 2C, 3C, 4C and 5C in relation to battery capacity [4]

- Three areas can be differentiated in the previous chart:
- in the first area, there is a sudden decrease of voltage when the battery is fully charged. The type of battery determines how vast is this area,
 - in the second area, there is a decrease of voltage during battery work. The decrease can continue until the level of nominal voltage is reached,
 - the third area relates to absolute discharge – voltage decreases rapidly.

It is reasonable to use a nonlinear empirical model based on the approximation of experimental characteristics. Fig. 6 depicts such a model for a prototypical battery set. The model was developed with the usage of Simulink programme, Lookup Table (n-D) procedure and trajectories obtained from researches, which were conducted for seven different temperatures between -25°C and 40°C .

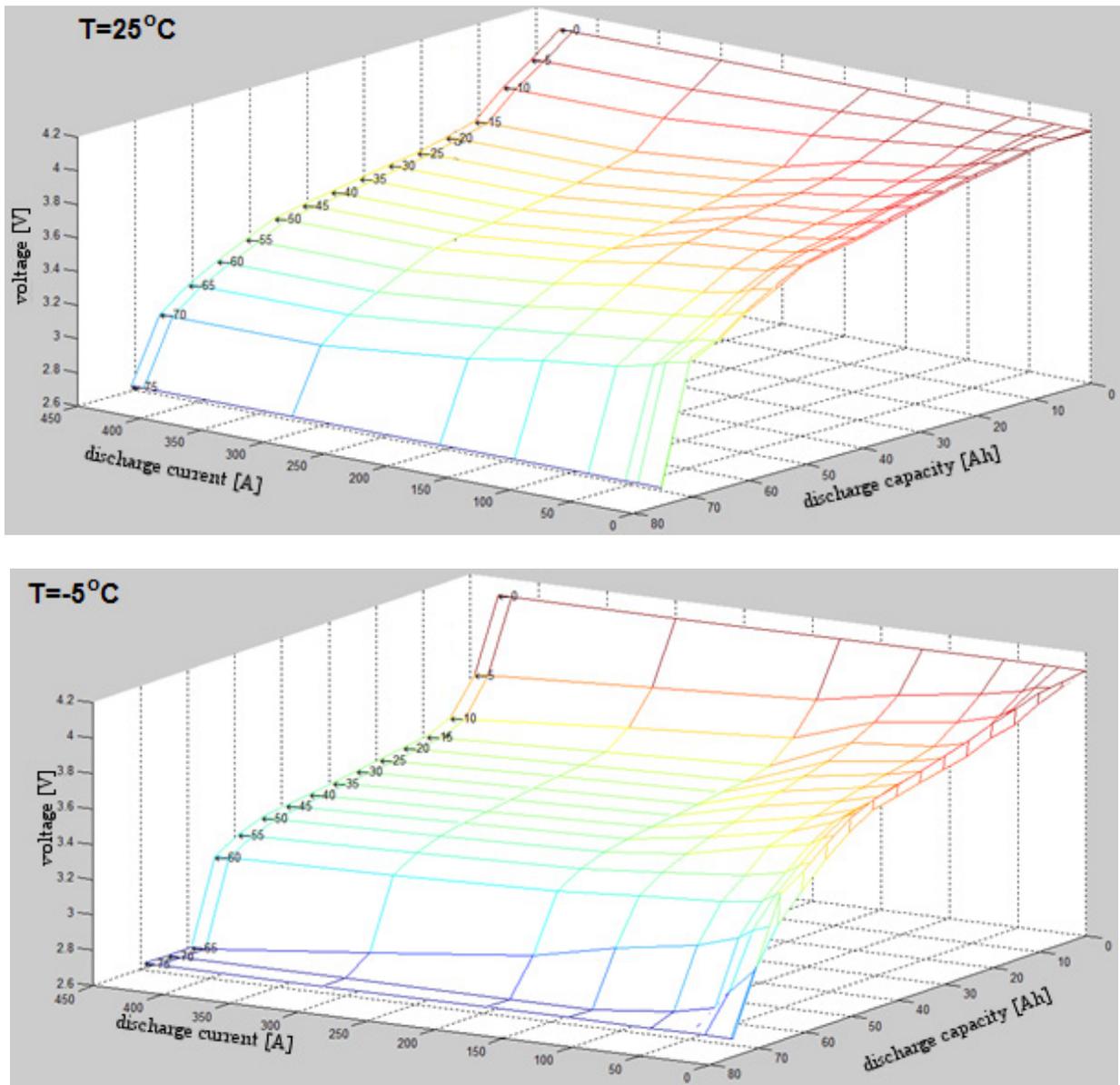


Fig. 6 (the yellow frame) depicts a battery model which consists of two sections: a section for determining the charge level SOC and a section for determining the voltage level U_{aku} . During a "trapezoidal" driving cycle (the blue line in the top left corner) the programme shows trajectories of: load current intensity I_{aku} (the yellow line in the top left corner), voltage U_{aku} (in the bottom left corner), capacity Q (in the top right corner) and charge level SOC (in the bottom right corner).

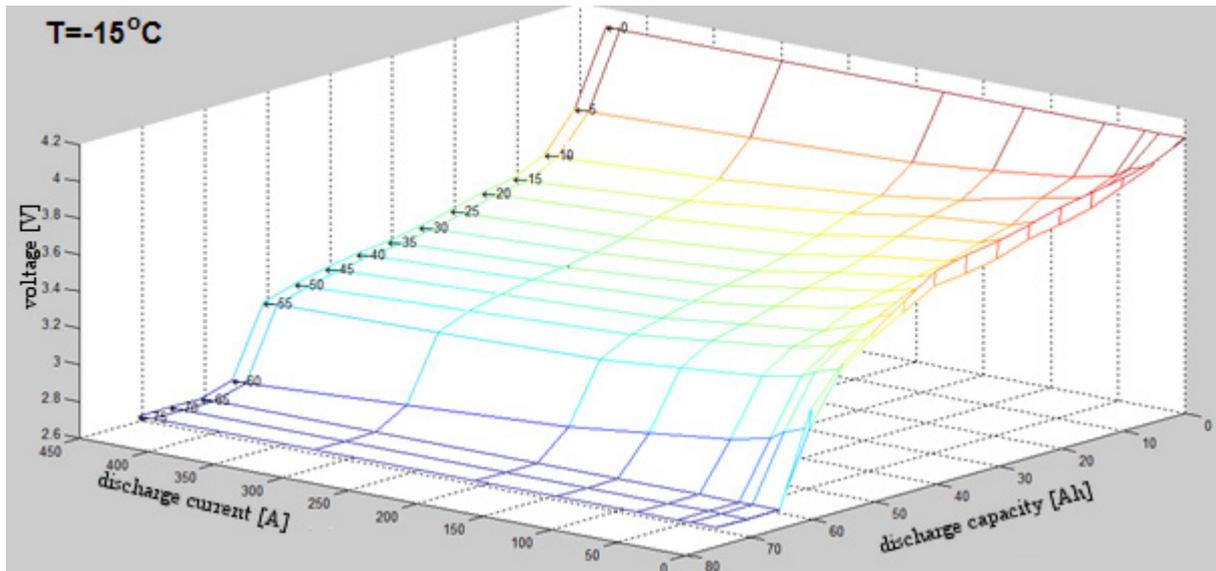


Fig. 5. Characteristics of battery discharge in relation to temperature, current and capacity

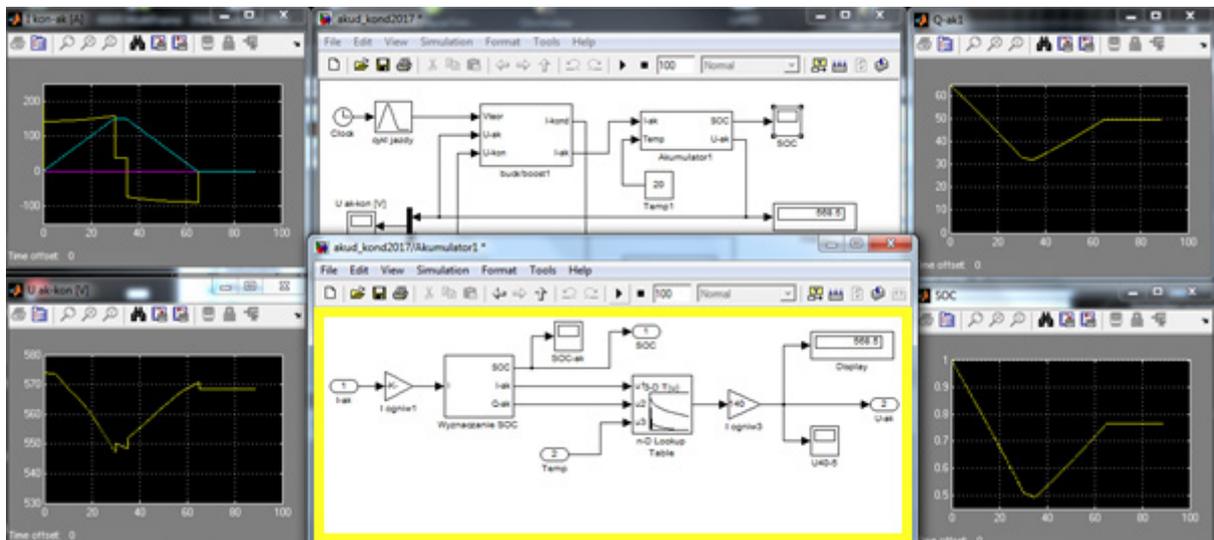


Fig. 6. Battery model and its parameters during a driving cycle

2. Supercapacitor model

Fig. 7 depicts a model of a supercapacitor battery shown as an equivalent circuit.

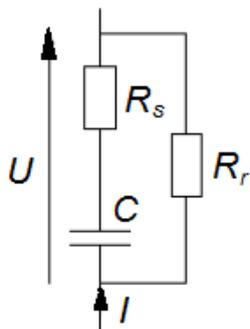


Fig. 7. Electrical equivalent circuit model of a supercapacitor: C – capacitance, R_s – series resistance, R_r – parallel resistance

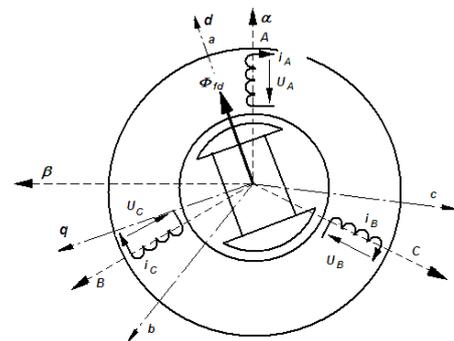


Fig. 8. Schematic view of a PM synchronous machine

Capacitance C has no constant value; therefore, it is described as a variable in relation to voltage:

$$C(u(t)) = C_0 + k_k U(t), \quad (12)$$

where:

C_0 – capacitance for the voltage 0 V,

k_k – coefficient of capacity (capacitance) in relations to voltage,

U – voltage.

The instantaneous current is described by the equation [4]:

$$I(t) = (C_0 + 2k_k U(t)) \frac{dU(t)}{dt}. \quad (13)$$

A method of determining coefficient k_k was presented in [4, 5], where $k_k = 1.6837$ F/V and capacitance $C_0 = 42$ in a set of Maxwell BMOD0350 supercapacitors.

The supercapacitor charge level is described as follows [4]:

$$k_k = \frac{\left(C_0 + \frac{4}{3} k_k U \right) U^2}{\left(C_0 + \frac{4}{3} k_k U_{\max} \right) U_{\max}^2}. \quad (14)$$

3. Model of a permanent magnet synchronous motor

Among electric machines, which are presently used in drive systems, permanent magnet synchronous motors achieve the highest power density and the highest efficiency as well. A model of the motor, depicted in Fig. 8 [7], is usually built under the following assumptions:

- a three-phase machine has a symmetrical construction,
- the resultant torque is equivalent to synchronous torque created by the machine,
- derivatives of permanent magnet current equal zero,
- trajectories of currents and voltages in the air gap are perfectly sinusoidal.

Magnetic fields generated by rotor and stator rotate with identical frequency in permanent magnet synchronous motors. With the usage of Clarke-Park transformation, it is convenient to create voltage formulas in d - q coordinate system, where coordinates d and q are related to a rotor, which is rotating with synchronous velocity. Moreover, d -axis coincides with the direction of permanent magnet magnetic flux Φ .

When foregoing assumptions are taken into consideration, the d - q system of equations is as follows [5]:

$$\begin{aligned} \frac{d}{dt} i_d &= \frac{1}{L_d} u_d - \frac{R}{L_d} i_d + \frac{L_q}{L_d} p \omega_r i_q \\ \frac{d}{dt} i_q &= \frac{1}{L_q} u_q - \frac{R}{L_q} i_q - \frac{L_d}{L_q} p \omega_r i_d - \frac{\phi_w p \omega_r}{L_q}, \\ T_e &= 1.5 p [\phi_w i_q + (L_d - L_q) i_d i_q] \end{aligned} \quad (15)$$

where:

Φ_w – constant magnetic flux, related to d -axis,

R_s – resistance of stator winding,

p – amount of magnetic poles pairs,

T_e – electromagnetic torque,

ω_r – rotor angular velocity,

$i_d, i_q, L_d, L_q, u_d, u_q$ – components of current, inductance, stator voltage, which are related to d-axis and q-axis respectively.

The first and the second equation show obvious cross coupling between the currents of d-axis and q-axis. It may be assumed that voltage u_d , voltage u_q and magnetic flux Φ_w deriving from permanent magnets are independent variables.

In the electromagnetic torque formula, there are two distinctive components: synchronous torque ($\Phi_w i_q$), deriving from magnetic flux of the permanent magnets, and reluctance torque ($(L_d - L_q) i_d i_q$), caused by magnetic asymmetry.

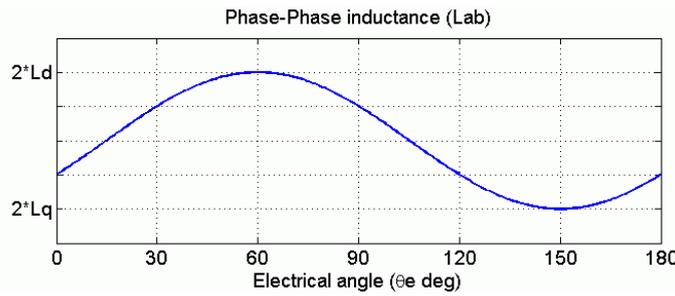
Inductance L_d and inductance L_q are in relation with the phase inductance and the rotor position inductance. For example, the inductance measured between phase a and phase b (phase c is open) is described by the formula:

$$L_{ab} = L_d + L_q + (L_q - L_d) \cos(2\Theta_e + \frac{\pi}{3}), \quad (16)$$

where θ_e represents electrical angle.

$$L_d = L_q = \frac{L_{ab}}{2} \quad L_d = \frac{\max L_{ab}}{2} \quad L_q = \frac{\min L_{ab}}{2}. \quad (17)$$

The following picture depicts changes of Phase-Phase inductance in relations to rotor angle θ_e [5]:



The dynamics equation for rotational motion is:

$$\begin{aligned} \frac{d}{dt} \omega_r &= \frac{1}{J} (T_e - F \omega_r - T_m) \\ \frac{d\Theta_e}{dt} &= \omega_r \end{aligned}, \quad (18)$$

where:

J – moment of inertia of the rotor and the load,

F – viscous-friction coefficient of the rotor and the load,

Θ_e – angular position of the rotor,

T_m – load torque,

ω_r – angular speed,

T_e – electromagnetic torque.

The power of the electric motor is given by the scalar product of voltage and electric current vectors:

$$P = \frac{3}{2} (u i) = \frac{3}{2} (u_d i_d + u_q i_q). \quad (19)$$

Assuming that the motor current regulator is ideal and that $i_d = \text{const} = 0$, the model of a synchronous motor takes an analogous form as the formulas describing the model of a direct-

current motor:

$$L_q \frac{di_q}{dt} + R_s i_q + \omega \Phi_w = u_q$$

$$T_e = \frac{3}{2} p \Phi_w i_q$$

$$P = \frac{3}{2} u_q i_q$$

When energy balance is taken into account, the foregoing PMSM model is a sufficient approximation of a real drive system.

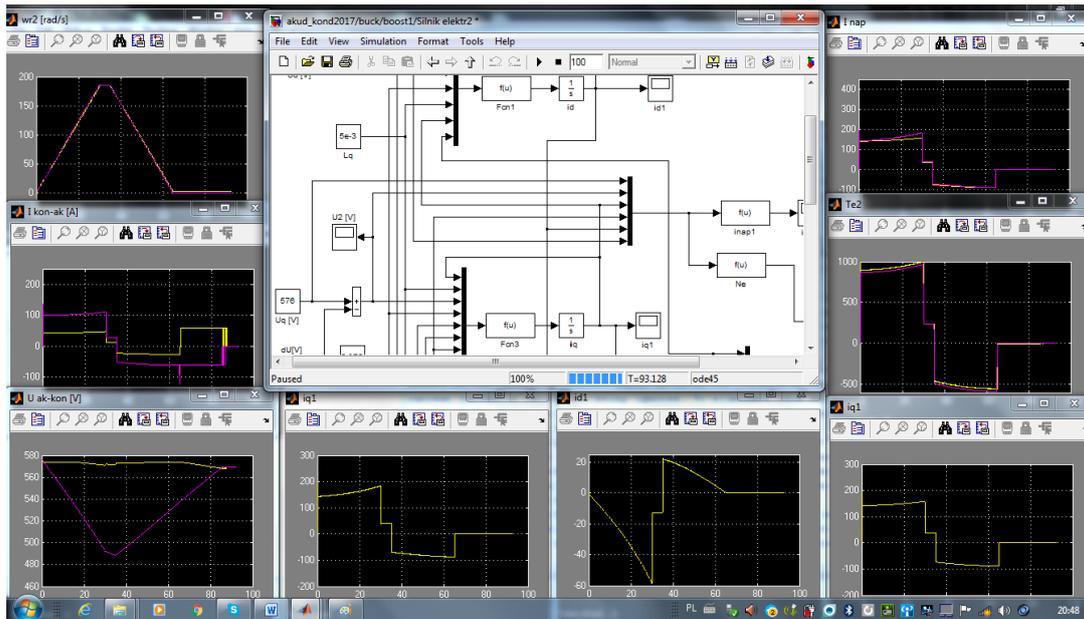


Fig. 9. PMSM model and trajectories of selected parameters during a trapezoidal driving cycle

4. Steering example

In the analysed electric drive system, there are three sources of energy: a battery, a supercapacitor and an electric machine, which works as a motor while propelling or as a generator while braking. The steering system should control the energy flow between these sources. One of many concepts of such kind of steering may be described with the usage of a didactic theoretical trapezoidal driving cycle: Fig. 9. – absorbing energy simultaneously from a battery and a capacitor, Fig. 10. – absorbing energy consecutively from a battery and a capacitor.

The trapezoidal cycle is characterized by the following phases: acceleration, constant velocity driving, braking and a standstill: Fig. 9 (in the top left corner) and Fig. 10 (in the left picture). In the pictures, there is also the theoretical cycle marked (the purple line) and its course (the yellow line).

At the beginning of the cycle, the battery and the capacitor have the same voltage value: Fig. 9 (the bottom left corner), Fig. 10 (the right picture). The energy stored in the battery and in the capacitor is used during vehicle acceleration in accordance with certain proportions, Fig. 9., or the energy stored in the supercapacitor is used at first, until the minimal voltage is reached, and then the acceleration is realized with the usage of the energy stored in the battery, Fig. 10.

The trajectory of capacitor voltage is marked with the purple colour and the trajectory of battery voltage is marked with the yellow colour. Fig. 9 (in the left middle picture) and Fig. 10 (in the middle picture) depict the intake of current from the supercapacitor and the battery.

The energy retrieved during braking is collected in the supercapacitor.

According to simulation calculations, voltages of the battery and of the capacitor are different

at the beginning of a standstill. Therefore, it is essential to equalise the voltages by steering the current flow between the battery and the capacitor.

Fig. 9 also depicts different values of current i_q and current i_d of a motor in a complete model (the second and the third bottom picture) and the values of current i_q in a simplified model for $i_d=0$ (the right bottom picture). There are minor differences visible in the trajectories of current i_q , driving torque of the rotor T_e and current I_{nap} .

Simultaneous intake of energy from the both sources reduces current obtained only from the battery, therefore the conditions of work become “gentler”.

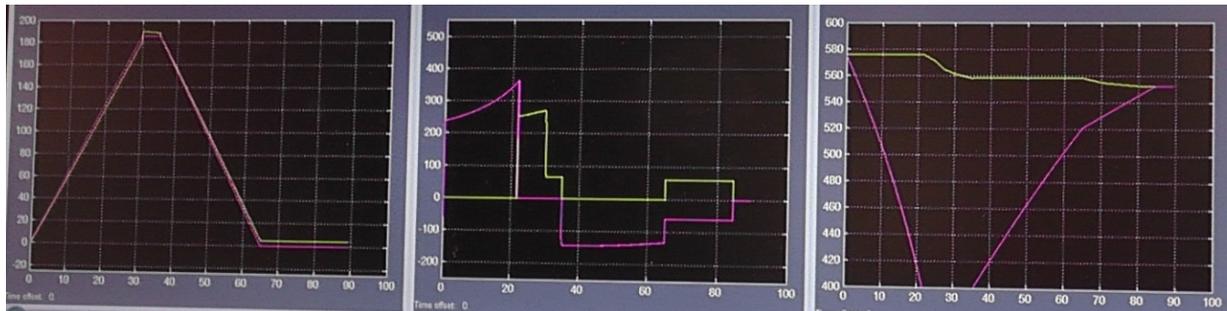


Fig. 10. Trajectories of selected parameters during a trapezoidal driving cycle

The presented simulation calculations lead to a conclusion that the described electric drive model can be accepted as correct. Furthermore, the model makes it possible to analyse different concepts of steering the whole system already in the initial phase.

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