

## ROBUST PREDICTIVE CONTROL OF AN OVERHEAD CRANE

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### Abstract

The predictive control scheme is developed for an overhead crane using the generalized predictive procedure applied for the discrete time linear parameter-varying model of a crane dynamic. The robust control technique is developed with respect to the constraints of sway angle of a payload and control input signal. The two predictive strategies are presented and compared experimentally. In the first predictive control scheme, the online estimation of the parameters of a crane dynamic model is performed using the recursive least square algorithm. The second approach is a sensorless anti-sway control strategy. The sway angle feedback signal is estimated by a linear parameter-varying model of an unactuated pendulum system with the parameters interpolated using a quasi-linear fuzzy model designed through utilizing the P1-TS fuzzy theory. The fuzzy interpolator is applied to approximate the parameters of a crane discrete-time dynamic model within the range of scheduling variables changes: the rope length and mass of a payload. The experiments carried out on a laboratory scaled overhead crane confirmed effectiveness and feasibility of the proposed solutions. The implementation of control systems was performed using the PAC system with RX3i controller. The series of experiments carried out for different operating points proved robustness of the control approaches presented in the article.

**Keywords:** overhead crane, predictive control, linear parameter varying model, recursive estimation, fuzzy interpolation

### 1. Introduction

Cranes are commonly used in many industries for transporting heavy and high-volume loads. There are different types of cranes, such as overhead, gantry, tower and boom cranes, which are widely used in factories, shipyards, warehouses and in construction of buildings. The under actuated nature of crane systems causes undesirable oscillation of an unactuated payload suspended on a flexible rope manipulated by the crane's drives. The oscillation of a payload adversely affects the accuracy of performed transportation tasks and may present a safety hazard to employees, transferred payload and surrounding objects. The trade-off between safety and efficiency of crane's operations is the challenging problem, which have been studied in numerous research works.

A thorough review of various methods reported in the literature for crane modelling and control is presented in [1]. A comprehensive review of crane control strategies developed during the years from 2000 to 2016 is discussed in [15]. Different modelling, control and measurement approaches developed for crane systems have been recently reported in the literature [5-9, 18, 19]. Crane control strategies can be broadly categorized into optimal control, input shaping and feedback control. Many solutions are developed using an input-shaping method, which may demonstrate the robustness in the presence of varying natural frequency of an oscillatory system [20]. Feedback control methods are adapted to ensure the robustness to external disturbances and model uncertainty in feedforward or optimal control strategies [21], and developed using gain-scheduling [22], or fuzzy logic technique [14, 16].

The article presents the model predictive controller developed using the generalized predictive control (GPC) procedure introduced by Clarke [4]. The idea of model predictive control (MPC),

which enables to optimize prediction of process behaviour with respect to constraints of process variables, has been recently applied in different crane control approaches. The MPC-based control scheme is developed for hydraulic forestry crane [10], boom crane [2], and laboratory models of a gantry crane [17] and overhead crane [11].

In this article, the GPC procedure is adapted to develop the robust predictive controller based on a linear parameter varying (LPV) model of a crane system. The two methods are used to estimate the parameters of a crane model. The first approach is based on the recursive least square (RLS) estimation that requires measuring the sway angle of a payload. In the second approach, the sway angle feedback signal is estimated by a discrete time model, and the parameters of a crane's model are estimated by a fuzzy interpolator. The P1-TS theory proposed in [12, 13] is applied to approximate the parameters of a crane discrete-time dynamic model within the range of rope length and mass of a payload changes. The robust predictive controller is developed with respect to the sway angle of a payload and control input signal constraints. The experiments carried out on a laboratory scaled overhead crane confirmed effectiveness and feasibility of the proposed solutions. The implementation of control systems was performed using the PAC system with RX3i controller.

The rest of the article is organized as follows. Section two introduces the planar model of a crane and the methods applied to estimate the parameters. The predictive controller is presented in section three, while the results of experiments are discussed in section four. Section five delivers the final conclusions.

## 2. Crane dynamic modelling

Consider a planar model of a crane transferring a payload (Fig. 1), which is assumed to be a point-mass suspended at the end of a massless rigid cable. The influence of the pendulum motion on the cart motion is neglected due to the assumption of large mechanical impedance in the drive system. Under these assumptions, the system is modelled as a cascade of the actuated cart (1) and the unactuated pendulum (2) formulated as the first-order and second-order discrete-time linear parameter varying models, respectively.

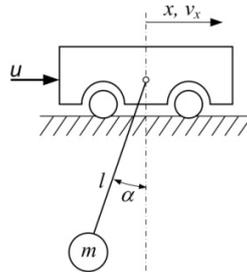


Fig. 1. Planar model of a crane, where  $m$ ,  $l$ ,  $u$  and  $\alpha$  are, respectively, mass of a payload, rope length, controlling signal corresponding to control force acting on a crane, and sway angle of a payload

$$A(z^{-1})v_x(t) = B(z^{-1})u(t-1), \quad (1)$$

$$C(z^{-1})\alpha(t) = D(z^{-1})v_x(t), \quad (2)$$

where  $A(z^{-1})$ ,  $B(z^{-1})$ ,  $C(z^{-1})$ , and  $D(z^{-1})$  are polynomials in the backward shift operator  $z^{-1}$ :

$$A(z^{-1}) = 1 + a_1 z^{-1},$$

$$B(z^{-1}) = b_0,$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + c_2 z^{-2},$$

$$D(z^{-1}) = d_0 + d_1 z^{-1}.$$

The two methods are proposed to estimate the model's parameters. The first method is based on the RLS algorithm that involves measuring the sway angle of a payload. The second method is based on a fuzzy interpolation that requires measuring the scheduling variables, such as rope length and mass of a payload, which correlate with the operating conditions of a crane system.

## 2.1. Recursive least square estimation

The models (1) and (2) can be equivalently presented as

$$\hat{v}_x(t) = \varphi_1^T \hat{\theta}_1(t), \quad (3)$$

$$\hat{\alpha}(t) = \varphi_2^T \hat{\theta}_2(t), \quad (4)$$

where the observation and parameter vectors are as follows

$$\varphi_1 = [u(t-1), -v_x(t-1)]^T,$$

$$\varphi_2 = [v_x(t), v_x(t-1), -\alpha(t), -\alpha(t-1)]^T,$$

$$\hat{\theta}_1(t) = [\hat{b}_0, \hat{a}_1]^T,$$

$$\hat{\theta}_2(t) = [\hat{d}_0, \hat{d}_1, \hat{c}_1, \hat{c}_2]^T.$$

The parameter vector can be estimated using the recursive algorithm with forgetting factor

$$\hat{\theta}_1(t) = \hat{\theta}_1(t-1) + \mathbf{P}_1(t) \varphi_1 (v_x(t) - \varphi_1^T \hat{\theta}_1(t-1)), \quad (5)$$

$$\hat{\theta}_2(t) = \hat{\theta}_2(t-1) + \mathbf{P}_2(t) \varphi_2 (\alpha(t) - \varphi_2^T \hat{\theta}_2(t-1)), \quad (6)$$

where the covariance matrixes are updated according to

$$\mathbf{P}_i(t) = \frac{1}{\mu} \left( \mathbf{P}_i(t-1) - \frac{\mathbf{P}_i(t-1) \varphi_i \varphi_i^T \mathbf{P}_i(t-1)}{\mu + \varphi_i^T \mathbf{P}_i(t-1) \varphi_i} \right), \quad i = 1, 2, \quad (7)$$

and  $\mu \in (0, 1]$  is the forgetting factor.

The crane dynamic model is identified on-line using the RLS algorithm based on the current and past measurements. Thus, the plant parameters such as rope length and mass of a payload are not necessary to be measured. However, the sway angle of a payload is required to be measured, that is the drawback of this approach (the additional cost of the sensing equipment, which should be installed in a crane system).

## 2.2. P1-TS fuzzy estimation

The fuzzy interpolation is proposed as the alternative method of parameters estimation, which does not require sensing the sway angle signal, but requires measuring the scheduling variables, such as rope length and mass of a payload, which correspond to the operating conditions. Assuming, that operating conditions vary within the known range of scheduling variables, the parameters of a crane dynamic model can be interpolated by a quasi-linear fuzzy model. Hence, a crane dynamic is approximated through interpolation between a set of local linear models determined through identification experiments at the local operating points selected within the bounded intervals of scheduling variables  $w_i \in [w_i^-, w_i^+]$  (where  $i = 1, 2$ , and  $w_1 = l$ ,  $w_2 = m$ ). Applying the P1-TS fuzzy theory proposed in [12, 13], a fuzzy quasi-linear interpolator can be developed by dividing an each interval  $[w_i^-, w_i^+]$  into  $n_i$  subintervals  $[\beta_{i,j}, \beta_{i,j+1}]$  (where  $\beta_{i,j} < \beta_{i,j+1}$ , and  $j = 1, 2, \dots, n_i$ ), that leads to obtain  $n_1 \cdot n_2$  fuzzy interpolation regions. For each interval, the linear membership functions are defined as follows:

$$N_{i,j}(w_i) = \frac{\beta_{i,j+1} - w_i}{\beta_{i,j+1} - \beta_{i,j}}, \quad P_{i,j}(w_i) = 1 - N_{i,j}(w_i). \quad (8)$$

The estimates of parameters are interpolated according to the formula:

$$\hat{\theta} = [\hat{a}_0, \hat{b}_0, \hat{c}_1, \hat{c}_2, \hat{d}_1, \hat{d}_2]^T = g^T \Omega Q_k, \quad (9)$$

where  $g$  and  $\Omega$  are called generator vector and fundamental matrix [12], respectively, which can be determined recursively, for  $i = 1, 2$  according to (10) starting from the initial generator  $g_0 = 1$  and fundamental matrix  $\Omega_0 = 1$ :

$$g_i = \begin{bmatrix} 1 \\ w_i \end{bmatrix} \otimes g_{i-1}, \quad (10)$$

$$\Omega_i = \frac{1}{\beta_{i,j+1} - \beta_{i,j}} \begin{bmatrix} \beta_{i,j+1} & -\beta_{i,j} \\ -1 & 1 \end{bmatrix} \otimes \Omega_{i-1},$$

where  $\otimes$  denotes the Kronecker product, and  $Q_k$  (where  $k = 1, 2, \dots, n_1 \cdot n_2$ ) is the matrix of model's parameters identified at operating points corresponded to the lower and upper bounds of interpolation intervals  $[\beta_{i,j}, \beta_{i,j+1}]$ .

### 3. Predictive control scheme

The GPC procedure developed by Clarke [4] is adapted for the anti-sway crane predictive strategy. The GPC procedure uses a CARIMA (Controlled Auto-Regressive and Integrated Moving-Average) model. The discrete-time models (1) and (2) are rewritten to the form:

$$A_1(z^{-1})x(t) = B_1(z^{-1})u(t-1) + \xi_1(t)/\Delta, \quad (11)$$

$$A_2(z^{-1})\alpha(t) = B_2(z^{-1})u(t-1) + \xi_2(t)/\Delta, \quad (12)$$

where  $x(t) = T_s v_x(t)/\Delta$ ,  $\Delta = 1 - z^{-1}$ ,  $T_s$  is a sample time,  $\xi_1$  and  $\xi_2$  are the uncorrelated random sequences, and the polynomials in the backward shift operator are related to (1) and (2) as follows:

$$\begin{aligned} A_1(z^{-1}) &= \Delta A(z^{-1}), \\ A_2(z^{-1}) &= A(z^{-1})C(z^{-1}), \\ B_1(z^{-1}) &= T_s B(z^{-1}). \\ B_2(z^{-1}) &= B(z^{-1})D(z^{-1}). \end{aligned} \quad (13)$$

The objective function to be minimized is formulated as the weighted sum of squared error of crane position (difference between the reference signal  $x_r$  and the crane position) and payload sway angle over the prediction horizon ( $N_p$ ), and the control increments within the control horizon ( $N_u$ ):

$$\min J = \sum_{j=1}^{N_p} (\hat{x}(t+j) - x_r(t+j))^2 + \sum_{j=1}^{N_p} \lambda_{1,j} (\hat{\alpha}(t+j))^2 + \sum_{j=1}^{N_u} \lambda_{2,j} (\Delta u(t+j-1))^2, \quad (14)$$

where  $\lambda_1$  and  $\lambda_2$  are the weighting coefficients and the  $j$ -step ahead predictors are derived from:

$$\hat{x}(t+j) = G_1(z^{-1})\Delta u(t+j-1) + F_1(z^{-1})x(t), \quad (15)$$

$$\hat{\alpha}(t+j) = G_2(z^{-1})\Delta u(t+j-1) + F_2(z^{-1})\hat{\alpha}(t), \quad (16)$$

where  $G_1(z^{-1})$ ,  $G_2(z^{-1})$ ,  $F_1(z^{-1})$  and  $F_2(z^{-1})$  are the polynomials recalculated through recursion of the Diophantine equation.

According to [4], the vectors of optimal output predictions can be formulated as follows:

$$\hat{\mathbf{x}} = \mathbf{G}_1 \tilde{\mathbf{u}} + \mathbf{f}_1, \quad (17)$$

$$\hat{\boldsymbol{\alpha}} = \mathbf{G}_2 \tilde{\mathbf{u}} + \mathbf{f}_2. \quad (18)$$

Taking into account the constraints for control signal ( $u_{min}$ ,  $u_{max}$ ) and sway angle of a payload, the cost function can be rewritten as:

$$J = (\mathbf{G}_1 \tilde{\mathbf{u}} + \mathbf{f}_1 - \mathbf{x}_r)^T (\mathbf{G}_1 \tilde{\mathbf{u}} + \mathbf{f}_1 - \mathbf{x}_r) + \lambda_1 (\mathbf{G}_2 \tilde{\mathbf{u}} + \mathbf{f}_2)^T (\mathbf{G}_2 \tilde{\mathbf{u}} + \mathbf{f}_2) + \lambda_2 \tilde{\mathbf{u}}^T \tilde{\mathbf{u}} + \mathbf{v}_1^T (\tilde{\mathbf{u}}_{max} - \tilde{\mathbf{u}}) + \mathbf{v}_2^T (\tilde{\mathbf{u}} - \tilde{\mathbf{u}}_{min}), \quad (19)$$

where:

$$\tilde{\mathbf{u}}_{min} = \max(u_{min} \mathbf{I} - z^{-1} \mathbf{u}, -\mathbf{G}_2^{-1} (\alpha_{max} \mathbf{I} + \mathbf{f}_2)),$$

$$\tilde{\mathbf{u}}_{max} = \min(u_{max} \mathbf{I} - z^{-1} \mathbf{u}, \mathbf{G}_2^{-1} (\alpha_{max} \mathbf{I} - \mathbf{f}_2)),$$

and  $\mathbf{v}_1$  and  $\mathbf{v}_2$  are the Lagrangian multiplier vectors. The optimization problem can be solved using the Lemkes algorithm [3] for the Kuhn-Tucker complimentary conditions formulated as:

$$\begin{cases} \nabla J(\tilde{\mathbf{u}}) = \mathbf{0}, \\ \mathbf{v}_1^T (\tilde{\mathbf{u}}_{max} - \tilde{\mathbf{u}}) = \mathbf{0}, \\ \mathbf{v}_2^T (\tilde{\mathbf{u}} - \tilde{\mathbf{u}}_{min}) = \mathbf{0}, \\ \mathbf{v}_1, \mathbf{v}_2 \geq \mathbf{0}. \end{cases} \quad (20)$$

#### 4. Experiments on a laboratory stand

The proposed control techniques was tested on a laboratory scaled overhead crane equipped with DC motors, and incremental encoders used for sensing the position of crane and sway angle of a payload. The control algorithm was implemented on PAC with RX3i controller using the structured text, and the measurement system was completed with the PC equipped with the PLC1710HG measurement card.

The objective of the control was positioning the crane to  $x_r = 1$  m and reducing the payload deflection within the tolerance  $\pm 0.02$  m, where the payload deflection was approximated as a product of rope length and sway angle of a payload ( $l\alpha$ ). The one-step ahead prediction strategy was tested with the experimentally selected weighting coefficients  $\lambda_1 = 2.4$ ,  $\lambda_2 = 0.3$ , sample time  $T_s = 0.1$  s, and for the control signal range  $-10 \leq u(t) \leq 10$ , while the payload deflection in the transient state was constrained to  $l\alpha_{max} = +/- 0.12$  m.

The two predictive control strategies were compared: (i) GPC with the RLS estimation with forgetting factor  $\mu = 0.99$ , (ii) GPC with the P1-TS fuzzy interpolator used to estimate the parameters of the crane dynamic model. In the first approach, the incremental encoder was used as a source of the sway angle feedback signal, while in the second case the model-based estimation of sway angle of a payload is applied. Hence, in the second approach, the P1-TS fuzzy model is used to interpolate the parameters within the range of scheduling variables:  $l = [1.0, 2.2]$  m and  $m = [10, 90]$  kg. To find the linear models of a system, the identification experiments were carried out at the operating points: (1.0 m, 10 kg), (1.6 m, 10 kg), (2.2 m, 10 kg), (1.0 m, 90 kg), (1.6 m, 90 kg), (2.2 m, 90 kg). Thus, the interpolation intervals with the linear membership functions (7) were set as  $[\beta_{1,1}, \beta_{1,2}] = [1.0, 1.6]$  m,  $[\beta_{1,2}, \beta_{1,3}] = [1.6, 2.2]$  m, and  $[\beta_{2,1}, \beta_{2,2}] = [10, 90]$  kg.

Figures 2 and 3 present the comparison between the GPC-RLS and GPC-P1TS methods. The experiments were conducted for rope lengths  $l = \{1.3, 1.9\}$  m and mass of a payload  $m = 50$  kg.

Both control techniques show similar performances, which proved the robustness of the proposed GPC system both, using the RLS method and P1-TS fuzzy interpolator. The settling time is between 4.9 and 5.0 seconds, and the payload deflection in transient states does not exceed the limit of  $\pm 0.12$  m. The GPC scheme ensures fast positioning of a payload and sway suppression within the assumed tolerance of a payload deflection  $\pm 0.02$  m. The both methods, the GPC-RLS (with sensor feedback of payload deflection) and GPC-P1TS (sensorless approach with the P1-TS fuzzy interpolator adapted to estimate the crane's dynamic model parameters) proved robustness against the operating conditions variation.

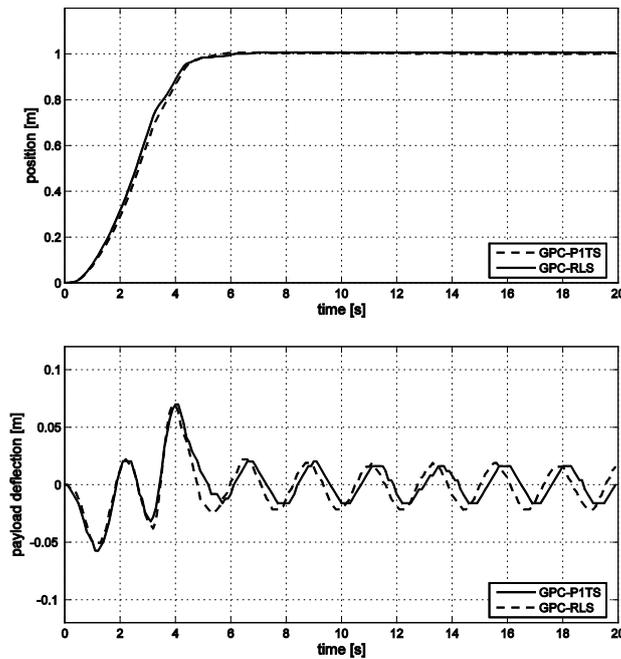


Fig. 2. Crane position and payload deflection – experiments for  $l = 1.3$  m and  $m = 50$  kg – comparison between GPC with RLS and P1-TS fuzzy estimation

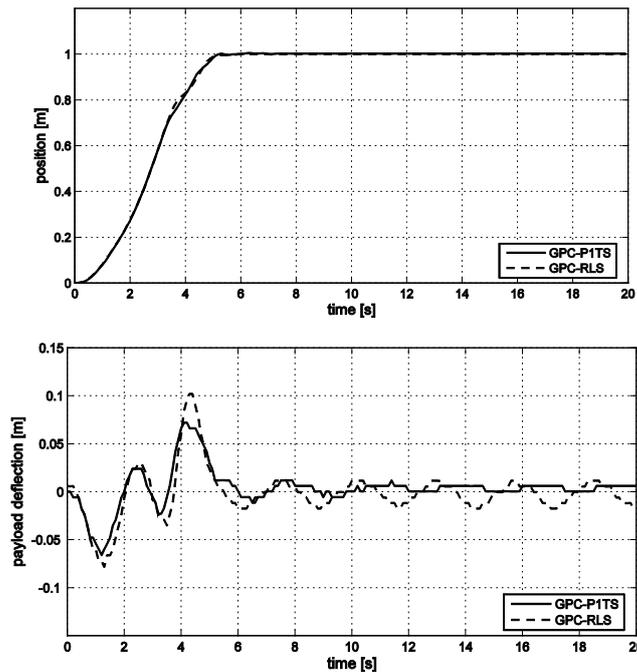


Fig. 3. Crane position and payload deflection – experiments for  $l = 1.9$  m and  $m = 50$  kg – comparison between GPC with RLS and P1-TS fuzzy estimation

## 5. Conclusions

The GPC-based control schemes coupled with the RLS estimation algorithm and fuzzy interpolation of the parameters of an LPV discrete time crane dynamic model are compared in the article. The robust control technique is developed with respect to the constraints on sway angle of a payload and control signal. The experiments carried out on a laboratory scaled of an overhead crane confirmed effectiveness and feasibility of the proposed solutions. The implementation of control systems was performed using the PAC system with RX3i controller. The GPC-RLS control approach was realized with using the incremental encoder utilized in the sway angle measurement system. In the sensorless control, approach (GPC-PITS) the sway angle is estimated by using the discrete time pendulum model of a crane with parameters estimated by the P1-TS fuzzy interpolator. The series of experiments carried out for different operating points proved robustness of the control approaches presented in the article.

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