

## PROPERTIES FLOW WORKING FLUID THROUGH A HYDRAULIC MICROGAP

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### Abstract

*This paper presents description the mathematical of a phenomenon accompanying the flow working fluid through a microgap. The laminar flow of liquids in crevices of smooth hydraulic resistance and theoretical model for the distribution of pressure in the gap and the flow rate through the slot hydraulic is presented. The theoretical models for the distribution of pressure and flow in the microgap on conventional hydraulic resistance of hydraulic joints have shape related to the errors of their execution. In deriving the theoretical models, by introducing a variable height of the gap in the initial episode was founded stream velocity profile variability in the hydraulic fluid retaining gap and zero values of local losses at the entrance to the slot. Diphase flow of liquid through the microgap is described. The colmatage process effect on the diphase liquid flow through the microgap is treated as that of the uniphase liquid flow through the variable geometric structure area. With assumptions, the colmatage process is defined as a stochastic Markov process. The particles being trapped in the microgap divide its area on separate segments. Their number and characteristics dimensions are interpreted as a random vector in space with a variable number and dimensions.*

**Keywords:** aviation, hydraulic drive, hydraulic resistance gap, fluid pressure, fluid flow, the gap

### 1. Introduction

Working fluid in fluid systems serves as an energy carrier and at the same time is a lubricating agent between moving elements often subject to substantial unit pressures or substantial relative velocities. It causes a change in the basic characteristic of hydraulic device machine such as the required linearity, assumed hysteresis and the leak to the outside or the working stability.

For many reasons a working fluid contains impurities leading to disturbances of normal operation or the wear of system parts [4, 5, 10, 16]. Experiments show that the presence of impurities in working fluids is the cause of material destruction due to friction, abrasion of material working surfaces and erosive wear of element surfaces [7, 10, 14, 15].

During lab research and operation experience with fluid-system based devices and machines it was established that using working fluids comprising a certain concentration of impurities causes a phenomenon of a gradual decrease in the active cross-section of a gap resulting from the absorption of particles dispersed in a fluid by gap walls. The research conducted at the Air Force Institute of Technology conformed the thesis that leaks through a gap vary with time as if the phenomenon of contamination of a gap with hard particles occurred [12, 13]. When fluid flows through a microgap, the phenomenon of stopping of the particles suspended in this gap and a change of the efficient flow surface of the gap occurs. In some cases, with time, fluid completely stops leaking through a microgap.

Phenomena associated with the flow of working fluids through microgaps are not yet sufficiently researched. So far, only few works dealing with this problem were published [1-3, 6].

The subject of the following study is the analysis and description of the flow of fluids through a microgap regarded as a hydraulic resistance element. Basic equations describing the motion of Newtonian fluid the hydraulic retaining slot due to three main principles of mechanics [8, 9, 11]:

the principle of conservation of mass (continuity equation), the principle of conservation of momentum and angular momentum and the principle of conservation of energy. These equations are not closed system. Therefore, they should be supplemented by additional equations expressing the state of density, viscosity and thermal conductivity as a function of pressure and temperature, as well as on the field of mass forces unit. The general solution of these equations, it is not known, and the designation of their solutions, which are functions of four independent variables and subject to specified initial conditions and boundary, encounters great difficulties. For the description of the flow of working fluids, fluid parameters of a dimensional span of suspension particles and the mechanism responsible for variations of the flow characteristic of a microgap as observed in daily practice were taken into account. Observations of phenomena taking place, when a fluid flows through a microgap, allows us a statement that the flow process of a fluid through a gap and stopping of hard particles in a gap should be treated as a stochastic one and be analysed from a probability viewpoint.

The results of the following analysis can serve to study the influence of impurities contained in a fluid on the change of basic characteristic parameters of a hydraulic device and those of an entire fluid system.

## 2. Assumptions for a physical model of the flow of working fluid through a microgap

In order to construe the process of fluid flow through a gap a physical model of the flow phenomenon the following assumptions was adopted.

- 1) An ordered pair of real numbers where one number is the value of the volumetric flow rate of fluid  $\theta$  flowing through a discussed gap and the other number is the pressure difference  $\Delta p$  across an element, will be called the hydraulic state of a microgap. The flow rate of a fluid flowing through a gap can be presented as a sum of volumetric unitary flow rates flowing through component ranges a gap is divided into, i.e. [2]:

$$Q = K_u^b \sum_{i=1}^N h_i^K, \quad (1)$$

where:

$K$  – coefficient depending on flow conditions and gap geometry, i.e. the coefficient taking into account the pressure drop, flow length, fluid viscosity and the geometrical gap length,

$h$  – gap height,

$N$  – lowest number of particles of an assumed size, indispensable completely to block the gap,

$u$  – flow-through length of a microgap,

$b$  – shape deformation factor.

- 2) In a hydro energetic element, a gap with stiff, impermeable and parallel walls is generated. This gap dimensions are as follows:  $l_o$  – microgap width in the direction perpendicular to the fluid flow,  $h$  – microgap height,  $m$  – geometrical length of a gap along the fluid flow as these dimensions. By our intuition we assume that every particle larger than size  $h$  of the gap is stopped in it, thus reducing the active flow width of the gap by the value of  $h$ .
- 3) A microgap is regarded a hydraulic resistance element. Seen from the aspect of energetic transformations a microgap is classed as a dissipative element. A characteristic of a gap seen as such can presented as:

$$\theta = G \Delta p, \quad (2)$$

where:

$\theta$  – fluid flow rate in the gap,

$G$  – gap conductivity,

$\Delta p$  – pressure drop across the gap.

- 4) A change of the gap state at a steady pressure drop during the working fluid flow through it can

only result from stopping solid particles dispersed in fluid on inlet edges of the gap exclusively due to different geometrical dimensions of particles and distances between the gap edges. A change of the microgap state at a steady pressure drop is a result of a change of the conductivity, this being a function of, among other things, the number, arrangement and properties of particles stopped across it. The conductivity of a microgap will change; it will decrease due to the effect of the gap blocking, then and only then, when there is a time interval  $\Delta t$  within an observation interval  $(0, t)$ , over which the probability of occurring solid particles of dimensions greater than the gap height  $h$  in a fluid is  $\neq 0$ ; we may express it as follows:

$$G_t^{\Delta p} \neq G_{t+\Delta t}^{\Delta p} \Leftrightarrow \bigcup_{\Delta t \in (0, T)} \bigcup_h \left[ \int_0^h f(x, t) dx (1) \right]. \quad (3)$$

- 5) At any moment at  $t > 0$  the most one solid particle is added to the gap and the sequence of moments of particle arrivals to the gap has no accumulation point.
- 6) At every moment ranges a microgap is divided into are of equal length  $m$ , and their cross-section is rectangular, with  $m > 0$  corresponding to the flow length of the gap. At the  $t > 0$  moment the gap cross-section is  $h \times l_0$ .
- 7) Every particle arriving at the gap arrives precisely at one of the ranges the gap was divided into before the particle arrival. The capacity of arresting by a singular range of particles contained in a working fluid is clearly defined by the characteristic dimension  $x$  of an arriving particle, where  $x > 0$ .
- 8) A single range of the  $h \times l_0$ ,  $h > 0$ ,  $l_0 > 0$  cross-section arrests an arriving particle with the characteristic dimension  $x$ , then and only then, if  $x \geq \min\{h, l_0\}$ . In case  $x \geq l_0$  a range is liquidated, and if  $h \leq x < l_0$  a range is divided into two sections with dimensions  $h \times l_1$  and  $h \times l_2$ , where  $l_1 + l_2 + x = l_0$  [10].
- 9) The flow rate through a single range with the  $h \times l$  cross-section and length  $m$  is equal to the stationary, steady-state flow through a rectangular orifice  $h \times l$  in a parallel-walled plate with the thickness  $m$ ,  $m \geq 0$ ,  $h > 0$ ,  $l > 0$ , where  $m$  and  $h$  are by assumption parameters characterizing the gap geometry. At any moment, the flow rate through a gap is equal to the sum of flow rates through individual ranges the gap at a given moment is into divided.

### 3. Pressure distribution and flow rate of liquids working gaps in typical hydraulic resistance

The flow of hydraulic fluid through a slot bearing the author has treated as a stationary flow between two flat and parallel plates located from each other at such a distance that they form a hydraulic bearing gap. As is known, the length of the initial section of the hydraulic retaining slots in which there is a stabilization of the flow (velocity distributions in any cross-sections are identical) is proportional to the Reynolds number and height of the hydraulic retaining slots. Due to the fact that the initial gap distance of the liquid velocity, profile in the gap is not formed by the reflection on the speed of the liquid in the initial section of the slot gap height change.

Taking into account the change of the flow channel (the hydraulic retaining slots) dependence on the flow of hydraulic fluid in the retaining slot has the form [8, 12]:

$$Q = \frac{5\Delta p}{8\mu m} \frac{4}{h_1^2 - l_1^2} \int_0^{h_1} \int_0^{l_1} (h_1^2 - y^2)(l_1^2 - x^2) dx dy = \frac{5}{72} \frac{\Delta p}{\mu m} \frac{l_s^3 h^3}{l_s^2 + h^2} \quad (4)$$

and dependence on the pressure distribution in the gap has the form [1]:

$$p(x, z) = \int_{\delta_1}^{\delta_2} x^i \left[ x_2 - \delta_1^2(z) - \frac{[\delta_2^2(z) - \delta_1^2(z)][F(x) - F(\delta_1)]}{F(\delta_2) - F(\delta_1)} \right] dx + G, \quad (5)$$

Integral constants E and G are determined using the continuity of velocity boundary conditions on the stationary walls of the form:  $u(\delta_1, z) = u(\delta_2, z) = 0$  and  $w(\delta_1, z) = w(\delta_2, z) = 0$  and pressure  $p(z)$  satisfying the conditions:  $p(x, 0) = p_0$  and  $p(x, m) = p_1$ . Error of the approximations, with  $l_s / h \rightarrow \infty$ , is 16%.

Entering into formulas (Eq 4) and (Eq 5) boundary conditions was determined depending on the pressure distribution in the gap and the fluid flow through the gap for typical hydraulic retaining slots. Shapes of typical hydraulic retaining slots are associated with the errors of their execution.

For hydraulic flat retaining slots of non-parallel walls schematically shown in Fig. 1a the distribution of pressure in the gap model is described:

$$p(x, z) = p_0 - \frac{\left[1 + \left(\frac{H-h}{h}\right)\right]^2}{2 + \left(\frac{H-h}{h}\right)} \frac{2\frac{z}{m} + \left(\frac{z}{m}\right)\left(\frac{H-h}{h}\right)}{\left[1 + \frac{z}{m}\right]^2} \Delta p \quad (6)$$

and fluid flow in the gap model:

$$Q \cong \frac{l_s h^3 \Delta p}{12 \mu m} \left[1 + \frac{3}{2} \left(\frac{H-h}{h}\right) + \left(\frac{H-h}{h}\right)^2 + \frac{1}{4} \left(\frac{H-h}{h}\right)^3\right]. \quad (7)$$

For these patterns can be introduced, dimensionless coefficient related to the geometry of the slot ( $l_s, H, h, m$ ) of the form:  $k_1 = \frac{H-h}{h}$ .

For hydraulic flat retaining slots of non-parallel walls schematically shown in Fig. 1b the distribution of pressure in the gap is described by the formula:

$$p(x, z) = p_0 - \frac{\left[1 + k_2 \frac{x}{m}\right]^2}{2 + 2k_2 \frac{x}{m}} \frac{2\frac{z}{m} + 2\left(\frac{z}{m}\right)\left(\frac{x}{m}\right)k_2}{\left[1 + k_2 \frac{x}{m}\right]^2} \Delta p \quad (8)$$

and fluid flow in the gap model:

$$Q \cong \frac{l_s h^3 \Delta p}{6 \mu m} \left[\frac{(1+k_2)^2}{2+k_2}\right], \quad (9)$$

where:  $k_2 = \frac{h-H}{h}$ .

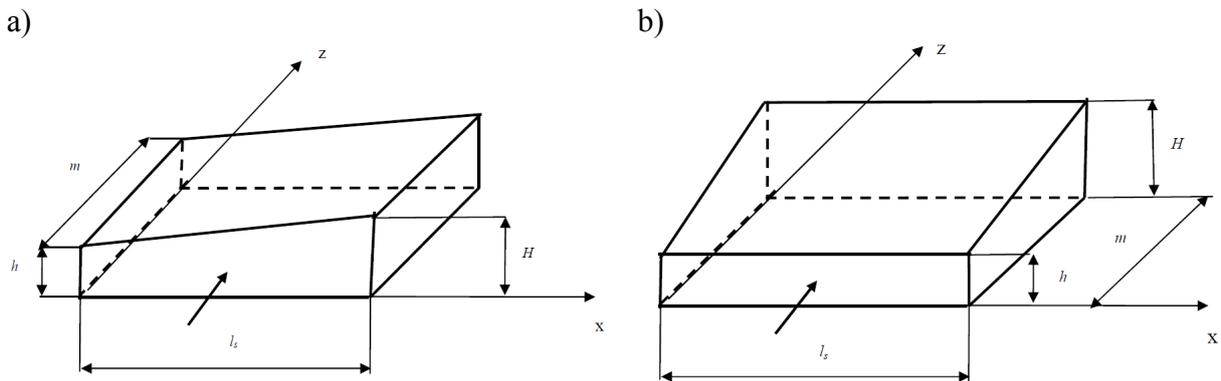


Fig. 1. Schematic hydraulic flat retaining slots of non-parallel walls

For the transverse annular, hydraulic retaining slots (front slit formed by two parallel plates round) schematically shown in Fig. 2 the pressure distribution in the gap model are described:

$$p(r) = p_0 - \frac{3\mu Q}{4\pi h^3} \ln \frac{r_1}{r_2} + \frac{3Q^2}{80\pi h^2} \left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right) \quad (10)$$

and fluid flow in the gap model:

$$Q \cong \frac{\pi r_1 h^3 \Delta p}{6 \mu m} \frac{1}{\sum_{n=0}^{2r_1} (-1)^n \frac{1}{n-1} \left(\frac{m}{r_1}\right)^n} \quad (11)$$

For the longitudinal eccentric annular retaining slot, hydraulic schematic shown in Fig. 3 the distribution of pressure in the gap model is described:

$$p(r) = p_0 \quad (12)$$

and fluid flow in the gap model:

$$Q \cong \frac{\pi r_1 (r_2 - r_1)^3 \Delta p}{6 \mu m} \quad (13)$$

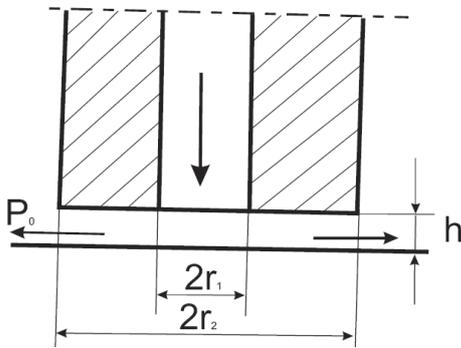


Fig. 2. Leading hydraulic resistance gap formed by two parallel circular plates

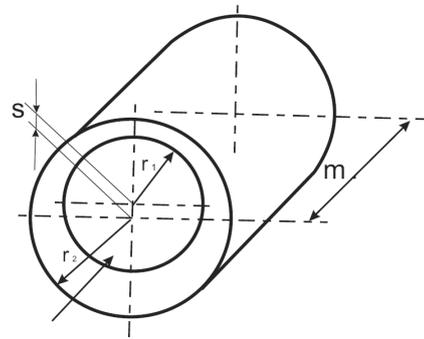


Fig. 3. Hydraulic longitudinal annular gap eccentric resistance

Although these models are designated with the linearized system of equations, using them, small ranges of variation in the flow cross the gap and small Reynolds numbers is warranted.

#### 4. Interpretation of the working fluid flow as a two-phase mix through a microgap as the step stochastic Markov process

The observation of phenomena taking place when a fluid flows through a microgap allows us to establish that the working fluid flow process and arresting hard particles in a microgap should be regarded as a stochastic one and discussed under a probability category.

We assume that the  $E_i$  state is a state in which at the  $t$  moment in a volume unit there are  $i$  particles arrested and which after a  $\Delta t \rightarrow 0$  time transforms into the  $E_{i+1}$  state. The probability of transition from the  $E_i$  state into the  $E_{i+1}$  state after the  $\Delta t$  time interval is given by the formula:

$$P_{i+1} = \lambda(i) \Delta t + Q(\Delta t), \quad (14)$$

where  $\lambda(i)$  is a function of the intensity of arresting particles in a microgap and  $\lim_{\Delta t \rightarrow 0} \frac{Q(\Delta t)}{\Delta t} = 0$ , meaning that the probability of transition of more than one particle into the  $E_{i+1}$  state approaches zero faster than  $\Delta t$ .

According to assumptions for the physical model, the following mathematical model for the flow of working fluid as a two-stage mix through a microgap was adopted.

The microgap state at any given moment  $t > 0$  is defined clearly by the system of random variables:

$$S(t) = \{R(t), L_1(t), \dots, L_{R(t)}(t)\}, \quad (15)$$

where correspondingly:

$R(t)$  – the number of component ranges the gap at the  $t$  moment is divided into,

$L_i(t)$  – the dimension of a rectangular range and of that range along the microgap for  $i = 1, 2, \dots, R(t)$ .

We assume that for  $R(t) \in \{0, 1, 2, \dots\}$ ,  $0 < L_i(t) < L_0$ ,  $L_0 < \infty$  and for every state of the gap  $s = \{r, l_1, l_2, \dots, l_r\}$  the fluid flow rate through individual ranges is defined with a probability level = 1 as:

$$Q_r = \{Q(l_1), Q(l_2), \dots, Q(l_r)\}, \quad (16)$$

where  $Q(l_i)$  is a function determining the flow rate through a singular range the characteristic dimension of which is given by the variable  $l_i$ , such, that a singular range of the  $h \times l_0$ ,  $h > 0$ ,  $l_0 > 0$  cross-section stops an arriving particle with the  $x$  dimension, then and only then, if  $x \geq \min\{h, l_0\}$ , and when  $x \geq l_0$  a range is liquidated and when  $h \leq x < l_0$  a range is divided into two ranges of the dimension  $h \times l_1$  and  $h \times l_2$ , where  $l_1 + l_2 + x = l_0$ . The  $Q(l_i)$  function determining the flow rate through a singular range is a continuous function and closely increasing, i.e.  $Q(0) = 0$ ;  $\lim_{L \rightarrow \infty} Q(L) \rightarrow \infty$ . The fluid flow rate through a singular range can be calculated from the following formula [3]:

$$Q = \frac{5\Delta p}{8\mu m} \frac{4}{h_1^2 - l_1^2} \int_0^{h_1} \int_0^{l_1} (h_1^2 - y^2)(l_1^2 - x^2) dx dy = \frac{5}{72} \frac{\Delta p}{\mu m} \frac{l_0^3 h^3}{l_0^2 + h^2}. \quad (17)$$

Let us assume the stochastic process  $S(\cdot) = \{S(t); t > 0\}$  is the homogeneous Markov process. Therefore, a probability structure of the  $S(\cdot)$  process is defined by a transition probability function, which describes the way that the changes of state occur. The probability structure of the  $S(\cdot)$  process is therefore given by the transition probability function describing how the changes of state occur. We designate the conditional distribution of random variables  $\{R(t), L_1(t), \dots, L_{R(t)}(t)\}$  as  $P_t^{(r)}(dl_1, dl_2, \dots, dl_r / l'_1, l'_2, \dots, l'_r)$ , however, on condition that  $R(0) = r' \geq 0$  and  $L_i(0) = l'_1, l'_2, \dots, l'_r(0) = l'_r$  over the  $\{R(t) = r\}$ ;  $r \geq 1$  subset. Talking about the  $r$  of random variables, we assume them determined in the same probability space and that their joint conditional distribution exists. In this case,  $L_k$  can be interpreted as variables in the space of samples. Without doubt, the fact that the changes of states are accompanied by the change of the number of dimensions of the space where random variables are determined is an aggravation of a notation for the discussed process. Of course, for the  $A$  subset contained in the space of samples the following notation:

$$P_t^{(r)}(A / l'_1, \dots, l'_r) = \int_A P_t^{(r)}(dl_1, \dots, dl_r / l'_1, \dots, l'_r), \quad (18)$$

is the probability of the  $R(t) = r$  occurrence and  $\{[L_1(t), \dots, L_r(t)] \subset A\}$  on condition that  $L_i(0) = l'_i$  for  $i = 1, 2, \dots, r'$ ,  $R(0) = r'$ , where  $r' \geq 0$ . For  $r = 0$   $P_t^{(0)}(A / l'_1, \dots, l'_r)$  is the conditional probability of the  $\{R(t) = 0\}$  occurrence, on condition that  $L_i(0) = l'_i$  for  $i = 1, 2, \dots, r'$ ,  $R(0) = r'$ .

On account of the assumption of the homogeneity of the transition function for the Markov process, the stochastic process  $S(\cdot)$  is clearly defined by the conditional distribution of random variables  $\{R(t), L_1(t), \dots, L_{R(t)}(t)\}$ . Let us assume that:

$$P_t^{(r)}(dl_1, \dots, dl_r / l'_1, \dots, l'_r) = \sum_{r_1} \prod_{k=1}^{r'} P_t^{(r_k)}(dl_{j_{k-1}}, \dots, dl_{j_k} / l'_k)(r_1, \dots, r_r), \quad (19)$$

$$r_1 + r_2 + \dots + r_{r'} = r; \quad j_0 = 0, \quad j_k = \sum_{i=1}^k r_i \text{ for } k = 1, \dots, r'.$$

$$\text{For } r = 0 \quad P_t^{(r)}(dl_1, \dots, dl_r / l'_1, \dots, l'_r) = \begin{cases} 1 & \text{for } r = 0, \\ 0 & \text{for } r = 1, 2, \dots, \end{cases} \quad (1 \text{ for } r = 0 \text{ and } 0 \text{ for } r = 1, 2, \dots).$$

The equation (19) means that  $r$  microgaps of dimensions  $l_1 \dots l_r$  are generated due to a stochastically independent division of each of the  $r'$  components of the connectivity with  $l'_1, \dots, l'_r$  dimensions into  $r_1, \dots, r_{r'}$  parts, where  $r_1 + r_2 + \dots + r_{r'} = r$ .

Making an assumption that during a flow particles are fully carried by the working fluid until they are possibly arrested, the transition probability functions of the change of state are as follows:

$$\begin{aligned} P_t^{(2)}(dl_1, dl_2 / l) &\cong NQ(l) t P_{podz}(l), \\ P_t^{(0)}(dl / l) &\cong NQ(l) t P_{likw}(l), \\ P_t^{(1)}(dl_1 / l) &\cong 1 - [P_t^{(2)}(dl_1, dl_2 / l) - P_t^{(0)}(dl / l)], \\ P_t^{(r)}(dl_r / l) &\cong 0 \quad \text{for } r = 3, 4, \dots, \end{aligned} \quad (20)$$

where  $N$  is the particle concentration in a working fluid and:

$$P_{podz}(l) = \begin{cases} F_x(l) - F_x(h) & \text{for } l > h, \\ 0 & \text{for } l \leq h, \end{cases} \quad (\text{for } l > h, \text{ for } l < \text{ or } = h), \quad (21)$$

$$P_{likw}(l) = 1 - F_x(l) \quad \text{for } l > 0.$$

$F_x$  denotes a cumulative distribution function of the random variable  $X$  corresponding to the characteristic dimension  $x$  of hard particles in a working fluid. To determine the properties of a step (discontinuous) process, where within small time intervals the system described by this process will certainly remain in its initial state or with a small probability, will change onto another state, it should be assumed that:

$$P_t^{(1)}(dl_1 / l) \xrightarrow{t \rightarrow 0} \delta_l, \quad (22)$$

where  $\delta_l$  designates a deterministic distribution concentrated at the point  $l$  for  $l > 0$ , and that:

$$\frac{P_t^{(2)}(dl_1, dl_2 / l)}{P_t^{(2)}(dl_1, dl_2 / l)} \xrightarrow{t \rightarrow 0} \xi(dl_1, dl_2), \quad (23)$$

where the left side represents a joint conditional distribution of random variables  $L_1(t), L_2(t)$  on condition that the number of component ranges a gap at the  $t$  moment is divided into is  $R(t) = 2$ ,  $R(0) = 1$ ,  $L(0) = l_1$ , and  $\xi(dl_1, dl_2)$  being the distribution of the variables  $L_1, L_2$ , which with the probability of 1 fulfil the condition:  $L_1 + L_2 = \max \{L - X_h, 0\}$ , where  $X_h$  designates a random variable having a cumulative distribution function:

$$F_{X_h}(x) = \begin{cases} 1 - \frac{F_x(l) - [F_x(x) \wedge F_x(l)]}{F_x(l) - F_x(h)} & \text{for } x > h, \\ 0 & \text{for } x \leq h \end{cases} \quad (24)$$

and within the  $(0, l - h)$  range the conditional distribution  $L_1$  will be unvarying.

The conditions (22) and (23) imply that a singular connectivity component of the characteristic

dimension  $l$  for a sufficiently small time  $t$  will remain unvarying with a probability close to 1 inside the  $(0, t)$  interval, however, if for a sufficiently short time  $t$ , even only once within the range  $(0, t)$  the gap divides, then it will be exactly one division into two parts. It will be a division with the conditional probability equal to 1, one achieves after placing a section of the random length  $X_h$  that have the  $F_{X_h}$  cumulative distribution function in the  $(0, l - X_h)$  range with an unvarying distribution of the position of the left end of this section. Please note that the  $F_{X_h}$  is a cumulative distribution function of the conditional distribution  $X$  on condition that  $l > x > h$ , i.e. on condition that a particle will be arrested due to its characteristic size being too large with respect to the gap height:  $x > h$ , however, too small to annul this connectivity component:  $x < l$ .

The Markov's condition imposes that the conditional distribution of random variables  $L_1(t), L_2(t)$  performs the following equation:

$$P_{t+s}^{(r)}(dl_1, \dots, dl_r / l'_1, \dots, l'_r) = \sum_{r'=0}^{\infty} \iint_{(l'_1, \dots, l'_r) \in L_k} P_t^{(r)}(dl_1, \dots, dl_r / l'_1, \dots, l'_r) P_s^{(r')} (dl'_1, \dots, dl'_r / l''_1, \dots, l''_r). \quad (25)$$

From the equation (15) follows that for every cumulative distribution function  $F_X$  and for every closely increasing function of the flow  $Q$  there is only one family of real measures  $P_t^{(r)}(dl_1, \dots, dl_r / l'_1, \dots, l'_r)$ ,  $r \geq 0, l'_1 > 0, \dots, l'_r > 0$ . In addition, the following equality is served:  $\sum_{r=0}^{\infty} P_t^{(r)}(dl_1, \dots, dl_r / l_1, \dots, l_r) = 1$ .

We call the Markov's process with transition probabilities described by the equation (25) and with a probability equal to 1 serving one initial condition  $S(0) = \{l, l_0\}$  a 'probabilistic model' of the working fluid flow through a gap  $l_0 > 0$  wide,  $h > 0$  high and  $m \geq 0$  long.

In order to assign values of the flow rate  $Q$  of the fluid flowing under the constant pressure drop  $\Delta p$  to successive states of the gap one must use the formula (17). After mathematical transformations, the formula (17) for the flow rate  $Q$  appears as:

$$Q(t) = Q_0 \exp \left\{ - Q_0 \frac{N \left( h + \frac{1}{\chi} \right) e^{-\kappa}}{L_0} t \right\}, \quad (26)$$

where  $Q_0 = \frac{5L_0 h^3 \Delta p}{72 \mu m}$  is the flow rate through a flat microgap at  $t = 0$ ,  $\chi$  is a parameter of assumed

exponential distribution of the process, and  $\kappa$  is a dimensionless parameter defining relations between the gap height  $h$  and the  $\chi$  parameter.

The relation (26) points to an exponential character of the working fluid flow process through a microgap as a time function. With the formula (26), one is able to determine the relation between the volume of the fluid flowing through a gap and the flow process time.

#### 4. Conclusions

The main goal of this article was to present the analysis of the process of fluid flowing through a microgap. The phenomenon of working fluid flowing through a microgap was treated as a flow in hydraulic elements taking into account parameters of a dimensional distribution of suspended particles and mechanisms responsible for the practically experienced varying flow characteristic of a microgap.

In the presented formulas for calculating pressure distribution and fluid flow in a typical hydraulic retaining slots do not take into account the impact of the initial segment, which forms a laminar velocity profile and the local losses are included only in the specified empirically pressure at the inlet.

At high values of the ratio of length to the height of the gap flow and low Reynolds numbers involved is insignificant pre-cutter and its omission does not lead to significant computational errors.

In the case of hydraulic resistance of the small joints of the flow parameters of the flow length is determined by a complicated analysis of the velocity field and pressure in the environment and the same slot. The values of liquid flow in the interstices of a low-resistance flow length calculated from formulas given in this article only determine the upper limit.

Basing on observations phenomena of working fluid flowed through a microgap, it was established that hard particles are arrested through a microgap in such gap. It should be regarded as a stochastic one and discussed from a probability point of view.

The mathematical model of the working fluid flow through a microgap was formulated in this article on the basis of an arbitrary assumption of solely mechanical causes for stopping hard particles contained in a working fluid. Despite the fact that the phenomenon itself proceeds discontinuously, this model has a continuous character. Taking into consideration the characteristic of the medium, in which fluid flows, we assumed that the process of arresting hard suspended particles is induced by two mechanisms: the first one consists in stopping particles of suspension solely due to dimensional differences between particles and the gap height, and the other one results from differences in size of arriving particles and characteristic dimensions of areas limited by the gap edges and previously stopped particles. The latter mechanism, as the dimensional spectrum of arrested particles shifts towards ever-decreasing sizes, causes the microgap flow-through characteristic to be unstable.

The microgap condition according to the proposed model is described by a system of random variables. It was assumed that the stochastic process  $S(\cdot) = \{S(t); t > 0\}$  is the homogenous Markov process. The probability structure of the  $S(\cdot)$  process was described with the aid of the transition probability function, which describes the way in which the changes of state occur. Observation of changes of flow rate of fluid flowing through a microgap means of course the observation of results of a process understood as the Markov process as it happens. A condition of a process at a given time point or after a certain amount of fluid has flowed through, does not determine clearly processes to follow, but solely a probability that a state of a system is one belonging to a subset of a range of states the flow rate values are assigned to. Owing to the description of the flow of working fluid through a microgap in a step-like, stochastic process of a Markov's structure it is possible now to clearly interpret the process of the clogging a microgap with regard to probability.

The results of the above considerations can be used to study the influence of contaminations on the process of gradual decrease in the effective cross-section of a microgap resulting from absorption of particles dispersed in a fluid by gap walls as well as to research into influence of contaminations on the operation of hydrostatic systems.

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