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OSCILLATION SYSTEMS FROM HYPER-DEFORMS ELEMENTS

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Abstract

In construction of machines more and more often are used materials called hyper-elastic, for example: foam structures, materials based on natural and synthetic rubbers and other materials subjecting large deformations. The porous structures are produced on the basis of different synthetic materials, thermoplastic polymers and thermosetting. Subject of porous materials is studied by many scientists around the world. Porous materials (also called cellular plastics, foam plastics or foamed plastics) include gas phase dispersed in a solid phase of polymeric material. Properties of such systems depends on the properties of the polymer warp and cell structure, comprising the gas phase. Approach previously used to anticipate and interpreting the behaviour of the hyper-deformable structures, which use the theory of hyperelastic materials, does not resolve the issue because it only describes the elastic properties. The article presents an original methodology developed structural identification the viscoelastic properties of hyperdeformable materials, and in particular modern construction materials from the group of plastics and composites of elastomers, foams, etc. The aim of the article was an oscillation analysis in systems from elements of incompressible and the description of phenomena setting in during of work of suchlike systems.

Keywords: *elastomers, incompressible materials, oscillation, very large reversible deformation*

1. Elastomers

This article aim to analyse vibrations in systems where there are elements of elastomer. In the analysis were used the assumptions used in the theory of hyperelastic materials [1] – elastomer is isotropic incompressible material with very large reversible deformations. It is the theory of finite deformations – on the major directions was assumed deformations (1):

$$\begin{aligned} e_{xx} &= (\partial u/\partial x)+\frac{1}{2}[(\partial u/\partial x)^2+(\partial v/\partial x)^2+(\partial w/\partial x)^2], \\ e_{yy} &= (\partial v/\partial y)+\frac{1}{2}[(\partial u/\partial y)^2+(\partial v/\partial y)^2+(\partial w/\partial y)^2], \\ e_{zz} &= (\partial w/\partial z)+\frac{1}{2}[(\partial u/\partial z)^2+(\partial v/\partial z)^2+(\partial w/\partial z)^2]. \end{aligned} \tag{1}$$

Hooke's law in the form of (2):

$$\begin{aligned} e_{xx} &= \frac{1+\nu}{E}\left[\sigma_{xx}-\frac{\nu}{1+\nu}(\sigma_{xx}+\sigma_{yy}+\sigma_{zz})\right], \\ e_{yy} &= \frac{1+\nu}{E}\left[\sigma_{yy}-\frac{\nu}{1+\nu}(\sigma_{xx}+\sigma_{yy}+\sigma_{zz})\right], \\ e_{zz} &= \frac{1+\nu}{E}\left[\sigma_{zz}-\frac{\nu}{1+\nu}(\sigma_{xx}+\sigma_{yy}+\sigma_{zz})\right], \end{aligned} \tag{2}$$

introduced designations (3):

$$\lambda_1^2 = 1 + 2e_{xx},$$
$$\lambda_2^2 = 1 + 2e_{yy}, \tag{3}$$
$$\lambda_3^2 = 1 + 2\varepsilon_{zz}.$$

The assumption of incompressible material leads to the conclusion that the Poisson's ratio is fixed at 1/2, it mean that in the entire body is a homogeneous condition: stress – hydrostatic pressure on values in accordance with dependence (4):

$$p = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}. \tag{4}$$

The volume of the body is the same; it means that the product of deformation λ is always equals 1 according to (5):

$$\lambda_1 \lambda_2 \lambda_3 = 1. \tag{5}$$

Using the above dependence, we are known with the theory of hyperelastic materials, subjection (6):

$$\sigma_{nom} = \frac{E}{3\left(\lambda_1 - \frac{1}{\lambda_1^2}\right)}, \tag{6}$$

in which σ_{nom} it is a nominal stress, acting strength we referring to the initial section.

The dependence represented by the formula (6) is the known theory of hyperelastic material and is often used – including in the description of experimental polymers creep [11]. It is a strongly nonlinear dependence – in Fig. 1 is a graph of σ–λ expected range of items work in this type of material – a deflection to half-length. For comparison, the dotted line represents a linear relationship.

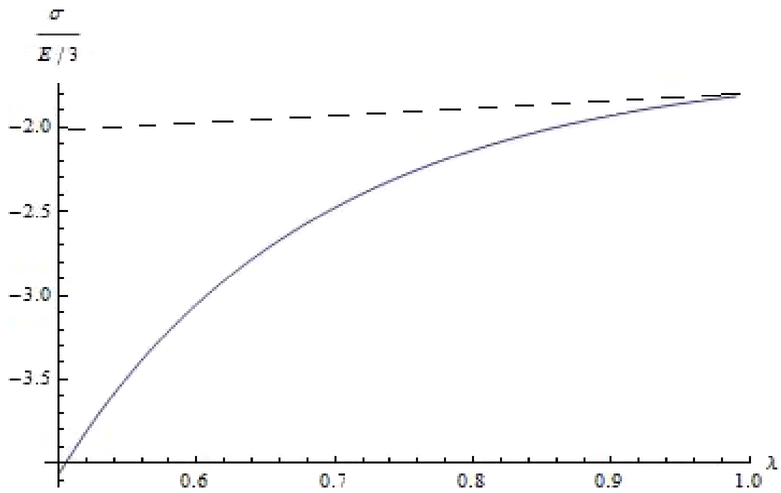

Fig. 1. σ–λ in the expected range of work

2. Scheme of oscillations

Let us consider the vibration in the arrangement illustrated in Fig. 2. It was assumed that the material properties are described as a parallel connection model nonlinear elasticity described as hyper-elastic materials and viscous damping – Fig. 3.

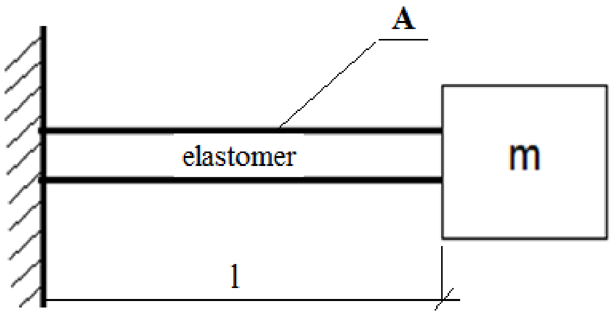

Fig. 2. Ideological notion of question

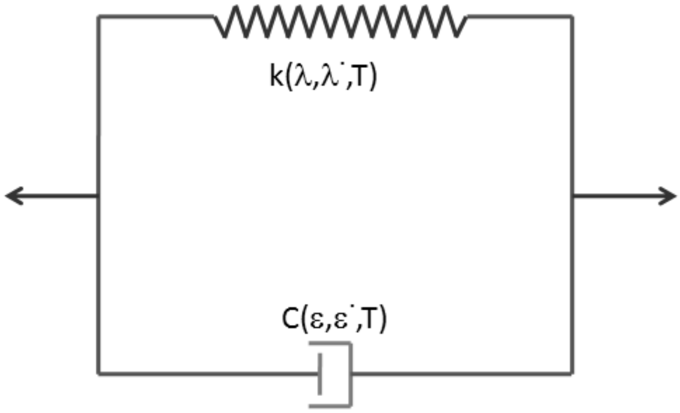

Fig. 3. Material model analyses

This model is analogous to the Kelvin-Voight model with non-linear elasticity. In order to determine the dimensional equations of vibration multiply the relation (6) through cross-sectional area and in the base equation (7):

$$\lambda_1 = \frac{x+l}{l}, \tag{7}$$

resulting from the dependence (8):

$$\lambda_1 = \varepsilon_1 + 1. \tag{8}$$

We get (9):

$$F = \frac{EA}{3\left[\frac{x+l}{l} - 1\Big/\left(\frac{x+l}{l}\right)^2\right]}, \tag{9}$$

that is we can at last record in equation such (10):

$$F = \frac{EA}{3l\left[x+l-\frac{l^3}{(x+l)^2}\right]}. \tag{10}$$

We receive the equation of oscillation in form (11):

$$m\ddot{x} + c\dot{x} + k\left[x+l-\frac{l^3}{(x+l)^2}\right] = F(t). \tag{11}$$

The coefficient of stiffness was marked as (12):

$$k = \frac{EA}{3l} \cdot \tag{12}$$

Equation (11) has a highly non-linear in nature resulting from the occurrence of the denominator exponential. The nature of vibration of this kind has not yet been described in vibration theory – his term requires the use of numerical methods. In order to derive the parameters describing the model – stiffness and damping performed experimental studies involving the compression of the cylindrical sample made of a rubber elastomer cross-linked. Characteristic $\sigma = f(\varepsilon)$ the grid elastomer of natural rubber – free grip was shown in Fig. 4.

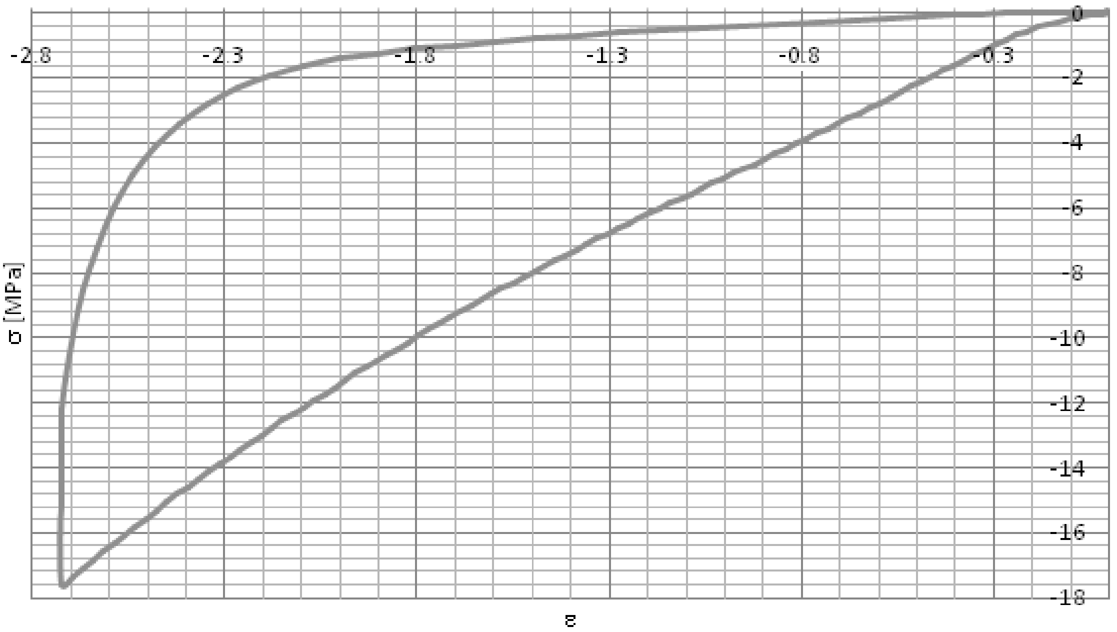

Fig. 4. Characteristic $\sigma = f(\varepsilon)$ the grid elastomer of natural rubber – free grip

3. Characteristics of experimental studies

The samples were prepared in the shape of a cylinder with a diameter of 23.5 mm and a thickness of 10 mm. The materials were tested using appropriate standards for extremely different operating states: the nominal operating temperature +23°C (hard forced elasticity) and glassy brittle: –15°C, different speed ranges, e.g. 1 mm/min and 1 mm/s, 6 mm/s. Attempts to research I made for compression: free and limited, compression with the force of cyclically variable.

Figure 4 shows the pronounced asymmetry of the hysteresis loop – back after the initial intensity of the load is greater than the initial load intensity. This phenomenon is non-linear viscoelastic polymeric materials found in [1]. The occurrence of such a phenomenon is confirmed by a series of experiments – the first made in the sixties of the last century by Landeman [2]. First, using the load line in Fig. 3, which is practically easy to define the value E of 2.71 MPa, further assuming $l = 1$, $A = 0.01$ m^2, we obtain $k = 9034$ N/m. This value was used for analysis of vibration in the system described by equation (11). Assumed: mass $m = 0.1$ kg, extortion corresponding to work of material, in which the test was performed. In view of the above, for the compression accepted (13):

$$F(t) = F \sin \nu t - F_0, \tag{13}$$

where $F_0 = 200$.

First constructed undamped vibrations assuming $C = 0$.

The calculations were made using the software package to support math MATHEMATICA v. 7.0.

It has been found that the oscillations are non-linear in nature – are phenomena typical of the nonlinear oscillation amplitude of forced vibrations undamped is over, the highest values occur at much consideration of frequency offset relative to the value. The resonant graph shown in Fig. 5.

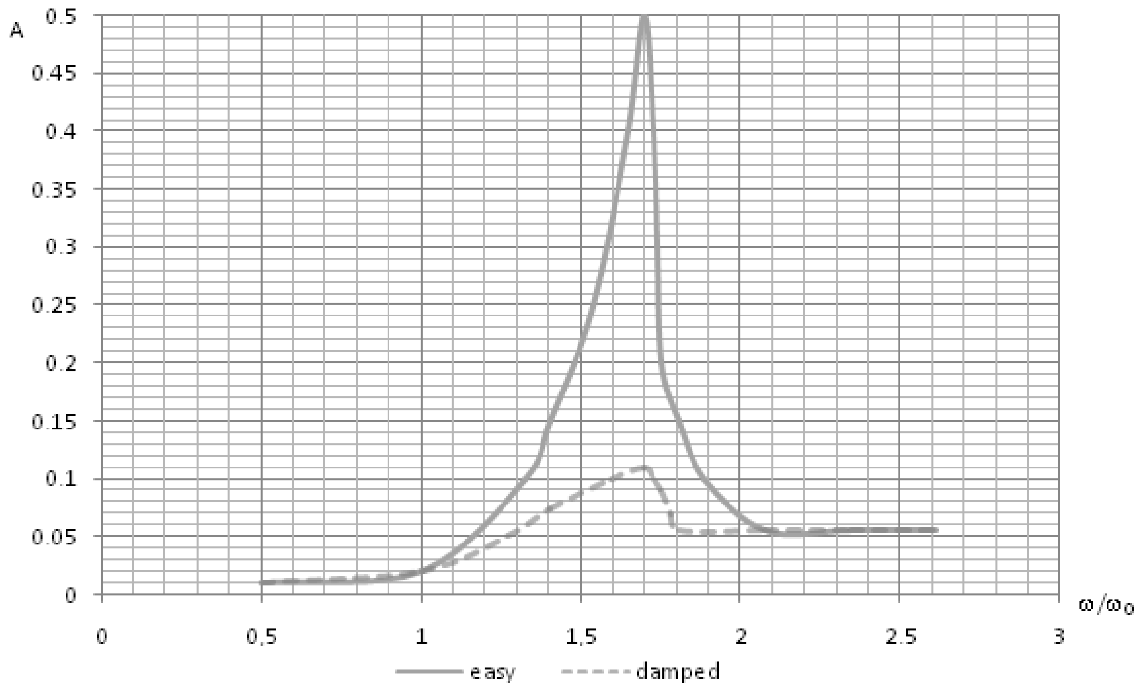

Fig. 5. The amplitude of forced vibration of harmonic oscillator in the dependence from of force input function frequency

To determine the level of attenuation used in the graph of the hysteresis loop shown in Fig. 4, due to the different viscoelastic properties of the sections of the loading and unloading adopted non-linear relationship as a function of speed. The equation analysis was (14) [15]:

$$m\ddot{x} + J(-\dot{x}) * C_1\dot{x} + J(-\dot{x}) * C_{01}\dot{x} + k\left[x + l - \frac{l^3}{(x+l)^2}\right] = F\sin\nu t - F_0\,, \tag{14}$$

where: $J = 1$ for $\dot{x} > 0$, or $J = 0$ for $\dot{x} \le 0$.

Was developed an additional program to simulate the compression tests in which data is entered as damping coefficients C_1 and C_{01}, is determined hysteresis loop. The coefficients C_1 and C_{01} are variable up to the deletion area of the loop, such as obtained in the experimental studies. Value was received: $C_1 = 0.9$ N·s/m, $C_{01} = 5$ N·s/m.

Figure 6 and 7 shows the analysis results for the initial conditions: initial time: $t_p = 0$, final time: $t_k = 0.12$ s, period: $T = 0.005$ s, $C = 0$ ($t_p = 0$, $t_k = 0.12$ s, $T = 0.001$ s, damping ratio: $C_1 = 0.9$, $C_{01} = 5$); a) – the change of strength in time, b) – the dislocation in time, c) – the change of speed in time, d) – the speed in function the dislocation.

The resonance curve of damped oscillations was introduced in Fig. 5 by dashed line. The dissipation level (the dispersion of energy) in the grid elastomer of natural rubber is considerable – resonance strengthener steps out is small (Fig. 5).

4. Conclusion

The study presents method for the analysis of vibration in a visco elastic element of incompressible hyper-elastic material.

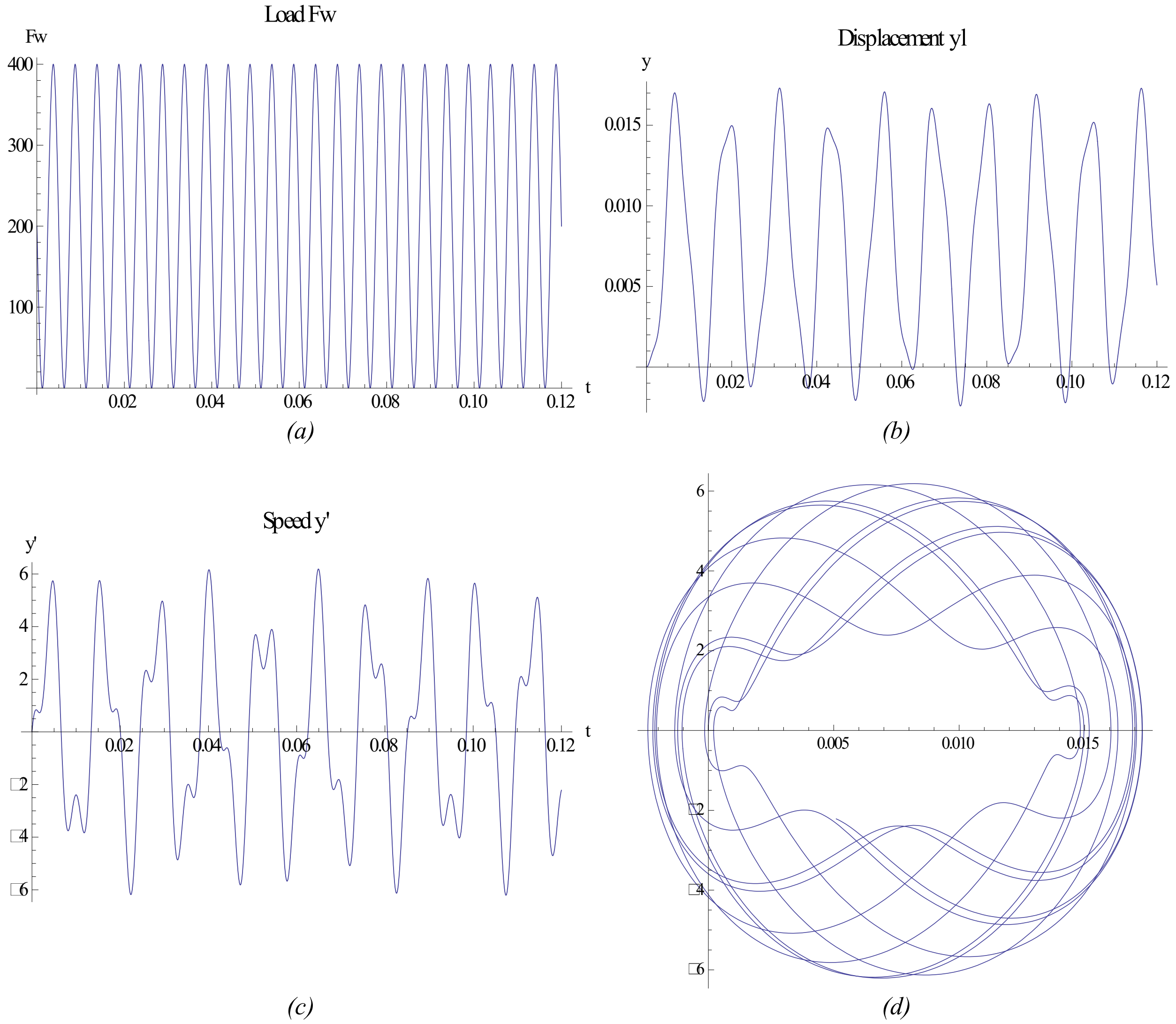

Fig. 6. Result analyse for initial conditions: $t_p = 0$, $t_k = 0.12$ s, $T = 0.005$ s, $C = 0$

It has been found that the oscillations are highly nonlinear nature of the resonance frequency is lower than the frequency of their own circuit. The measured level of attenuation in the material is significant – there is a strong reducing vibration.

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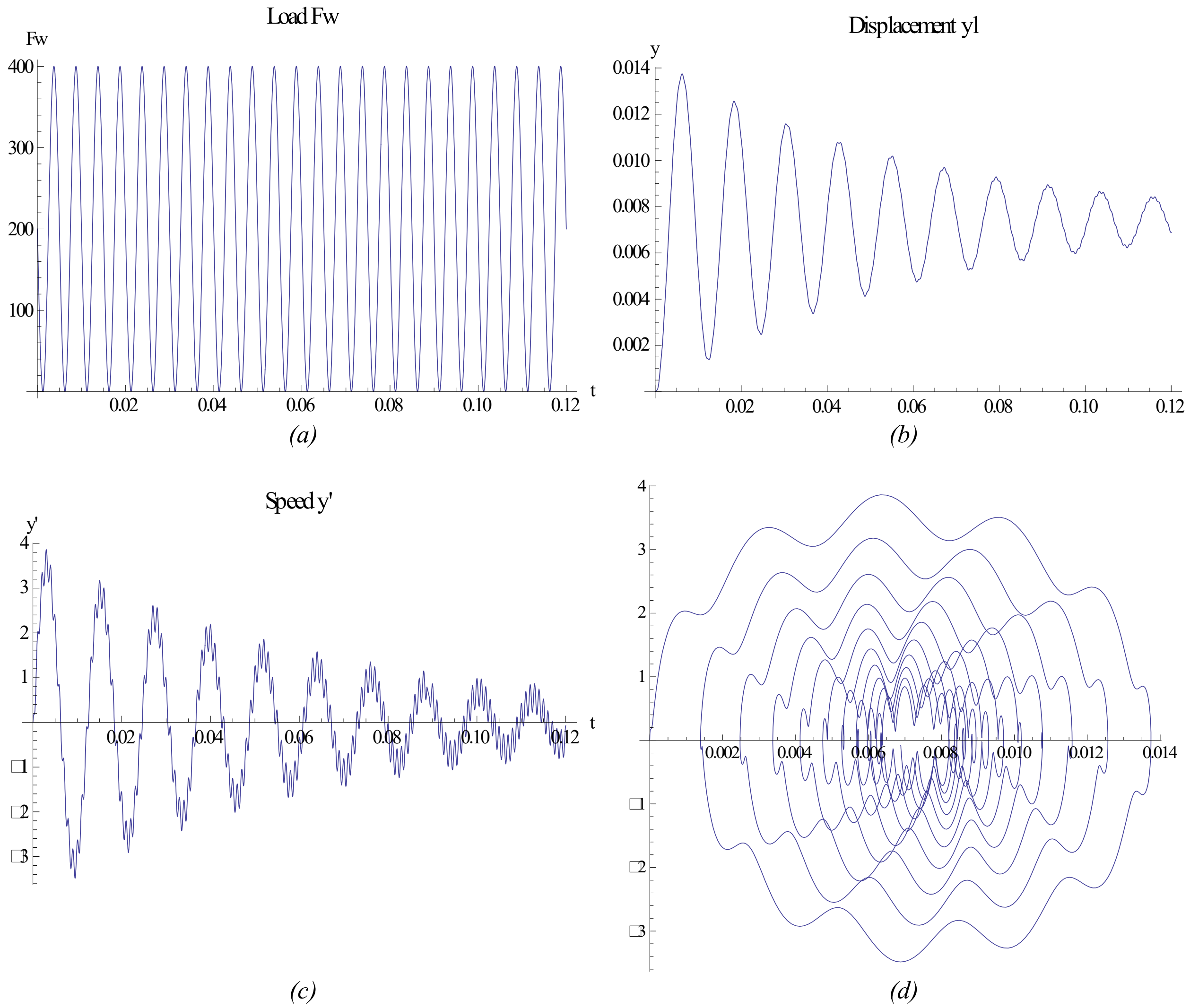

Fig. 7. Result analyse for initial conditions: $t_p = 0$, $t_k = 0.12$ s, $T = 0.001$ s, $C_1 = 0.9$, $C_{01} = 5$