DETERMINATION OF NONDIMENSIONAL ARGUMENTS
IN DIMENSIONAL FUNCTIONS OF SHIP PROPULSION
ENGINE OPERATION

Jan Roslanowski, Adam Charchalis

Gdynia Maritime University, Faculty of Marine Engineering
Morska Street 81-87, 81-225 Gdynia, Poland
tel.: +48 58 6901323, fax: +48 58 6901399
e-mail: rosa@am.gdynia.pl, achar@am.gdynia.pl

Abstract

The following article describes different ways for determining dimensionless arguments in dimensional functions of ship propulsion engine operation. Dimensional space has also been characterized in relation to properties it possesses. An attention has been paid to the fact that it creates Abelian group, where involution fulfills multiplication axiom by scalar, and positive numbers that belong to dimensional space create dimensionless subspace. The conception of dimensional dependence has also been explained. There are also described conditions, which should be fulfilled by dimensional quantities to be dimensionally independent. Fundamental theorems of dimensional analysis have also been characterized. There is also given the definition of dimensional function of ship propulsion engine operation concerning values and dimensional arguments. It has been explained what requirements are to be fulfilled. One can also learn what limitations are imposed on dimensional function of ship propulsion engine operation concerning dimensional homogeneity and invariance. The ways of dimensional function transformation into a numerical one have been described. In addition, some conditions have been given which must be applied at given method of dimensional function transformation. An attention has been paid to the fact that dimensionless arguments are similarity invariant as a result of transformation by the similarity method of mathematical model of ship propulsion engine operation. In this study, the ship propulsion engine performance is expressed by product of Joule and second interpreted as transfer of energy in the form of work. Identification of ship propulsion engine operation by dimensional analysis performed in terms of its usefulness for diagnosis of ship propulsion engines. The basic problem of marine diesel exploitation is the monitoring of its technical condition. Diagnosis of marine diesel propulsion increases the safety of the ship and thus protects the marine environment against pollution.

Keywords: ship propulsion engine operation dimensional analysis, dimensionless arguments in dimensional functions

1. Introduction

Operation of ship propulsion engine consists in processing and transfer of energy. Firstly, there comes along in engines the exchange of chemical energy contained in fuel for thermal energy during heat generating process in engine cylinders. Generation of heat in engine cylinders is being transformed into mechanical energy in the form of indicated work. Separation of work indicated and performed by gases in cylinders takes place in piston- con rod system of the engine. One part of this work is lost, first, for propulsion of timing gear mechanisms, second, for overcoming of friction resistance in bearings, third, overcoming of piston friction against cylinder walls as well as, ventilation resistance. On the hand, second part of indicated work is being transferred to the propeller as effective work of engine operation, so called, power. The above operation causes the torque of the crankshaft at a definite rotation velocity of the engine.

Effective operation of the engine is a kind of energy conversion and determines its performance. Engine operation, interpreted as energy conversion in the form of useful work, is expressed by the following formula [1, 2, 3]:

\[ D = 2\pi \int_{0}^{t} n(\tau) \cdot M_{0}(\tau) \cdot \tau \cdot d\tau, \] (1)
where:

\( D \) – propulsion engine operation in [Js],
\( n(\tau) \) – function of rotating velocity of an engine from time \( \tau \) in [1/s],
\( M_0(\tau) \) – crankshaft torque function from time \( \tau \) in [Nm],
\( \tau \) – operation time of propulsion engine in [s].

Engine operation understood as energy release at a definite time, can be evaluated. Evaluation consists in comparison of engine operation with dimensional physical quantity of Joule second as a measure unit [1-3].

Dimensional analysis can be used for this purpose, as it gives a chance to go from qualitative to numerical specifications. Thus, it enables us to analysis propulsion engine operation, by means of functions of many variables. However, it requires the knowledge of regularities taking place among dimensional quantities, which characterize engine operation.

2. Ship propulsion engine operation as dimensional function

Engine operation as dimensional quantity together with other quantities of this type which characterize the ship motion, belongs to elements of dimensional space. Elements products of dimensional space form Abelian group together with involution of real exponent. It allows describing dimensional space of engine operation by means of positive real numbers. These numbers from subspace of dimensional space of engine operation. It means that dimensional space of engine operation includes both nondimensional and dimensional quantities, which can be dimensionally independent.

Each system of nondimensional quantity cannot be dimensionally independent. Out of dimensional space elements, one can choose a defined by space dimension, a certain amount of dimensionally independent quantities, called a space base. All bases of a given dimensional space are of equipotent character.

Elements of the same dimensional space can be arguments of the function of engine operation, defined in this space. In such a way, defined function of engine operation is not an ordinary numerical function and is called a dimensional function. Dimensional function of engine operation must identically well describe its performance in each set of units, and thereby must be invariant, in relation to dimensional transformation. Apart from this condition it must fulfil the condition of dimensional homogeneity i.e. it must not change a dimension of its value, in a situation when its arguments do not change them. Not in all cases dimensional functions of engine operation, are correct. Correctness condition of defined function with arguments creating a configuration dimensionally independent in dimensional space \( n \) is fulfilled precisely by one solution of \( n \) equation set with a number of unknowns being equal to its arguments. This condition allows us to check if the structure of described propulsion engine operation was set correctly or not [2, 3].

Generally speaking, arguments of engine operation function can be dimensionally independent and dependent. So on the basis of Buckingham theorem [3] the last can be expressed, by means of numerical function. The numerical function of engine operation can be determined on the basis of an experiment exact to a constant parameter.

Dimensional analysis does not deliver any information about a numerical form of engine operation function. Such information can be only obtained on the basis of engine performance during the ship motion.

3. Structure of dimensional function of propulsion engine operation

Analysis of propulsion engine operation requires the knowledge of dependences between the quantities that determine the engine performance and dynamic features of propulsion system of the ship.
Ship propulsion engine operation during its work is determined by the following dimensional quantities:
- torque of the engine $M$,
- rotational velocity of the engine $n$,
- consumption of fuel volume by the engine $G_v$,
- pressure of charging air $p$,
- time of engine performance $t$.

Dimensional form of the function of propulsion engine operation $D$ can be defined on the grounds of function dependence between, the above mentioned, quantities. They have adequate dimensions in an accepted system of measure units SI.

Taking into account the above-mentioned premises, one looks for numerical forms of the following dimensional functions of ship propulsion engine operation $D$:

$$D = \Phi(M, n, G_v, p, t),$$

where:
- $\Phi$ – symbol of dimensional function,
- $M$ – rotational moment of the engine in $[\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}]$,
- $n$ – rotational velocity of the engine in $[1/\text{s}]$,
- $G_v$ – consumption of fuel volume by the engine in $[\text{m}^3/\text{s}]$,
- $p$ – pressure of charging air in $[\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}]$,
- $t$ – time of engine operation in $[\text{s}]$,
- $D$ – operation of ship propulsion engine in $[\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1}]$.

Function of ship propulsion engine operation (2) is described in dimensional space of the third grade, which means that among arguments of the function, three of them are dimensionally independent and create, so called, dimensional base. There are ten possibilities of their choice in a given function, but three of them are irregular. Chart 1 presents correct possibilities of arguments choice, dimensionally independent, so called, dimensional bases in the function of ship propulsion engine operation [3].

<table>
<thead>
<tr>
<th>Ordinal number</th>
<th>Form of dimensional function</th>
<th>Dimensional base</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$D = f\left(G_v \cdot \phi_i\right) \cdot \frac{M}{n}$, $\phi_i = \frac{G_v \cdot p}{M \cdot n}$, $\phi_i = n \cdot t$</td>
<td>$M, n, p$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$D = f\left(p, \phi_i\right) \cdot \frac{M}{n}$, $\phi_p = \frac{p \cdot n \cdot G_v}{M}$, $\phi_i = n \cdot t$</td>
<td>$M, n, G_v$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$D = f\left(G_v, \phi_i\right) \cdot \frac{G_v \cdot p}{n^2}$, $\phi_M = \frac{M \cdot n}{p \cdot G_v}$, $\phi_i = n \cdot t$</td>
<td>$n, G_v, p$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$D = f\left(G_v, \phi_i\right) \cdot \frac{M^2}{G_v \cdot p}$, $\phi_a = \frac{n \cdot M}{G_v \cdot p}$, $\phi_i = \frac{G_v \cdot p \cdot t}{M}$</td>
<td>$G_v, p, M$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$D = f\left(G_v, \phi_i\right) \cdot \frac{M^2}{G_v \cdot p}$, $\phi_a = \frac{n \cdot M}{G_v \cdot p}$, $\phi_i = \frac{G_v \cdot p \cdot t}{M}$</td>
<td>$G_v, p, M$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$D = f\left(G_v, \phi_i\right) \cdot M \cdot t$, $\phi_a = n \cdot t$, $\phi_p = \frac{p \cdot t \cdot G_v}{M}$</td>
<td>$p, t, M$</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$D = f\left(G_v, \phi_i\right) \cdot M \cdot t$, $\phi_a = n \cdot t$, $\phi_p = \frac{p \cdot t \cdot G_v}{M}$</td>
<td>$M, t, G_v$</td>
<td></td>
</tr>
</tbody>
</table>
For a defined dimensional base given in chart 1, one can determine a general form of dimensionless arguments as independent variables of numerical function of propulsion engine operation, exact to constant coefficients. Stable coefficients can be determined because of parameter measurement of stabilized operation of ship propulsion engine, by means of multiple regression. To measure parameters of engine operation one can, best of all, use control and measuring devices, installed as standard equipment of the ship. Drawn and chartered dependences of nondimensional arguments as variables dependent on independent ones enable us to determine in the field of real numbers, numerical forms of function operation, by method of multiple regression and to define its constant coefficients. Adopted dimensional base decides on a dependence character among nondimensional arguments that form numerical functions of ship propulsion engine operation. Acceptance of dimensional base $p, t, M$, is the reason why the character of dependence between dimensionless arguments is linear. Nondimensional arguments of engine operation function calculated on the grounds of accepted dimensional base $p, t, M$, in case of linear function calculation (6) given in chart 1 are shown in Fig. 1. Matching of nondimensional arguments of engine operation function presented in Fig. 1, was carried out by means of linear regression, according to STATISTICA programme, receiving the equation (3) [2].

Exemplary measurements results of engine operation were carried out on the Sulzer engine of 5RD68 type, which was a propulsion engine of the bulk-carrier of 5500 DWT displacement.

Figure 1 presents coordinates of dimensionless arguments of numerical function in propulsion engine operation with clearly visible linear dependence:

$$\frac{D}{M \cdot t} \cdot 10^6 = 10^{-6} \cdot \frac{p \cdot G_v \cdot t}{M} + 6.27 \cdot 10^6 \cdot n \cdot t - 0.001,$$

where denotations as formula (2).

Correlation coefficient of the above fitting to straight line is equal to:

$$r = 0.99999572 \Rightarrow r^2 = 0.99999144,$$

which results in standard mistake of estimation equal to 0.00952.

Acceptance of another dimensional base, for example, $M, n, p$ results in the fact that corresponding with the base dimensionless arguments of dimensional function of engine operation in three-dimensional set of coordinates an not so much linear, as in the case of dimensional base equal to: $p, t, M$.
Nondimensional arguments of engine operation function, calculated on the grounds of accepted dimensional base $M, n, p$ in the case of numerical function determination (1) given in chart 1, are shown in Fig. 2. Matching of nondimensional arguments of engine operation function, presented in Fig. 2, was carried out by the method of nonlinear regression, according to STATISTICA programme, receiving the equation [2]:

$$\frac{D \cdot n}{M} = -0.01799 + 6.2985 \cdot n^2 \cdot t^2 - 0.0341 \cdot n \cdot t + 1.4752 \frac{G \cdot p}{M \cdot n},$$

(3)

where denominations as in formula (2).

Fig. 2. Dimensionless arguments of ship propulsion engine function in an accepted dimensional base $M, n, p$ of the form $Z = f(Y, X)$ [3, 4] (look chart 1, position 1). Explanations: $Z = (D \cdot n)/M$ - dimensionless indicator of engine operation, $X = n \cdot t$ - similarity invariant of rotation engine speed, $Y = (G \cdot p)/(M \cdot n)$ - invariant of volumetric similarity of fuel consumption by the engine, the remaining denominations as in formula (2)

Nondimensional arguments of numerical function concerning propulsion engine of the ship, received on the grounds of parameter measurements of its operation, have been mapped on the cylindrical surface described by the equation (4) and presented in Fig. 3. Correlation coefficient of the above matching equals to $r = 0.999996$ which bears evidence of good matching of roller surface (4) to dimensionless quantities obtained on the grounds of parameter measurement of engine operation.

Fig. 3. Cylindrical surface describing scatter of nondimensional quantities determined on the grounds of parameter measurement of ship propulsion engine operation [2-4]
Structures of numerical functions (3) and (4) differ from one another, because of the fact that different choices of dimensional bases have been used in them.

Forms of dependence among nondimensional arguments of numerical functions of propulsion engine operation can be defined on the grounds of parameter measurements of engine operation. They can be true only for the engine on which performance is just being tested.

Dependences among nondimensional arguments of numerical functions of propulsion engine operation can be considered only as proper suggestions, as regards dimensional aspect. The best dependence is the one that has a simple form and easy physical interpretation.

4. Summary

The possibility of applying dimensional analysis depends on dimensioning of all quantities that characterize engine operation in an accepted system of units. Application of this system of units in problems connected with a description of propulsion engine operation is indispensable to build, if possible, its full mathematical description.

It is necessary also to carry out measurements of engine operation, in order to receive exact parametric functions of its operation, on the grounds of utilized dimensional quantities.

Received dependences among nondimensional arguments of numerical functions of engine operation are defined exact to constant coefficients calculated on the grounds of parameter measurements of its operation.

Nondimensional arguments of numerical functions of propulsion engine operation are characterized by the fact that they take into account essential quantities that describe its operation, according to time. Therefore, they are of dynamic character, and in consequence, of this, they can be used for diagnostic and prognostic purposes.

References