

OPERATION VALUATION OF VOLUMETRIC COMPRESSORS BY MEANS OF DIMENSIONAL ANALYSIS

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Abstract

The following article introduces determination method of displacement compressors basing on its work parameters, by means of dimensional analysis. Operation of displacement compressors as volumetric compressing devices, according to J. Girtler, has been treated as a new physical quantity of [J·s] dimension.

Such operation can be interpreted as mechanical energy transfer, through working organ, to gas factor volume in working space of the compressor in a definite time.

Operation of volumetric compressor in which energy transfer, takes place, can be an information carrier of its technical condition.

It expresses transformation of energy delivered to the gas, being forced through, by the compressor. This quantity can be determined, on the basis of, algebraic diagram of dimensional analysis, constructed by S. Drobot.

The above diagram allowed us to control correctness of inference rules, in relation to mathematics, used in numerical functions of volumetric compressors operation, fitted in installations.

Constructional solutions of compressors depend on the kind of compressed factor, the way of cooling, size of compressor and on the kind of propulsion and destination of compressor.

Keywords: operation of volumetric compressor, algebraic diagram, dimensional analysis physical dimension quantity, Joule multiplied second

1. Introduction

Volumetric compressors belong to compressing devices of compression higher than two, whose aim is to increase pressure of gas or steam factor and dislocate it from space of lower pressure to space of higher pressure.

Suction, compression and pressure drive in such compressor takes place, owing to increasing and decreasing of working space volume, through moving working organ. Gas factor also takes place, in the above mentioned, working space. Compression is carried on by way of pulsation, and static pressure of gas factor of constant mass increases, owing to decreasing of its volume.

The shape of working space of volume compressors and its space changes is solved in a different way. Most popular solution is represented by piston compressors whose working organ, for example piston, makes reciprocating movements. Membrane compressors belong to this group of compressors. The next group is represented by rotary compressors whose working organs carry out rotational movements.

Operation of volumetric compressor is the cause, why a constant number of gas molecules, taking place in its working space, owing to periodical movement of working organ, changes its volume in a reproducible way.

According to the above, it makes sense to investigate the operation of volumetric compressor, defined by energy delivered to it and by means of working organ, transferred to the volume of gas factor in working space.

Operation valuation of energetic devices like volumetric compressors, is proposed by J. Girtler in

work [2] the compares it with dimensional physical quantity of measure unit, called Joule/ second.

Working parameters of volumetric compressor depend on installation characteristic, in which it is installed.

2. Operation of volumetric compressor in dimensional space

Operation of volumetric compressor consists in transport of mechanical energy from any external source to gas volume in its working space through moving working organ. This energy is used up for changes of gas factor volume in working space of compressor and for pulsatory increases of static pressure of gas during its forcing through. It causes oscillations of pressure in inlet piping and conduit piping of compressor. Size of this work depends on working process, which consists of cycles of the following phases:

- sucking gas from suction space to working space,
- gas compression in working space from suction pressure to forcing through pressure,
- gas pressure drive from working space to pressure space,
- expansion of gas remainder left in working space from pressure drive to suction pressure.

Volumetric compressors are characterized by forcing through from suction side to pressure side precisely defined weight of pressed doses of gas factor. However, the work character is discontinuous and pulsatory. It is caused by periodically repeated increase and decrease of working space volume.

Gas energy increase in volumetric compressor is defined by maximal change of its working space volume falling on one cycle of work. In volumetric compressors of steady rotations, pressure change only unnoticeably influences efficiency of the compressor.

Quantity changes of characteristic volumetric compressors are presented graphically, depending on their efficiency, where rotational speed of the compressor is limited by magnitude of inertia forces and work of automatic valves.

Utilization degree of energy delivered to compressor's drive is defined as a quotient of power on compressor's shaft and capacity in an adopted time unit. The above degree is connected with compressor operation and depends on the factors:

- assumed method of compressor operation,
- state of environment in which work is carried out,
- condition of compressor's mechanism itself [5, 6].

Assuming that the method of compressor operation and environment conditions are constant and invariable, then a degree of energy utilization delivered to compressor's drive, depends exclusively on its technical condition.

Energy delivered to working organs of the compressor in the form of work, causes an increase of pressure energy, overcoming of altitudes difference and friction resistance between inlet and outlet intersections of the compressor [5, 6]. And thus, it contributes to an increase gas energy flowing through the compressor.

Operation of volumetric compressor is characterized by quantities, which are elements of dimensional space. Such space is nothing else but a set of elements which are physical dimensional quantities, which on account of operation called product, create a commutative group, called "abelian group". In this group, they define multiplication of positive real numbers. Besides, involution of real exponent is also defined in, the above mentioned, dimensional space.

Positive real numbers describing dimensional space and possessing multiplication and involution properties create subspace of dimensional space π_0 . It means that subspace of dimensional spaces create dimensionless quantities.

Every measure of dimensional quantity can be expressed as a product of numerical value and a measure unit. Dimension of dimensional quantity is a feature, which can enable us to compare it with other quantities. Dimensions of different quantities stay with each other in connections

resulting from their definitional equations, for example, the torque is defined as a product of force and a radius of its operation.

Operation of volumetric compressor and energy delivered to it is expressed by means of the following formula:

$$D = \int_0^t q_v \cdot e_{rv} \cdot \tau^2 \cdot d\tau, \quad (1)$$

Where:

- q_v – actual performance of the compressor $\left[\frac{m^3}{s}\right]$,
- e_{rv} – utilization degree of energy delivered do compressor's $\left[\frac{kg}{m \cdot s^2}\right]$,
- τ – working time of compressor [s],
- D – operation of volumetric compressor [Js].

Therefore, operation of volumetric compressor (1) can be expressed as a dimensional function of many variables. In this way, we can obtain a general, physical equation of compressor operation in different forms:

$$D = q_v \cdot e_{rv} \cdot \tau^2 = \Phi(p_t, p_{ss}, N_e, \tau), \quad (2a)$$

$$D = q_v \cdot e_{rv} \cdot \tau^2 = \Phi_1(q_v, p_t, p_{ss}, N_e, \tau), \quad (2b)$$

where

- p_t – forcing pressure of compressor $\left[\frac{kg}{m \cdot s^2}\right]$,
- p_{ss} – suction pressure of compressor $\left[\frac{kg}{m \cdot s^2}\right]$,
- N_e – effective power of compressor $\left[\frac{kg \cdot m^2}{s^3}\right]$,
- remaining denotations as in formula (1).

The above dimensional functions (2) are defined in dimensional space π , whose arguments are elements of this space. They are not numerical functions and therefore must fulfil additional conditions of invariance and dimensional homogeneity. These conditions do not restrict the forms of numerical functions, i.e. such, whose both, arguments and function value are dimensionless quantities [1, 3, 4, 7].

It is worth to add, that the condition of invariance results from a possibility to describe physical dimensional quantities in different unit arrangements. In an adopted system of basic measure units SI matrix of involution exponents of function arguments (2) is of the third order. It means that in case of dimensional function:

- (2 a) – three quantities out of four arguments are dimensionally dependent on one of them,
- (2 b) – three quantities out of five arguments are dimensionally dependent on those two.

That is why from dimensional functions (2), one can chose arguments dimensionally independent and separate them from the remaining ones, i.e. to adopt dimensional base for these functions, depending on the fact how many independent dimensional arguments it has. And so for the forms:

(2 a) for four ways (combination $\binom{4}{3}=4$),

(2 b) for ten ways (combination $\binom{5}{3} = 10$).

Not all randomly adopted dimensional bases, will be proper, which results from dimensional independence of unit system: kg, m, s . From random choice of dimensional function bases of volumetric compressor operation, only determined number is correct, as regards mathematics. For the function (2 a) only two bases out of four are correct, and for the function (2 b) only five bases out of ten, are correct. Correct dimensional bases, in relation to mathematics, functions (2a) and (2b) are mentioned in Tab. 1.

Tab. 1. Choice possibilities of arguments dimensionally independent, so called dimensional bases in function of volumetric compressor operation

Ordinal number	Form of dimensional function	Dimensional base	remarks
$D = \Phi(p_t, p_{ss}, N_e, \tau)$			
1	$D = f(\xi_{p_{ss}}) N_e \cdot \tau^2$ $\varphi_{p_{ss}} = \frac{p_{ss}}{p_t}$	p_t, N_e, τ	
2	$D = f_1(\xi_{p_t}) N_e \cdot \tau^2$ $\xi_{p_t} = \frac{p_t}{p_{ss}}$	p_{ss}, N_e, τ	
$D = \Phi_1(q_v, p_t, p_{ss}, N_e, \tau)$			
1	$D = f_2(\varphi_{p_{ss}}, \varphi_{p_t}) N_e \cdot \tau^2$ $\varphi_{p_{ss}} = \frac{p_{ss} \cdot q_{rv}}{N_e}; \varphi_{p_t} = \frac{p_t \cdot q_{rv}}{N_e}$	N_e, q_{rv}, τ	
2	$D = f_3(\varphi_{q_{rv}}, \varphi_{p_t}) N_e \cdot \tau^2$ $\varphi_{q_{rv}} = \frac{q_{rv} \cdot p_{ss}}{N_e}; \varphi_{p_t} = \frac{p_t}{p_{ss}}$	p_{ss}, N_e, τ	
3	$D = f_4(\varphi_{q_{rv}}, \varphi_{p_{ss}})$ $\varphi_{q_{rv}} = \frac{q_{rv} \cdot N_e \cdot \tau^2}{p_t}; \varphi_{p_{ss}} = \frac{p_{ss}}{p_t}$	p_t, N_e, τ	
4	$D = f_5(\varphi_{p_{ss}}, \varphi_{N_e}) q_{rv} \cdot p_t \cdot \tau^2$ $\varphi_{p_{ss}} = \frac{p_{ss}}{p_t}, \varphi_{N_e} = \frac{N_e}{q_{rv} \cdot p_t}$	q_{rv}, p_t, τ	
5	$D = f_6(\varphi_{N_e}, \varphi_{p_t}) q_{rv} \cdot p_{ss} \cdot \tau^2$ $\varphi_{N_e} = \frac{N_e}{q_{rv} \cdot p_{ss}}, \varphi_{p_t} = \frac{p_t}{p_{ss}}$	q_{rv}, p_{ss}, τ	

According to works [1, 6] one can assume that the best dimensional base of these functions will be the following dimensionally independent quantities: p_{ss}, N_e, τ .

After making use of Buckingham theorem [1, 3, 4] in function (2) and adopting the above dimensional base, we can obtain the following equations:

for function (2a):

$$D = q_v \cdot e_{rv} \cdot \tau^2 = f(\xi_{P_T}) \cdot N_e \cdot \tau^2, \quad (3a)$$

for function (2b):

$$D = q_v \cdot e_{rv} \cdot \tau^2 = f_1(\phi_{P_T}, \phi_{q_{rv}}) \cdot N_e \cdot \tau^2, \quad (3b)$$

where:

– f_i, f – numerical functions,

- $\xi_{P_t} = \phi_{P_t} = \frac{P_t}{P_{ss}}$ – complete compression of the compressor (dimensionless argument of numerical functions),
- $\phi_{q_{rv}} = \frac{q_{rv} \cdot P_{ss}}{N_e}$ – discriminant of compressor capacity being dimensionless argument of numerical function f_1 ,
- the remaining denotations as in formula (2).
- Formulas (3) can be converted into the following forms:

$$\Phi_D = \frac{D}{N_e \cdot \tau^2} = f\left(\frac{P_t}{P_{ss}}\right), \quad (4a)$$

$$\Phi_D = \frac{D}{N_e \cdot \tau^2} = f_1\left(\frac{P_t}{P_{ss}}, \frac{q_{rv} \cdot P_{ss}}{N_e}\right), \quad (4b)$$

where:

- Φ_D – dimensionless discriminant of volumetric compression operation,
- the remaining denotation as in formulas (2) and (3).

One cannot see however, that in discriminant of volumetric compressor operation, in formula (4), density of gas does not appear, hence the conclusion, compressor operation does not depend on density of gas.

Also from the above dependence, it appears, that the operation of volumetric compressor of constant power supply, changes proportionally to the time square of its operation.

From formulas given in Tab. 1 (formulas 2 and 3) it follows that if the same dimensional quantities are adopted in dimensional functions, as independent of the remaining ones, then the adequate variable arguments of their numerical functions will be equal, that means $\xi_{P_t} = \phi_{P_t}$. Therefore their name as similarity invariants used in literature [3, 4, 7].

It is worth mentioning that dimensional analysis does not supply any information of numerical function form f . According to Drobot in his work [1], dimensional analysis ensures only dimensional correctness of description, not intervening in physical description of the word. With its aid, we can obtain operation functions of volumetric compressors, determined exact to a constant coefficient, if only the arguments of dimensional functions of compressor operation are dimensionally independent [1, 3, 4]. Constant coefficient can be determined only on the basis of quantities measurement characterizing operation of volumetric compressor during its work.

In the simplest case, numerical forms of dimensional functions, under discussion, can be approximated with linear functions. Such approximations are presented in Tab. 2. One should treat them as rough definition of dependence between quantities describing operation of volumetric compressors. As the best estimation of compressor operation function, they consider such estimation, whose sum of squares of difference between this quantity and a measured one is the least. The value of dimensional function of compressor operation cannot depend on adopted system of dimensionally independent quantities, i.e. dimensional base. On the other hand, the form of numerical function of compressor operation, which best of all, describes this operation, can differ, depending on the correct choice of this form.

Dimensionless arguments presented in Tab. 1, and taking place in dimensional functions, are called similarity invariants [3, 4, 7]. If similarity invariants are such, that for each point of work that belongs to dimensionless base of compressor operation, are fulfilled assigned limitations, then its operation is determined for all point of work, that belong to dimensional space.

Knowledge of numerical function concerning compressor operation, allows us to define its operation not only during working process, on the basis of measured parameters, but also anticipate them in assigned limitations of dimensional space.

Tab. 2. Linear estimations of numerical functions concerning the operation of volumetric compressor

Ordinal number	Linear estimations of dimensional functions	Dimensional base
$D = \Phi(p_t, p_{ss}, N_e, \tau)$		
1.	$D = A_o \cdot \frac{p_t \cdot N_e \cdot \tau^2}{p_{ss}} + B_o \cdot N_e \cdot \tau^2$	p_{ss}, N_e, τ
$D = \Phi_1(q_v, p_t, p_{ss}, N_e, \tau)$		
2.	$D = A_1 \cdot q_{rv} \cdot p_{ss} \cdot \tau^2 + B_1 \frac{p_t \cdot N_e \cdot \tau^2}{p_{ss}} + C_o \cdot N_e \cdot \tau^2$	p_{ss}, N_e, τ
3.	$D = A_2 \cdot N_e \cdot \tau^2 + B_2 \cdot p_t \cdot q_{rv} \cdot \tau^2 + C_1 \cdot q_{rv} \cdot p_{ss} \cdot \tau^2$	q_{rv}, p_{ss}, τ
A _i ; B _i ; C _i ; – real numbers determined on the basis of measurements (i = 0...4).		

3. Diagnostic properties of dimensional functions of volumetric compressor operation

Knowledge of function course of compressor operation in a given technical state, allows us to determine it, in other technical states of different compressor of the same type, on condition that similar processes of compression, take place in it.

From description of compressor, operation follows that pressure of suction and pumping, effective power and real capacity of compressor, constitute basic parameters of its work and operation. They define each state of compressor's work and therefore they can be used as diagnostic symptoms of its technical condition.

Besides, its technical state can be defined by similarity invariants, called discriminants: $\xi_{p_t} = \phi_{p_t}$ – complete compression of compressor, $\phi_{q_{rv}}$ – compressor capacity, ϕ_{N_e} – effective power, Φ_D – volumetric compressor operation and time of its operation. These discriminants have constant values, defined by the values of geometric, kinematic and dynamic parameters of the compressor.

It means that on the basis of these values deviations of the compressor just being tested. It was possible to make a diagnosis concerning its technical condition and compare to the model, in excellent condition. In order to determine an estimation form of numerical function, one should carry out diagnostic examination of volumetric compressor, in defined conditions of its work. Basing on these measurements, one can assume some conditions as for the form of numerical function.

Measurements results of work parameters that define dimensionless arguments of dimensional functions concerning compressor operation of the same type, in its different technical states, can be collected, in so-called, information bank of operation [3]. Such information bank of operation should have at its disposal analytical and qualitative record of numerical functions, received on the basis of dimensional functions. Qualitative records of numerical functions concerning volumetric compressor operation are presented in table 3.

Information bank of operation can be used, for both scientific-technical information needs and conducting next diagnostic research, taking advantage of collected information. Such bank will be able to import following information:

- is identification survey of technical state of volumetric compressors of definite type necessary,
- is compressor operation survey carried out earlier,
- isn't dimensionless base located in limitations, for which numerical function of compressor operation, has already been determined,
- are technical states of volumetric compressor operation sufficiently described in dimensional meaning, so that, making a diagnosis will not need identification.

Tab. 3. Qualitative records of numerical functions of compressor operation

Register of basic measure units	Dimensional quantities describing parameters of work at inlet and outlet of compressor		Operation of volumetric compressor in a given technical state	Information about technical state of compressor
1	2		3	4
Class percent	Inlet parameters	Outlet parameters	Operation features	
kg ; m ; s	$\xi_{p_t} = \frac{p_t}{p_{ss}}$ $\varphi_{q_{rv}} = \frac{q_{rv} \cdot p_{ss}}{N_e}$ $\varphi_{N_e} = \frac{N_e}{q_{rv} \cdot p_{ss}}$	$\Phi_D = \frac{D}{N_e \cdot \tau^2}$	$D = f_1(\xi_{p_t}) N_e \cdot \tau^2$ $D = f_3(\varphi_{q_{rv}}, \varphi_{p_t}) N_e \cdot \tau^2$ $D = f_6(\varphi_{N_e}, \varphi_{p_t}) q_{rv} \cdot p_{ss} \cdot \tau^2$	Short description of damage type of compressor

On the basis of relative quantities concerning energetic losses, taking place in volumetric compressor, one can also estimate its general technical state. Therefore, operation of volumetric compressor is a diagnostic symptom [5, 6].

Investigation of relative variability of energetic losses leads to determination of the function f_3 . In order to determine analytical form of numerical function of operation (3 b) one should carry out diagnostic testing of volumetric compressor in definite conditions of its work. Basing on these measurements one can assume some conditions concerning the form of numerical function.

4. Recapitulation

Analysing procedures of creating dimensional functions of volumetric compressor operation and their transforming into numerical functions, one can state:

1. diagnostic investigation is carried out in advance, in order to make a diagnosis concerning technical condition of the compressor, basing on measurements of work parameters during their operation,
2. it is necessary for the dimensional function to fulfil interpretation rules adopted with notions that describe operation of volumetric compressors,
3. selected dimensional quantities interfere significantly in description of volumetric compressors operation and restrict considerably a possibility of its description,
4. operation of volumetric compressors can be described, by means of such parameters of its work as:
 - operation of volumetric compressor D ,
 - suction pressure of compressor p_{ss} ,
 - pumping pressure of compressor p_t ,
 - effective power of compressor N_e ,
 - real capacity of compressor q_v ,
 - operation time of volumetric compressor τ .
5. on the basis of linear estimations of numerical functions given in table 2, it will be possible to define compressor's operation and make a diagnosis about its technical condition,
6. measurements results of parameters describing operation of volumetric compressors of definite technical state, taking place in dimensionless numerical arguments of function of their operation, can be collected in so called, information bank of volumetric compressor operation.

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