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MODULATION EFFECT OF VIBRATION FREQUENCY OF AN UNSPRUNG HEAVY MACHINE UNDER THE VARIABLE ROAD ADHESION CONDITIONS

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Abstract

Mobile heavy machines as unsprung vehicles exhibit low dissipation ability, hence the ride even at low speeds may give rise to intensive vibration. Particularly dangerous situations occur when the road wheels break away from the road surface due to the 'galloping' effect, being the result of excited vertical and angular vibration of the machine frame in the vertical plane of symmetry. That implies a major restriction on the ride velocity, which negatively impacts on the machine performance. Vibrations thus produced are mostly in the low-frequency range and hence energy dissipation in tyres will reduce the vibration intensity in a minor degree only. The motion of tired wheels will always involve some slipping. While investigating the feasibility of increasing the efficiency of the vibration reduction systems, one ought to take into account the variable adhesion of road wheels due to different dynamic loading acting on the vehicle axles during the ride. Observations of unsprung machines during the ride suggest the occurrence of self-excited vibration. Mobile machines constitute dynamic systems, which can be governed by nonlinear, sometimes non-stationary differential equations of motion. Their stability also depends on intensity of external vibrations. This study investigates the motion of unsprung mobile machines, taking into account the dynamic processes in the driving system under the conditions of the variable adhesion of road wheels. The model of interaction between a tired wheel and the terrain takes into account the relationship between the road wheel adhesion factor and the slipping action. Mathcad supported by Matlab-Simulink and in the frequency domain – simulations in the time domain. The purpose of the simulation procedure was to find the causes of the vibration modulation frequency and determine the conditions triggering the occurrence of self-excited vibrations. Simulations are supported by the analysis of motion stability.

Keywords: *mobile heavy machines, unsprung vehicles, 'galloping' effect, adhesion factor, slipping action, frequency modulation of vibration*

1. Physical model of an unsprung wheeled machine

In the context of the potential occurrence of self-excited vibrations during the ride of a wheeled machine, the plane model of an excavator-loader (Fig. 1) is implicated for further considerations. The excavator – loader is an unsprung machine in which the frame is rigidly connected to the axles. At that stage, the flexibility of fluid cells in the machine equipment is neglected. It means $\varphi_0 = \varphi_1 = \varphi_2$. The vertical response of the roadway surface to loads exerted by wheels is shifted in relation to the geometric axis of the wheel rotation [2, 6]. The magnitude of this eccentricity – e_{i} is a variable quantity as it is related to the dynamic wheel radius – r_{iD} through a dimensionless resistance factor f_{i}:

$$f_{\mathrm{i}} = \frac{e_i}{r_{\mathrm{iD}}}, \quad \text{where: } \mathrm{i} = 1, 2., \tag{1}$$

The value of the rolling resistance of a tired wheel depends on the roadway surface and ride velocity. In a narrow range of velocities, the rolling resistance is taken to be constant, dependent solely on the roadway surface conditions.

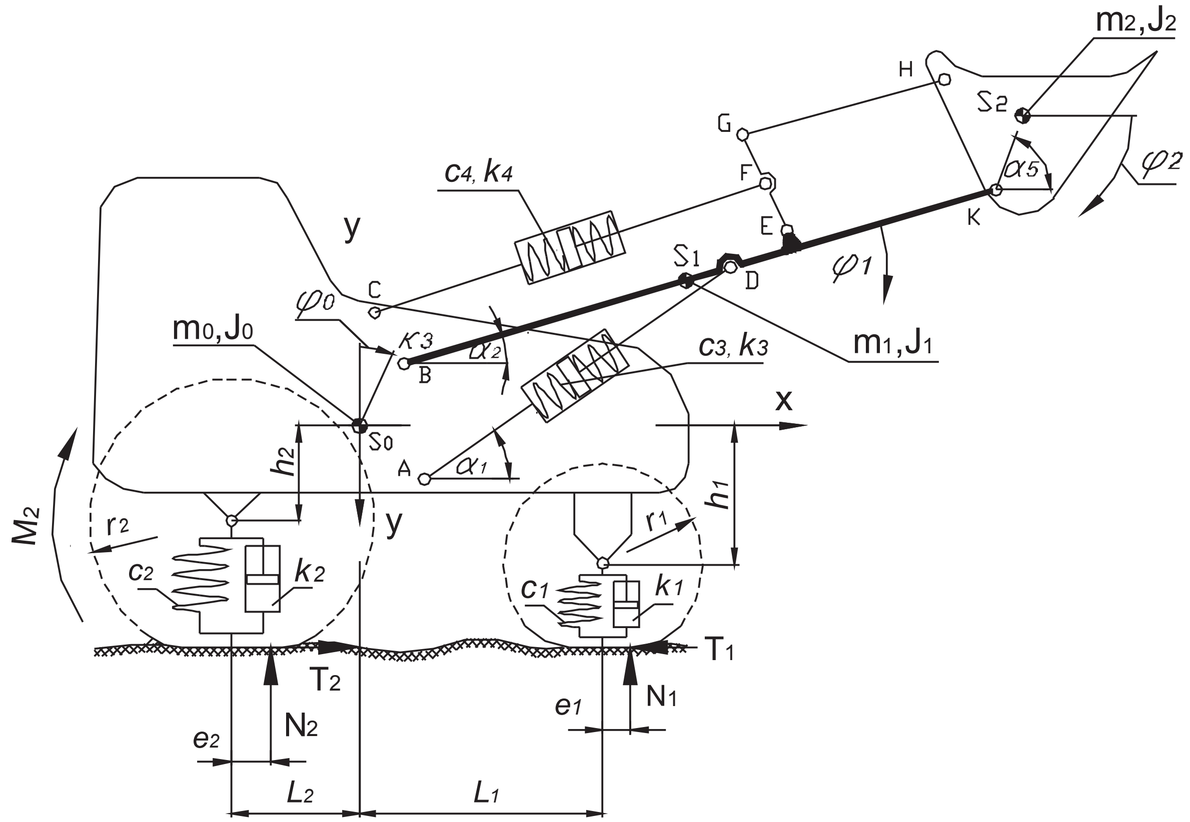

Fig. 1. Physical model of a two-axle wheeled machine

The notation of damping forces acting in the radial and circumferential directions takes into account the dependence between the damping factor in the tire and vibration frequency – ω [5]:

$$k = c\frac{\delta}{\omega}, \tag{2}$$

where:

δ – coefficient of energy dissipation related to the type of tires, typically $\delta \in [0.1\text{–}0.2]$,

c – stiffness of wheel tires of a given axle in the radial /circumferential direction.

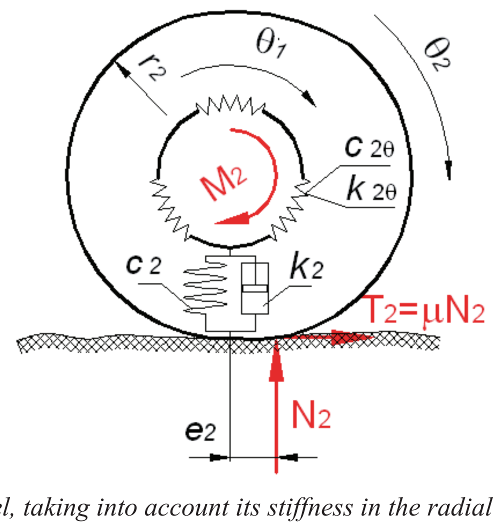

Fig. 2. Model of a tired wheel, taking into account its stiffness in the radial and circumferential direction

Neglecting the inertia of the outer part of the driving wheel's tire tread, differential equations of motion of a vehicle shown in Fig. 1 can be written as:

$$\begin{bmatrix} m_C & 0 & 0 & 0 & 0 \\ 0 & m_C & 0 & 0 & 0 \\ 0 & 0 & J_{C0} & 0 & 0 \\ 0 & 0 & 0 & m_C r_{2s}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} d^2x/dt^2 \\ d^2y/dt^2 \\ d^2\varphi_0/dt^2 \\ d^2\theta_1/dt^2 \\ d\theta_2/dt \end{bmatrix} = \begin{bmatrix} \mu(S)N_2 - f_1 N_1 \\ m_C g - N_1 - N_2 \\ (x_1 + f_1 y_1)N_1 + [x_2 - \mu(S)\, y_2]N_2 \\ M_2(\dot{\theta}_1) - [f_2 - \mu(S)]N_2 r_{2D} \\ c_{2\theta}(\theta_1 - \theta_2) + k_{2\theta}\dfrac{d}{dt}(\theta_1 - \theta_2) - [f_2 - \mu(S)]N_2 r_{2D} \end{bmatrix}, \quad (3)$$

where:
m_C – total mass of the machine,
J_{C0} – total inertia moment of the machine with respect to the centre of gravity,
N_i – load of i-th axle,
x_i, y_i – geometric coordinates of the i-th axle in the mobile coordinate system associated with the machine frame,
r_{2s} – static radius of the driving wheel,
$\mu(S)$ – coefficient of road adhesion associated with roadway surface and the wheel slip in the circumferential direction S.

The (circumferential) slip rate expresses the relative difference between linear velocities of the axles in the horizontal direction resulting from the rotating motion (no slippage) and the real velocity [1, 3, 5, 6]. In the case when $d\theta_2/dt > 0$:

$$S = 1 - \frac{\dfrac{dx}{dt} - y_2 \dfrac{d\varphi_0}{dt}}{r_{2D}\dfrac{d\theta_2}{dt}}. \quad (4)$$

The relationship between the coefficient of road adhesion and the slip rate for various roadway conditions is modeled by a bi-parametric function $\mu(S)$.

This function can be determined as long as the maximal coefficient of road adhesion – μ_{max} (reached under the critical slippage conditions – S_{kr}) can be found for the given roadway surface.

$$\mu(S) = \frac{\mu_1 S}{(\mu_2 + |S|)^2}, \quad (5)$$

$$\mu_{max} = \mu(S_{kr}) \Rightarrow \begin{cases} \mu_1 = \mu_{max} 4S_{kr}, \\ \mu_2 = S_{kr}. \end{cases} \quad (6)$$

Selected plots of the function $\mu(S)$ for diverse road conditions are shown in Fig. 3.

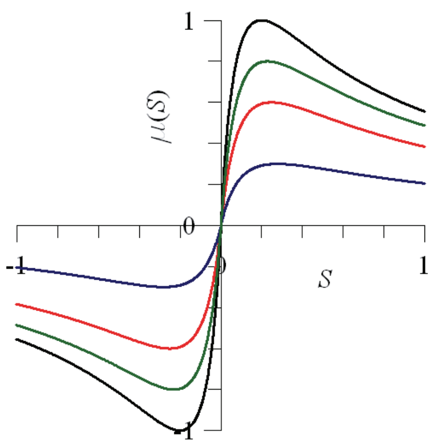

Fig. 3. Coefficient of road adhesion vs. slip rate for various roadway surfaces

The decrease of the circumferential adhesion of the tires during heavy slippage creates favorable conditions for self-excited vibrations to arise.

The rear axle wheels are driven by the applied torque M_2. The angular velocity of the driving wheel is dependent on the motor loading. The torque vs angular velocity characteristic of a diesel engine, reduced to the drive shafts, is shown in Fig. 4. The plot graphed with heavy line $\alpha = 1$ represents the maximal torque developed for the full dose of fuel. For smaller loads, the desired ride velocity is reached with a smaller dose of fuel supplied by the feed pump. The load characteristics obtained for smaller fuel doses in relation to the maximal one, designated as $\alpha = 0.9$, $\alpha = 0.7$, $\alpha = 0.6$, are graphed with fine lines [2].

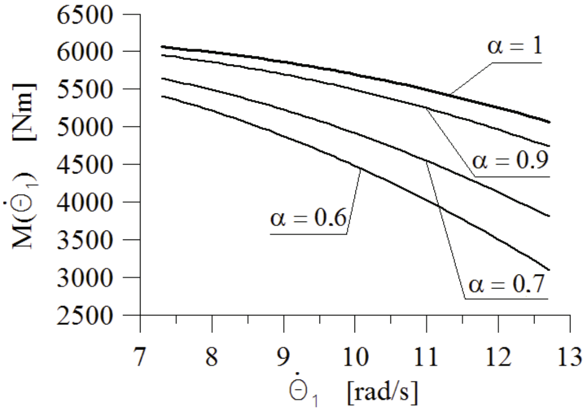

Fig. 4. Torque vs. angular velocity of an engine reduced to the drive shafts, for variable fuel doses

2. Vibration of an unsprung heavy machine

Computer simulations, supported by Matlab-Simulink and Mathcad, were performed for the applied kinematic excitations and for the full bucket load 2000 kg. The average angular velocity of the driving wheel was taken to be 10 rad/s. Parameters of the machine configured as in Fig. 1 are: $m_C = 9420$ kg, $J_{C0} = 10427$ kgm^2, $L_1 = 0.9509$ m, $L_2 = 1.1491$ m, $h_1 = 0.5095$ m, $h_2 = 0.3095$ m, $r_{s1} =$ $= 0.4335$ m, $r_{s2} = 0.6701$ m, $c_1 = 7.676 \cdot 10^5$ N/m, $c_2 = 1.383 \cdot 10^6$ N/m, $c_\theta = 4.0 \cdot 10^6$ Nm/rad, $f_1 = f_2 =$ $= 0.03$, $\delta = 0.2$. The tire stiffness in the radial and circumferential direction was obtained by applying procedures outlined in [1, 4].

Tab. 1. Natural frequency related to the maximal value of the road adhesion coefficient

μ_{max} [–]	f_{e1} [Hz]	f_{e2} [Hz]	f_{e3} [Hz]	f_{e4} [Hz]
0.3	0	0	1.9092	2.8782
0.4	0	0	1.9096	2.8919
0.5	0	0	1.9101	2.9067
0.6	0	0	1.9106	2.9226
0.8	0	0	1.9117	2.9581
1.0	0	0	1.9131	3.0075

Tire stiffness in the circumferential direction and road adhesion conditions are found to have a minor influence on the natural frequency – see Tab. 1. The equation of motion (3) is transformed to yield the equation of state in the matrix format. Underlying the stability analysis of the machine's motion during the ride is the Lapunov's method I, illustrated in Fig. 5 and 6.

The roots of the characteristic equation for wheel tires flexible in the radial direction are designated by circles, those obtained for inflexible ones - by crosses. It was established that the motion in the vertical plane of the machine while it moves forward is implemented in the conditions of the stability limit. Five roots of the characteristic equations are found on the imaginary axis of the complex plane, no matter what the road adhesion conditions and tire flexibility along its circumference. Stability tends to deteriorate with the worsening of the road adhesion conditions, and when the circumferential flexibility of the tires decreases.

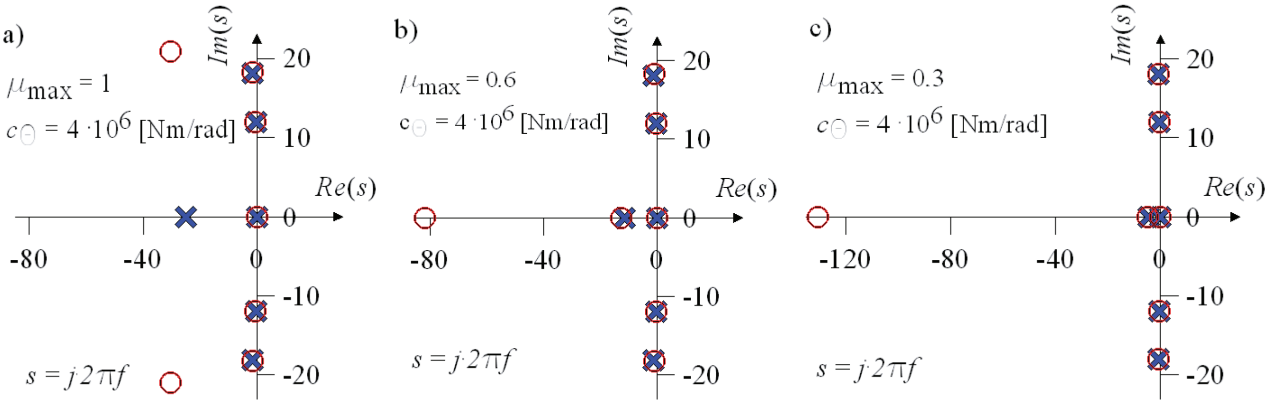

Fig. 5. The influence of road adhesion conditions on stability of the machine's motion when it moves forward

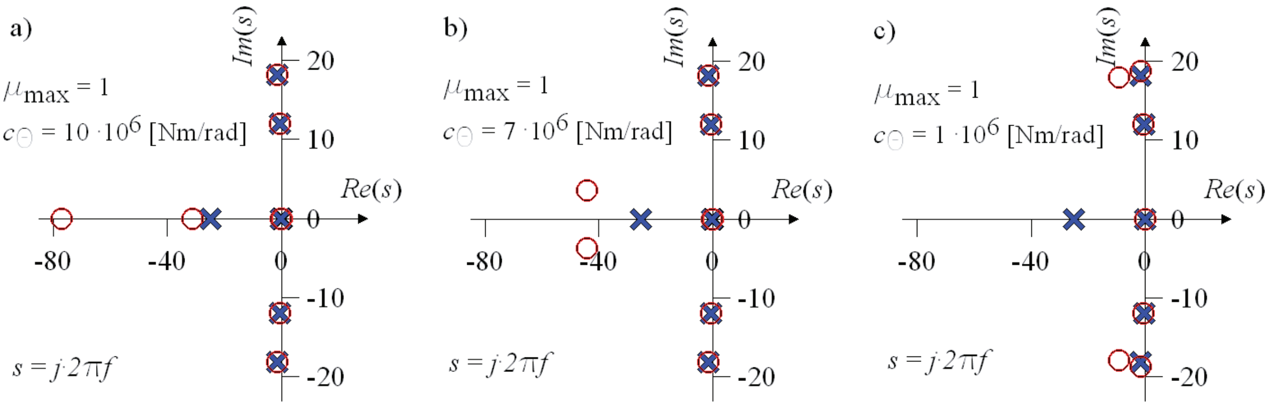

Fig. 6. The influence of circumferential flexibility of drive wheels on stability of the machine's motion during the forward ride under the maximal road adhesion conditions (μ_{max} = 1)

Amplitude-frequency characteristics are obtained for all degrees of freedom (DOF-s) and for inflexible tires (line 1) and those resilient in the circumferential direction (line 2).

Selected characteristics obtained for the vibration acceleration shown in Fig. 7-12 have relevance to the maximal road adhesion conditions (μ_{max} = 1) and amplitude of the unevenness profile Z = 10 mm. In each case, the frequency modulation effect was observed, most intensive with respect to vertical and angular vibrations of the vehicle, particularly in the resonance frequency range. As regards the angular acceleration of the drive shaft, the intensity of modulation tends to decrease after the resonance frequency is exceeded and filtering capabilities of wheel tires, resulting from their flexibility along the circumference, are best revealed.

4. Conclusion

The vertical motion of an unsprung heavy machine while it moves forward is implemented under the condition of stability limit, mostly due to stiffness of the dynamic system in the horizontal direction. The stability of the machine's motion in the horizontal direction is neglected at this stage. Simulations revealed a strong tendency of rear wheels to break away from the roadway surface under the excitations in the frequency range near to f_{e4} – see Fig. 12.

The ride with no payload enhances the risk of the manoeuvrability loss caused by the front wheels being lifted, further enhanced by the „galloping effect".

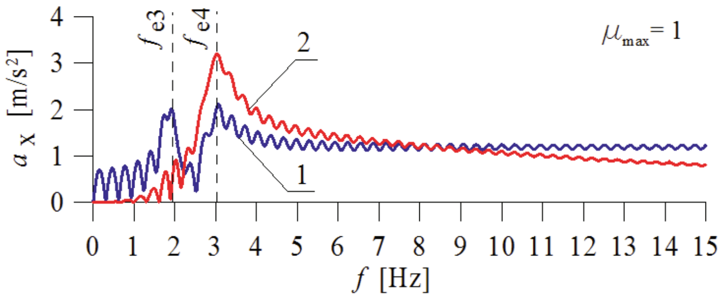

Fig. 7. Amplitude-frequency characteristic for the vehicle's acceleration during the progressive motion

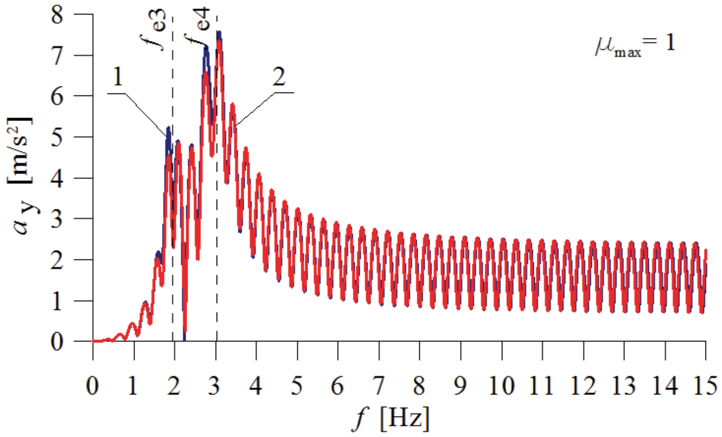

Fig. 8. Amplitude-frequency characteristic for the vehicle's acceleration in the vertical direction

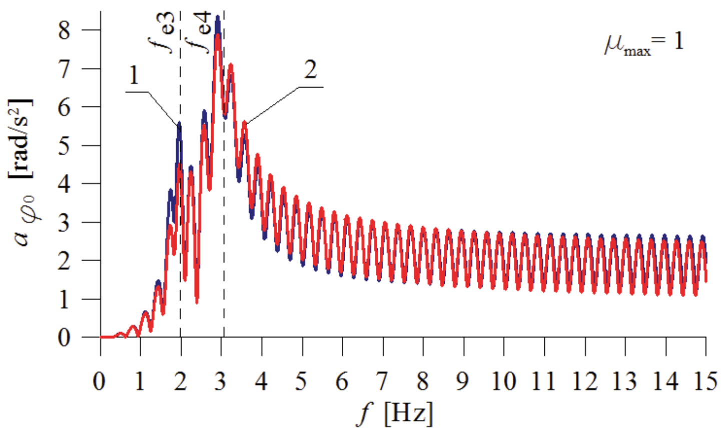

Fig. 9. Amplitude-frequency characteristic for the angular acceleration of the vehicle

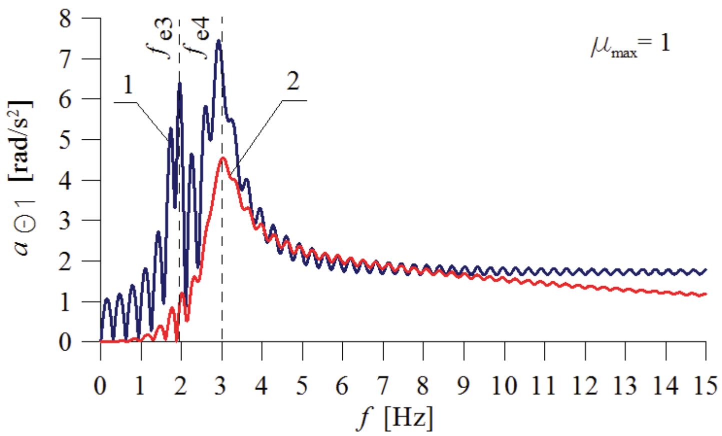

Fig. 10. Amplitude-frequency characteristic for angular acceleration of the drive shaft

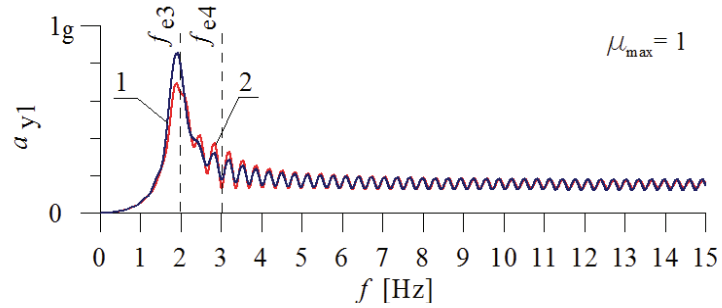

Fig. 11. Amplitude-frequency characteristic for the front axle acceleration in the vertical

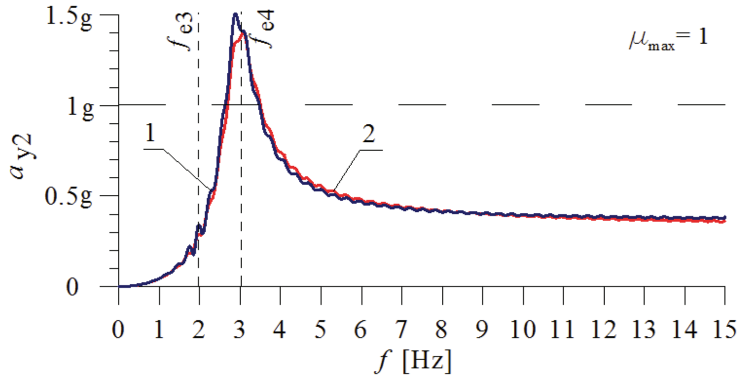

Fig. 12. Amplitude-frequency characteristic for the rear axle acceleration in the vertical

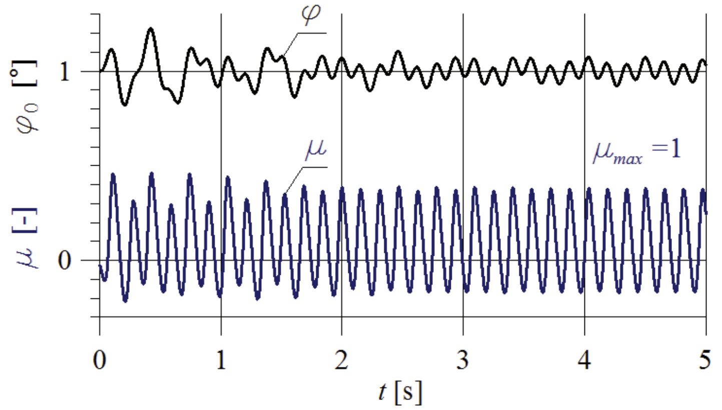

Fig. 13. Angular vibration of the machine and coefficient of road adhesion in the function of time

In a minor degree only do axle loads transmitted onto the base machine depend on the road adhesion conditions and wheel stiffness along their circumference, which implicates little influence that slipping has on angular and vertical vibration of the machine when the flexibility of fluid cells in implements is neglected in the ready-for transport configuration.

Flexibility of wheel tires significantly reduces the dynamic loading of the driving system.

Kinematical interactions of tires with the roadway surface are implemented in the conditions of variable road adhesion and slip rate. During one cycle of vibration, both the coefficient of road adhesion and the slip rate change both their magnitude and sign, which is illustrated in Fig. 13. Combined with strong nonlinearity of the dynamic system and circumferential friction models, that gives rise to the frequency modulation effect, at the frequency range nearing f_{e4}.

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