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THE MULTI-OBJECTIVE OPTIMIZATION OF THE AIR TRANSPORTATION FLEET STRUCTURE

Andrzej Majka

Rzeszow University of Technology, Department of Aircrafts and Aircraft Engines
Powstancow Warszawy Street 12, 35-959 Rzeszow, Poland
tel.:+48 17 8651604, fax: +48 17 8651942
e-mail: andrzej.majka@prz.edu.pl

Abstract

One of the most important factors in the development of modern civilization has become the increase in the significance of mobile and fast means of transportation of people and cargo. Aircraft as the fastest available means in the entire transportation system is the basis in places where the distance in the transportation of people is long and the travel time is short. The present work focuses on passenger transport in the European area using light transport aircraft, which may complement existing air transport and can become a real alternative to those who travel by other means of transport. The system of local transportation will become competitive in relation to other means of transport and will find its place on the market only when it has the highest indicators of efficiency. It can be reached by optimally obtaining passenger aircrafts fleet structure performing transportation tasks and optimizing its functioning. One of the basic problems is the rational selection and use of aircrafts, i.e. the minimization of their quantity while at the same time guarantee complete performance of transporting tasks. The selection of the best method of using the means of transport is connected with a large number of alternative variants, which makes it necessary to use special methods of searching for optimal solutions. The quality of the aircraft fleet should be estimated on the basis of several criteria when taking into account the difference in performing tasks. The way to design a competing aircraft fleet is to choose its characteristics by using advanced methods of multiple objective optimization. This work presents the methodology of the optimal designing of the aircraft fleet using the multitask character of the matter which is based on multiple objective mathematical programming in the concept of the set theory.

Keywords: *air transport, small aircraft, aircraft selection*

1. Introduction

1.1. Context and background

Some parts of Europe have a badly developed road and motorway network, as well as railroad and aviation lines. No doubt, there is a necessity to develop a transport infrastructure (especially motorways and airports) which is in line with fast economic progress. In those regions, the existing motorway sections regarding their partition do not guarantee long-range inter-regional communications. One of the ways leading to improvement of the situation is development of local aviation services. Air transportation as a part of the passenger carriage sector is characterized by a much higher average speed of transportation, which speaks to its undoubted advantage in relation to other means of transport. The infrastructural requirements are limited mostly to the airports as the isolated infrastructure. In order to use the mobility and potential of transportation performed by aircraft to the full, it is necessary to determine possible available places to perform take-offs and landings, i.e. operational-technical data, possibility to use them etc. The cost of creating a system enabling correct functioning of local aviation services, in spite of common opinions, is not quite high. Modernization of small airports enable using them in bad weather conditions would be limited to building asphalt or concrete runways with lightening. Thus, creating a network of airports an access to which will not be more than 20 to 30 minutes from each point of the country does not require a significant expenditure in comparison to the sums of money allocated to the development of the transport infrastructure.

A typical trait of European market is co-existence of a few, however, large transportation hubs that perform transcontinental flights and a dense net of local routes between the majority of small cities and tourists centres. Europe possesses an enormous number of partially unused airports and landing fields (Fig. 1) which can become a basis for creating competitive travelling offers around Europe using light commercial aircraft and less busy airports as well as relevantly adjusted and re-qualified landing fields addressed to those travelling by passenger airplanes so far. In the integrated system of communication of Europe, aviation has a chance to perform a substitution role in the places where road or rail infrastructure is poorly developed and its modernization and extension would demand significant financial costs. In the present regional structure, the air services play an integrating role and eliminate a disproportion in the level of economic development between the regions.

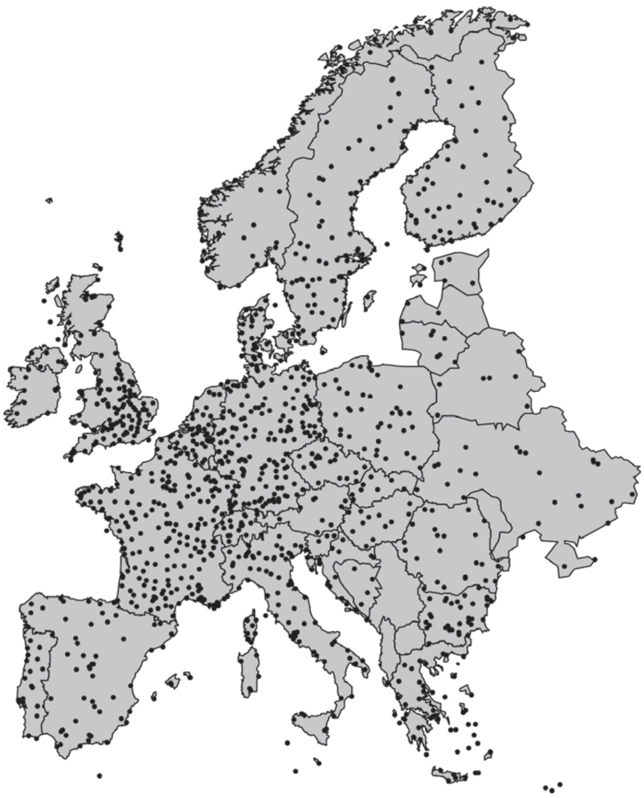

Fig. 1. The map of Europe with the marked airports [Source: Own elaboration]

1.2. Analysis methodology

The characteristic feature of the aircraft is their multipurpose and multitask character. This property concerns a single object as well as their sets, which constitute a certain fleet. It reveals in different aims which this fleet of objects is to fulfil (e.g. a fleet of transport aircraft) and in different conditions of its functioning. This defines the multipurpose (universal) character of the aircraft use. A lot of tasks performed using the aircraft determine the necessity of using different factors to estimate their effectiveness. Quality assessment of reaching the aim on the basis of these factors has such a feature that for a aircraft with the determined parameters (geometric, aerodynamic and performance) the highest quality is reached, as a rule, in a single task. Although, when performing all other tasks, homogeneous or non-homogeneous, the aircraft always loses the quality from the point of view of reaching the aim in comparison to its highest value. This type of loss characterizes the level of universalism when performing certain tasks. The way to increase the effectiveness of achieving the aim is to use the aircraft not in the whole range of possible applications but in a narrower range (specialization), i.e. in the optimal fields of specialization of the aircraft. From the aircraft fleet properties [4]:

- existence of different conditions of functioning and task performing,
- using many quality coefficients to estimate the aircraft fleet,
- the complex aircraft fleet structure consisting of many different aircrafts (autonomous elements) between which a particular task performing is divided.

Follows that, the mathematical model of the aircraft fleet can be a multitask system [4]. The selection of the aircraft fleet structure is based on the results of distributing the tasks from the alternative fields between the "competing" aircraft and determining the fields of the most effective use for each of them. In order to make an optimal choice of the fleet structure a complete computational model of an aircraft was made (aerodynamic, power unit, performance and cost) and then the method allowing choosing the aircraft types regarding multiple estimating criteria of the obtained solutions.

2. Multiple objective optimization

The way to design a competing aircraft fleet is to choose its characteristics by using advanced methods of multiple objective optimization. Choosing the best and optimal solution plays an important role in many engineering issues. In practical problems of the optimal choice of the technical system parameters we usually deal with the case in which the choice does not depend on one criterion (objective function), but on many criteria simultaneously. In aircraft design, the optimal values of weight, lift to drag ratio, selected performance parameters, cost and etc. are usually sought [1, 3, 6, 15-18]. This simultaneous optimization, which takes into account several criteria, can be expressed by a multi objective optimization (MOO) problem formulation that can be regarded as a generalized single objective optimization problem. In recent years all over the world, a lot of methods of solving the MOO problem have been discussed. Among them, it is possible to distinguish: Weighted Objectives Methods [2], Hierarchical Optimization Methods [19], Trade-Off Methods [10], Global Criterion Methods [8], Distance Functions and Min-Max Methods [14], Goal Programming Method [9] and more willingly unconventional approach to the MOO is used using Evolutionary Algorithms [5, 7, 13].

An MOO problem is connected with the search of the extreme values (minimum and maximum) of several objective functions. Additionally the MOO problem may contain a number of constrains which must be fulfilled for optimal solutions. As the objectives can be minimized or maximized, the general formulation of the MOO problem in a mathematical sense can be as follows [13]:

$$\text{Minimize/Maximize } \mathrm{f_n}(\boldsymbol{x}) = [\mathrm{f_1}(\boldsymbol{x}), \mathrm{f_2}(\boldsymbol{x}), \dots, \mathrm{f_k}(\boldsymbol{x})], \tag{1}$$

subject to:

$$\mathrm{g_i}(\mathrm{x}) \leq 0, \mathrm{i} = 1.2, \dots, \mathrm{m}, \tag{2}$$

$$h_i(\boldsymbol{x}) =, i = 1.2, \dots, p, \tag{3}$$

$$x_i^{(L)} \leq x_i \leq x_i^{(U)}, \tag{4}$$

where: $\boldsymbol{x} = [x_1, x_2, \dots, x_n]^T$ is the vector of decision variables. The solution which fulfils the constraints (2) and (3) and limitations put on the decision variables (4) defines the search space $\boldsymbol{S}$, where $f_i : \boldsymbol{R}^n \to \boldsymbol{R}$, $i = 1,\dots,k$ are the objective functions and g_i, h_j: $\boldsymbol{R}^n \to \boldsymbol{R}$, $i=1,\dots,m$, $j=1,\dots,p$ are the constraint functions. The feasible design space is the set of all design points represented by design variable vectors that satisfy constraints (2), (3) and (4). In general, for MOO, the task is to identify Pareto set points. Pareto set points are a set of Pareto optimality. A vector of decision variables $\boldsymbol{x}^*$ is Pareto optimal, if there is no feasible vector $\boldsymbol{x}$ that would improve some objective function without effecting simultaneous deterioration of at least one objective function.

2.1. The multiple objective optimization of a multitask system

Three main elements - set ***A*** called a set of system elements, task set ***Y*** and integral function E(***y***), constitutes of multitask system: <***A***, ***Y***, E(***y***)>. The vector of quality of the multitask system can be defined as follows [4], [11]:

$$F = F[\boldsymbol{A}, \boldsymbol{Y}, E(\boldsymbol{y})]. \tag{5}$$

Putting the mathematic multitask system into the notion of local quality function $f[\boldsymbol{x}, \boldsymbol{y}, \mu(\boldsymbol{D})]$ of the field of specialization $\boldsymbol{D}_i$ of the system element (e.g. aircraft in aviation system) $\boldsymbol{x}_i \in \boldsymbol{A}$, it is possible to express the coefficient of the multitask system quality (5) in terms of its values in particular fields of specialization $\boldsymbol{D}_i$ of certain elements $x_i \in \boldsymbol{A}$:

$$F[\boldsymbol{A}, \boldsymbol{Y}, E(\boldsymbol{y})] = \sum_{i=1}^{m} \sum_{y_j \in D_i} f\left[x_i, y_j, \mu(\boldsymbol{D}_i)\right] \; and \; \boldsymbol{Y} = \bigcup_{i=1}^{m} \boldsymbol{D}_i, \tag{6}$$

where $\mu(\boldsymbol{D}_i)$ – field of specialization measure $\boldsymbol{D}_i$.(e.g. aircraft utilization index, etc.)

The desire to take into consideration when estimating the effectiveness of the multiplicity of the solutions and heterogeneity of their traits and performance properties leads to different criteria and as a result to heterogeneous estimation of the efficiency. It leads to vector optimization task for which the methods of the solution seeking are known. The quality of solution $\boldsymbol{x} \in \boldsymbol{X}$ will be estimated by a certain set n element of the scalar factor which is usually interpreted as a certain vector of objective function values F. It is necessary to choose in the accepted set $\boldsymbol{X}$ the best in some respects variant $\hat{\boldsymbol{x}}$. It is known that the mathematical vector optimization model corresponds to this formulation

$$F(\hat{\boldsymbol{x}}) = opt_{\boldsymbol{x} \in \boldsymbol{X}}\big(F(\boldsymbol{x})\big), F = \{f_1, \dots, f_n\}, \tag{7}$$

where: *opt* – is the operator of the optimization of the vector of objective function values which specifies the principle of the solution variants preference. For the purpose of a single-valued choice of the solution an additional principle (a rule or hypothesis) is introduced to the analysis which specifies the compromise scheme between the factors $f_1, \dots, f_n$. This principle in the mathematical view is given by the functional of the components of the vector of objective function values (i.e. the component synthesis method)

$$F = \varphi[f_1, \dots, f_n]. \tag{8}$$

The introduction of the principle of the synthesis φ converts the vector of objective function values into a scalar functional and brings the process of the solution choice to the case of the scalar optimization. The most widely known method of the synthesis is linear synthesis of the components $f_1, \dots, f_n$ to the scalar criterion of the function

$$F(\boldsymbol{x}) = \sum_{i=1}^{n} w_i f_i(\boldsymbol{x}), \tag{9}$$

where w_i – weights which model the accepted system of the preferences fulfilling the conditions:

$$\sum_{i=1}^{n} w_i = 1, w_i \geq 0, i = 1, \dots, n. \tag{10}$$

These conditions highlight in the zone of the weights a certain area W which will be interpreted as a set of uncertainty. Fig. 3 on the subspaces w_1, w_2 presents sets $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ for the cases $w_1 > w_2$, $w_2 > w_1$ correspondently. Point $w_1 \cong w_2$ reflects the situation of preference equivalence. The weights w_1 and w_2 determines the slope of this line and establishes the essence of the compromise between the elements f_1 and f_2 (Fig. 2). The solution in this case is led to the choice of the certain values of $w_i \in [0, 1]$ made by a group of experts, and the principle φ separates the set of possible ratios for the uncertainty of the efficiency estimation. In many tasks there appears the necessity to establish desirable value limits of the factors themselves.

$$f_i \in [f_{i\,min}, f_{i\,max}], i = 1, \dots, n. \tag{11}$$

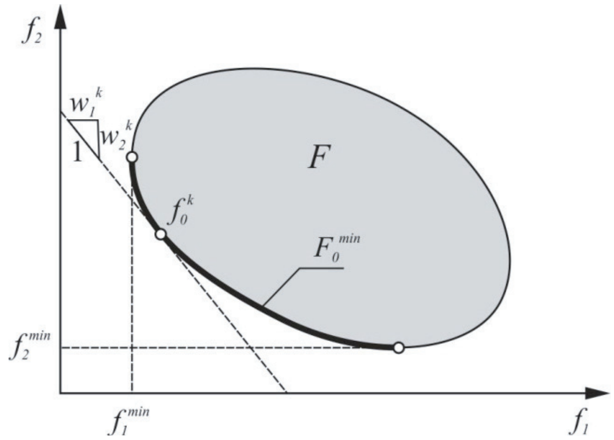

Fig. 2. Pareto set - minimization of the factors f_1, f_2

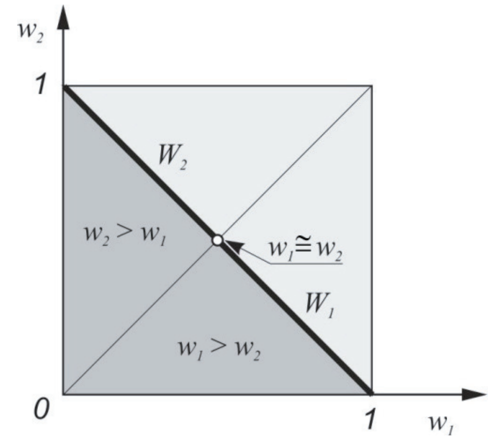

Fig. 3. Sets W_1 and W_2 for 2 criteria

For example, for aviation multitask system, these are the limits of the aero-technical, aerobatic, aerodynamic and weight values and other characteristics of parameters of the aircraft. On this basis it is possible to determine the limits of the values of the weights:

$$w_i \in [w_{i\,min}, w_{i\,max}], i = 1, \dots, n. \tag{12}$$

Thus, the question of taking into account the uncertainty in the efficiency estimation and the choice of a relevant solution leads to construction of the weights set and summing of products $w^T \cdot f(x, y)$.

3. Task division

Using the algorithm described in [12] the tasks were distributed between the aircrafts of the transport system based on operating cost [17] criteria and transport effectiveness [11, 12]. Operating cost defines total cost of all tasks realization, and transport effectiveness shows the aircraft quality of utilization.

The calculations were conducted with the following simplifying assumptions:

- a single task realized by the aircraft during a continuous mission. It excludes inter-landings and the possibilities to perform several tasks in a multistage flight,
- there is no time limit to perform a particular transport task. It means that passenger flow is not taken into account in particular lines. This approach is dictated by a complete lack of information about this topic and it is in accordance with the previous assumption,
- the number of each type of the aircrafts is unlimited (unlimited resource for each type). Simplifying means that we have so many aircrafts as we need.

The result obtained regarding the abovementioned assumptions does not allow estimating the quantity demand for small aircraft in Europe, although it helps determine the preferable types depending on the area of use.

4. Problem formulation

In this example of computation the system consisting of four types of aircrafts with different dimensions and transport capacity was assumed (Tab. 1). The calculations were made for a 50 different tasks generated with the assumed schedule, typical for the European transportation (Fig. 4). Each transportation task included caring an assumed number of passengers n_{pax} from the initial airport to the destination airport, in distance L_Z. The schedule of the transportations in the function of the route length for all tasks is presented in Fig. 5. The task was to optimal distribute tasks between aircraft of the fleet to find areas of specialization and the number of copies for each type for two selected criteria.

Tab. 1. Aircraft data

Description	Aircraft No 1	Aircraft No 2	Aircraft No 3	Aircraft No 4
Wing span, m	12.00	14.94	17.24	22.06
Wing area, m^2	18.60	30.20	34.27	39.72
Length overall, m	9.30	10.86	14.86	16.38
Engine power, kW	2 x 149	2 x 298	2 x 462	2 x 820
Take-off weight, kg	2084	3175	5300	7500
number of passengers, -	6	9	17	19

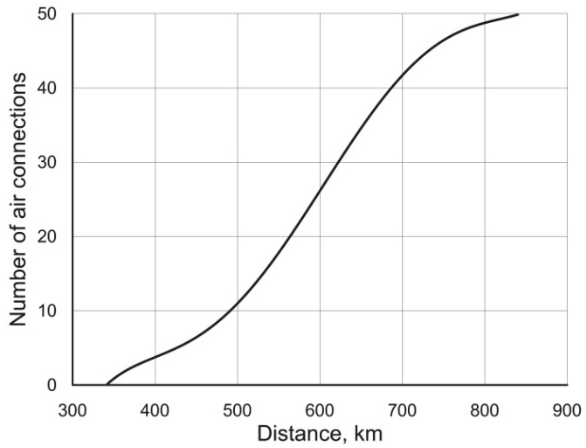

Fig. 4. Cumulative distribution function of the air connections typical for short haul flights in Europe

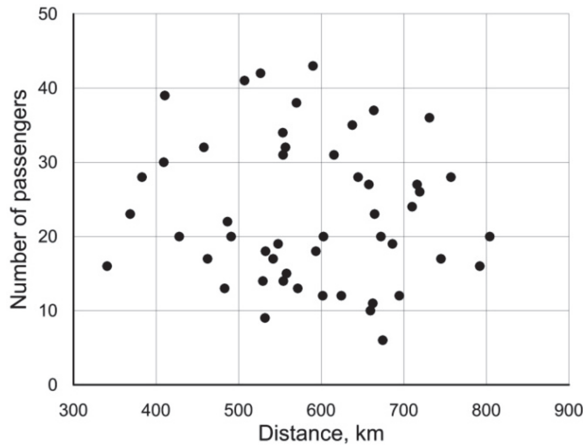

Fig. 5. The schedule of the transportation in the function of the route length

Figure 6 shows a set of compromise solutions. For the chosen weight factors (w_i) a compromise solution can be obtained depending on the accepted system of preferences. If both criteria are equivalent, the optimal type of the aircraft is number 1 (the smalest aircraft). For weight factor $w_1 = 0$ the result is obtained with taking into consideration only one criterion – transport effectiveness. In this case is preferred aircraft number one (the smallest) with small share of other aircrafts. For weight factor $w_1 = 1$ the result is also obtained with taking into consideration only one criterion – operating cost. In this case is preferred aircraft number three with small share of the aircrafts 1 and 4. If transport effectiveness is more important ($w_2 > w_1$) the smaller aircrafts are prefered, for cases when costs are more preferred, larger types of aircraft are justified.

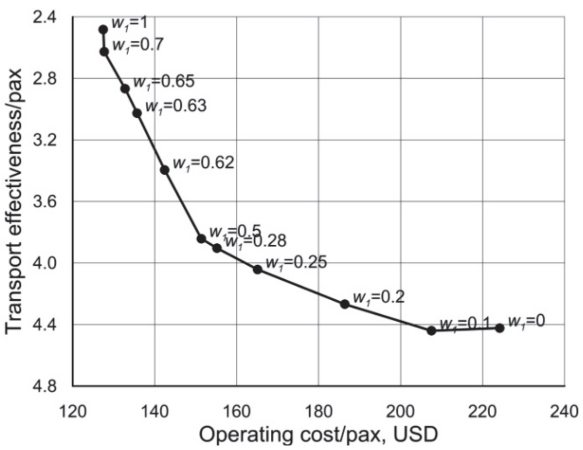

Fig. 6. The envelope of the best values of the analysed criteria (Pareto set) for different weight factors

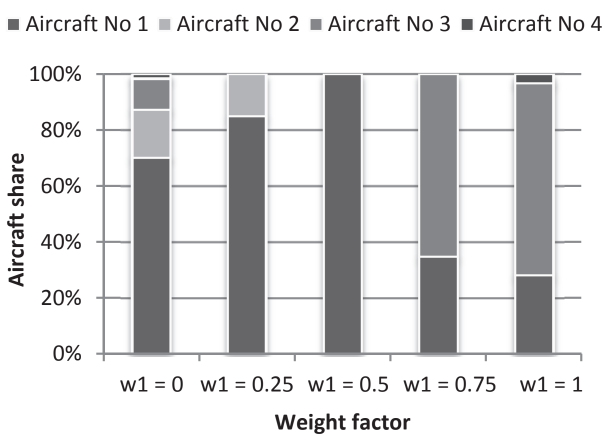

Fig. 7. Aircraft share in task realization for different weight factors

4. Summary

Among the transport aircraft the light airplanes find a wider application especially regarding their low cost of use and high flexibility in adjusting to the changing market conditions. Regardless of the fact that it will never be a common means of travelling, transportation of passengers by light planes can significantly help and complement the existing air transportation system. The basic means of transport would be small airplanes able to use small airports which are not currently in use to service the passenger or goods flow. Implementation of the system would help smaller societies gain an access to the air transport, which would positively influence the economic revitalization of the regions, especially those with poorly developed road infrastructure. The way to design a competing aircraft is to choose its technical parameters and area of use, by using the advanced methods of multiple objective optimization. The main aim of the work was to demonstrate a method of aircraft selection at the preliminary design stage of the tranportation fleet. The work focuses on the choice optimal type of the aircraft and number of aircrafts minimazing selected criteria. The method could be also helpful at preliminary design stage of a new aircraft to selection basic design parameters.

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