THE HYPERDEFORM MATERIALS IN SYSTEMS OF VIBRATIONS MACHINES

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Abstract

In construction of machines more and more often be practical used materials name hyper-elastic for example: foamy structures, materials of base natural and synthetic rubbers and another of large reversible deformations. To description of elastic proprieties, it is possible to use the worked out theories well in which the most important they are the multinomial models and them special coincidence: Mooney–Rivlin, Yeoh and Ogden. Precise description propriety in questions dissipation the energy, dumping of vibration is a fault currently. The aim of the article is the analysis oscillation in systems from elements of incompressible and the description of phenomena setting during of work of such like systems.

In this paper, is presented built methodology of elasticity propriety materials with large deformation. An assumption is a dissipation function we have present in analogous figure to elasticity function, well known with theory of hyper-elastic materials. A description of model of internal damping was proposed with the proof the influence of component part of polynomial: figure and volumetric. The numeric permissive on comparison of received conception of description of hyperelastic material worked out in support about theoretical foundations with results of experimental investigations analyses' be become executed.

***Keywords**: incompressible materials, elastomers, oscillation, very large reversible deformations, vibration*

1. Introduction

Dumping vibration problems are questions of mechanics, in exploitation of different devices [1], in motor industry, machine, air. A shock absorber is use when, the dumping across selection of masses and dimensions it is not used or from different regards were have given up him [2]. In case of effect dynamic load about large changeability of parameters desirable using the systems of adaptive absorption of energy clever to quick change his dynamic characteristic [3]. A basic question in dynamic load analysis on objects is the dissipation the kinetic energy in time of impact [4].

We can show absorbers which constructions be subordinate on: destruction kinematics process, the kinematics of process of destruction, method of action, the mechanism of dissipation, progress of cracks, delamination, exchange of energy of impact in kinetic energy of liquid and the warmth, exchange of energy of impact in kinetic energy of rotary motion [8]. In constructions of polymer composite structures the sources of energy dissipation are: matrix of polystructural viscoelasticity, morphology of material, defect of structure, thermoplastic and viscoplastic dumping [5, 6]. We in the paper will find the thesis [7], that proper on dumping can be relation by ratio of: dissipation energy to accumulated energy of deformation.

2. Vibration systems with hyperelastic elements

It foundations in analysis were accepted was the practical in theory of hyperelastic materials [9] – elastomer is a isotropic incompressible material about very large reversible deformations. It takes root theory complete deformations it – on principal directions takes root deformations (1):

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right],$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right], \tag{1}$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right].$$

It was introduced symbols (2):

$$\begin{aligned} \lambda_1^2 &= 1 + 2\varepsilon_{xx}, \\ \lambda_2^2 &= 1 + 2\varepsilon_{yy}, \\ \lambda_3^2 &= 1 + 2\varepsilon_{zz}. \end{aligned} \tag{2}$$

Foundation about incompressibility of material allows to ascertainable that Poisson's number is constants in whole body steps out homogeneous state: tension – hydrostatical pressure about value peaceably with dependence (3):

$$p = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}. \tag{3}$$

The volume of body is invariable, a product of deformations always even 1 is peaceably from (4):

$$\lambda_1 \lambda_2 \lambda_3 = 1. \tag{4}$$

Applying, we receive the above-mentioned replacement the well known with theory of materials the viscoelasticity dependence (5):

$$\sigma_{nom} = \frac{E}{3}\left(\lambda_1 - \frac{1}{\lambda_1^2}\right), \tag{5}$$

in which σ_{nom} it is a nominal stress, we take back working strength to initial section.

The introduced with example dependence (5) the substitution well known in theory of visco-elasticity materials has and it is used often – among other things in description of experimental investigations of creep of polymers [10]. It is then the dependence the factorials non-linear – in Fig. 1 was introduced graph $\sigma - \lambda$ in prospective range of work of elements with suchlike materials – the deflector about half of length. Dashed line for comparison was introduced linear dependence.

The models hyperelastic materials are in practical use to analysis in spatial states of stress systems – leading out the equation of direct vibration, it should be accept simplifying foundations. In description, the energy of deformation two parts is separated: it figures – near foundation of incompressibility structure and volumetric – relationships among deformations definite with Poisson's number. The deformations in case of bodies about isotropic proprieties were accepted dependence (6):

$$\begin{aligned} \varepsilon_2 &= -\nu\,\varepsilon_1 = -\nu\,(\lambda_1 - 1) = -\nu\,\lambda_1 + \nu, \\ \varepsilon_3 &= -\nu\,\varepsilon_1 = -\nu\,(\lambda_1 - 1) = -\nu\,\lambda_1 + \nu. \end{aligned} \tag{6}$$

After regard above-mentioned reductions, it was should execute differentiation with dependence peaceably (7):

$$\sigma_1 = \frac{\partial W}{\partial \lambda_1}. \tag{7}$$

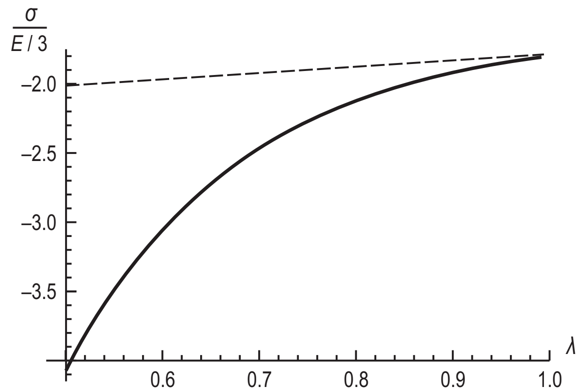

Fig. 1. $\sigma - \lambda$ in the expected range of work

If we want receive the equation of movement described with main deformations λ_i, for undimensional variable λ_1, we put dependence (8):

$$\lambda_1 = \frac{x+l}{l}, \tag{8}$$

where:
x_i – length called out the data with load change,
l_i – initial length,
i – it takes value: 1, 2, 3,
consequential with equation (9):

$$\lambda_1 = \varepsilon_1 + 1, \tag{9}$$

what it's carry on in consequence to equation with undimensional co-ordinate x.

3. Vibration analyses in system with hyperelastic element

The clear asymmetry of hysteresis loop is visible in Fig. 2. The initial intensity of recurrence after unloading is larger the than initial intensity the load. This phenomenon is non-linear viscoelasticity occurrent in polymers materials [11]. The row of experimental investigations occurrence suchlike phenomenon be confirmed – first executed by Leaderman [12].

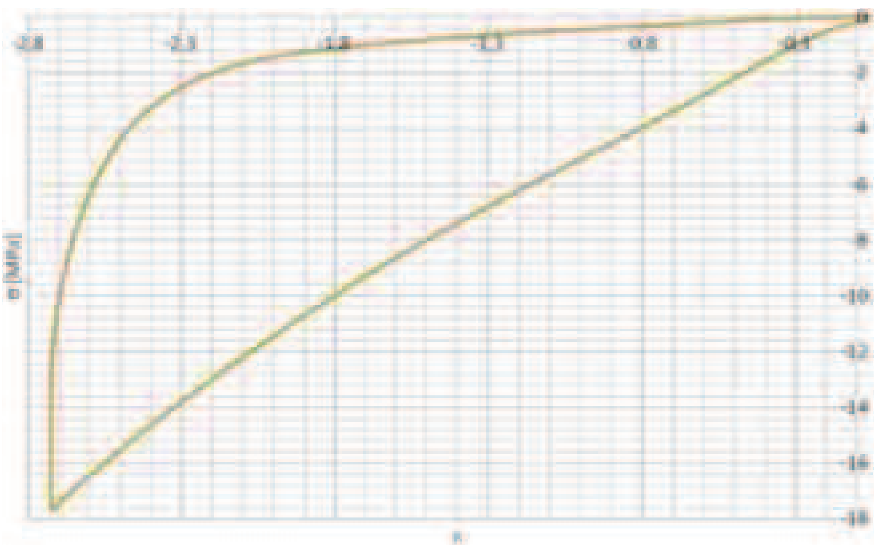

Fig. 2. Characteristic $\sigma = f(\varepsilon)$ the grid elastomer of natural rubber – free grip

An analysis was execute with used real part proprieties of hyperelastic materials. It required put in to equation differentiating function on the loading and relieving stages. Multinomial functions in describer both stages equations be accepted (models Mooney–Rivlin and Yeoh) or exponential – models Ogden and his modifications.

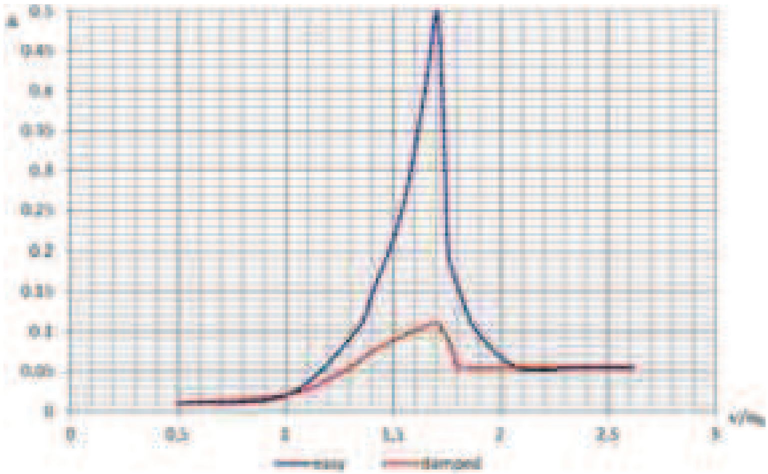

Fig. 3. The amplitude of forced vibration of harmonic oscillator in the dependence from of force input function frequency

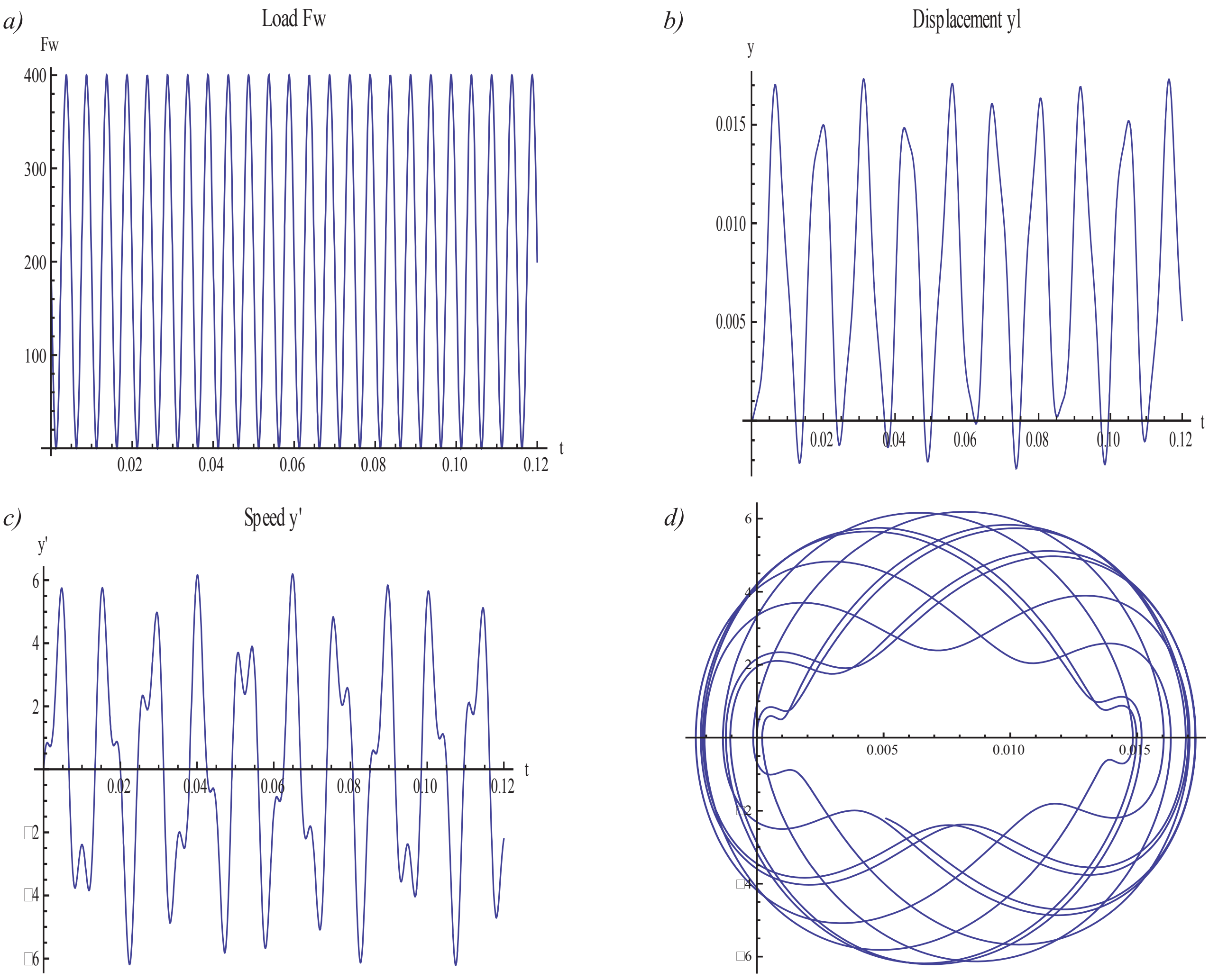

Fig. 4. Result analyse for initial conditions: t_p = 0, t_k =0.12 s, T = 0.005, C = 0

Equation to analysis describes vibration system executed from elastic incompressible elastomer was accepted (10):

$$m * x''[t] + 2k\,l^{-2} * (x[t] + l - l^{3} * (x[t]+l)^{-2}) = F[t]. \qquad (10)$$

It was also accepted that viscous dumping is visible. Dumping will be described (11):

$$m * x''[t] + b * x'[t] + 2k\,l^{-2} * (x[t] + l - l^{3} * (x[t]+l)^{-2}) = F[t]. \qquad (11)$$

Equation (11) is factorials non-linear. It results from occurrence in nominative of exponential dependence. The character of oscillation of suchlike did not it become in theory oscillation described so far – his qualification requires the uses of numeric methods. It in aim of removal of parameters describer the model values the stiffness and suppression – were executed was the experimental depending on grip the cylindrical sample investigations executed from the grid elastomer of natural rubber. The calculations were executed using packet of programmes helping the calculation mathematical MATHEMATICA v. 7.0.

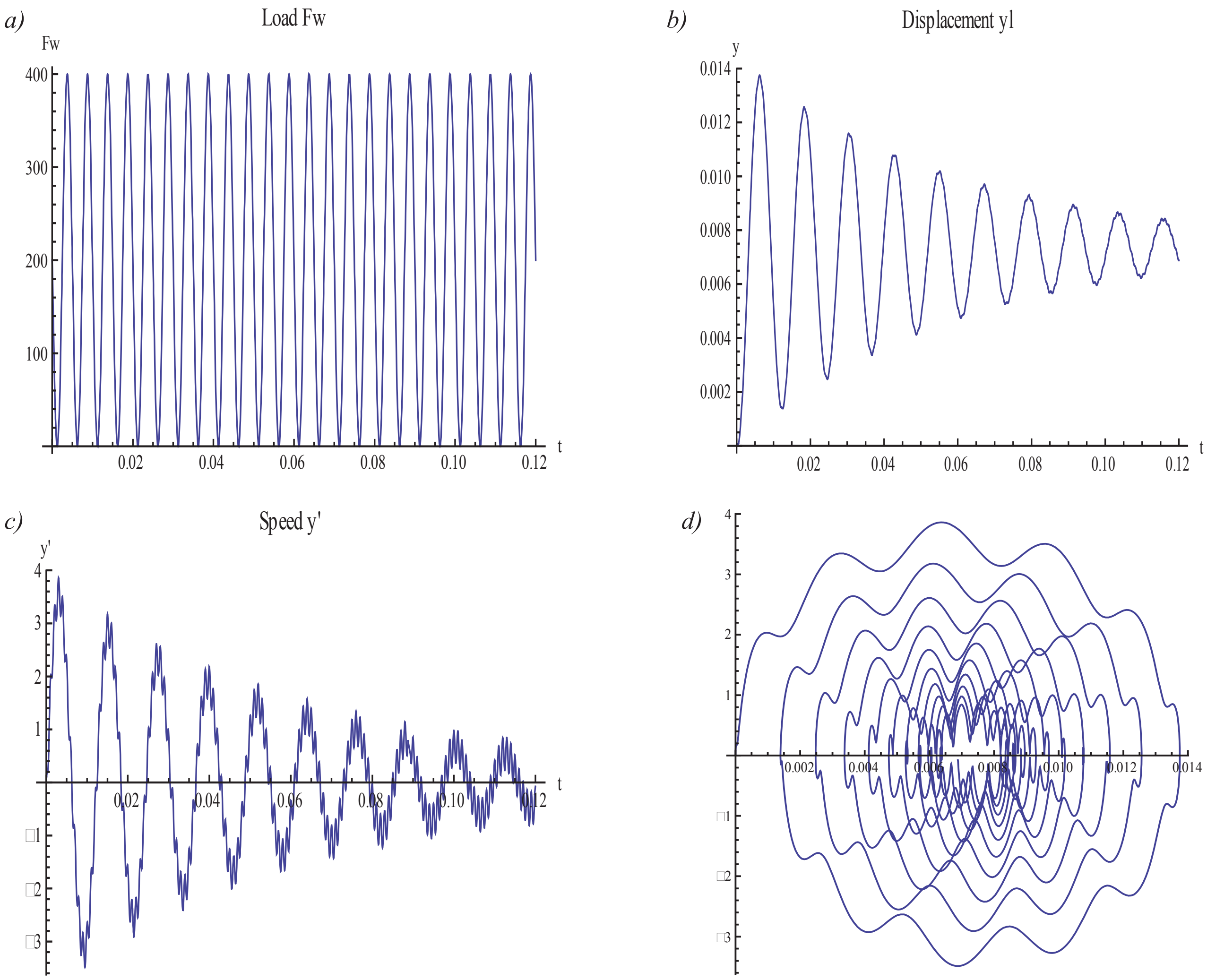

Fig. 5. Result analyse for initial conditions: $t_p = 0$, $t_k = 0.12$ s, $T = 0.001$ s, $C_1 = 0.9$, $C_{01} = 5$

It was affirmed that oscillation have non-linear character – typical phenomena step out for oscillation non-linear: the amplitude of affected oscillation is undamped complete, the largest values step out near regard considerably frequency biased in relation to value $\nu = \omega$. Resonance graph was introduced in Fig. 3.

It to qualification of value of level the suppressions was used the graph of hysteresis loop introduced in Fig. 3. Because the loadings and relieving different viscoelastic proprieties in sections were accepted the non-linear dependence in function of speed. It equation to analysis was accepted was (12):

$$m\ddot{x} + J(-\dot{x}) * C_1 \dot{x} + k\left(x + l - \frac{l^3}{(x+l)^2}\right) = F\sin \nu t - F_0 , \qquad (12)$$

where $H = \begin{cases} 1 & \text{for } \dot{x} > 0, \\ 0 & \text{for } \dot{x} \le 0. \end{cases}$

It was wrote additional program to simulation of test grip in which dumping ratio as data be introduced C_1 and C_{01}. The hysteresis loop is appointive. The coefficients C_1 and C_{01} are the variables until to draught of field of surface of loop such how received with experimental investigations. Value was received: $C_1 = 0.9$ N·s/m, $C_{01} = 5$ N·s/m.

Figure 4 and 5 presents result analyse for initial conditions: initial time: $t_p = 0$, final time: $t_k = 0.12$ s, period: $n = 0.005$ s, $C = 0$ ($t_p = 0$, $t_k = 0.12$ s, $n = 0.001$ s, damping ratio: $C_1 = 0.9$, $C_{01} = 5$): a) – the change of strength in time, b) – the dislocation in time, c) – the change of speed in time, d) – the speed in function the dislocation.

The resonance curve of damped oscillations was introduced in Fig. 3 by dashed line. The dissipation level (the dispersion of energy) in the grid elastomer of natural rubber its considerable – resonance strengthener steps out is small.

4. Conclusion

In work was introduced the method of analysis oscillation in system from viscoelaticity element from incompressible material the hyperelastic.

It was affirmed that oscillations have factorials non-linear character. It resonance was steps out from own frequency of arrangement lower frequency. The measured level of damping in studied material is considerable The dumping of oscillation steps out was strong.

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