A BOUNDARY CONDOTIONS AT MODELING 1-D PULSATING FLOWS IN PIPES ACCORDING TO THE METHOD OF CHARACTERISTICS

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Abstract

At this paper, there were compared three conceptions of boundary conditions and their definition at partially opened pipes. This is typical at inlet and exhaust pipes of internal combustion engines and piston compressors.

At an introduction there were listed the current state of affairs at a one dimensional modelling pulsations according to the Method of Characteristics (MOC). Subsequently, there was presented the main idea of MOC. At the third chapter there were presented general guidelines at initial and boundary conditions definition. Next there are three descriptions of the analyzed problem. Firstly, there was presented partially open end phenomena at space and stage plane. Secondly, there was shown the theory of acoustic wave propagation with description of stage waves at analyzed ends of pipe. Lastly, there is a new conception of constant pressure valve boundary conditions, which is the synthesis of the both mentioned earlier.

At the fourth paragraph there was shortly presented existing test rig dedicated to research of pulsating flow in pipes. This test rig was built at Flow Metrology Division of Lodz University of Technology [9]. At the second part of this chapter there were shown main assumption of elaborated simulation algorithm at Matlab Simulink environment. Lastly there were discussed simulation results compared with presented conceptions of boundary conditions.

Keywords: *1D simulations, Valve boundary conditions, method of characteristics, pulsating flow at pipes*

1. Introduction

The method of characteristics is mathematical technique for solving a hyperbolic type of partial differential equations. The method allows reducing partial differential equations (PDE) into family of ordinary equations enabling to integrate a solution from initial data. This is type of finite difference methods, which can be divided into implicit and explicit. A first order of accuracy explicit method was introduced in 1952 by Courant for use with the Characteristics form of the hyperbolic type PDE. In 1952 Hartee proposed schemes with second order accuracy, and in 1964 Benson proposed method based on Courant schemes dedicated into simulating internal combustion engines and reciprocating compressors. The method compares well with other first order of accuracy explicit methods and has advantages over them dealing with boundary conditions.

In 1960 Lax and Wendorff introduced a second order of accuracy explicit methods: a Two Step version developed later by Richmyer. In the 80's and 90's the development of Total Variation Dimensioning schemes and Flux Correted Transport has allowed the practical use of conservations scheme such as two step Lax Wendorff LW2 or MacCormack methods for intra pipe modelling. The association of TVD and LW2 or MC method is nowadays a recognized solution for efficient 1D manifold modelling, and is embedded into research (GASDYN) or commercial (GT-POWER) codes. Polish authors have also consequently used MOC according to fluid dynamics: Jungowski [2], Wisłocki [3], Prosnak [4], Puzyrewski and Sawicki [5] and at manifold modelling: Jungowski, Kordziński [6] and Sobieszczański [7].

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BOOST, FLUENT with modules: GT Turbo, Maxisim, Polyflow, WAVE, VRCTICS from Ricardo, KIVA or FIRE where KIVA is most flexible and popular at research codes). Besides them, there should be mentioned: PROMO method elaborated by Seifert. Lastly, MOC was modified according to specialized tasks: tapered pipes, boundary conditions (turbocharged engines, shock waves – valve impact, bending).

2. The main idea of M.O.C. (Method of Characteristics) [10]

2.1. Conservations laws

Flows are assumed to be one dimensional, ideal compressive gas is assumed to be inviscid.

Continuity equation: the rate of change of mass within the control volume is equal to net mass flow rate through the element:

$$\frac{\partial\rho}{\partial t}+\frac{\partial\rho u}{\partial x}+\frac{\rho u}{F}\frac{dF}{dx}=0\,, \tag{1}$$

a) Momentum equation: the rate of change momentum within the control volume is equal to the sum of forces applied on the control volume. This covers friction between the flow and the wall, a shear stress opposed to the flow.

$$\frac{\partial\rho u}{\partial t}+\frac{\partial\rho u^2+p}{\partial x}+\frac{\rho u^2}{F}\frac{dF}{dx}+\rho G=0\,, \tag{2}$$

$$G=\frac{1}{2}u\left|u\right|f\frac{4}{D}, \tag{3}$$

b) Energy equation is derived from the first law of thermodynamics applied to control volume:

$$\frac{\partial\rho e_0}{\partial t}+\frac{\partial\rho u h_0}{\partial x}+\frac{\rho u h_0}{F}\frac{dF}{dx}-\rho q=0\ . \tag{4}$$

The governing equations can be rewritten at vectorial form, where there are particular vectors: state vector of conservative variables (mass, momentum and total energy), flux vector and correction.

2.2. Principles of M.O.C

There were used Benson's [11] non dimensional notation at Fig. 1. For first step approximation, there are often leaved heat transfers and friction in pipes. Analysed flow can be defined as a homentropic. Additionally, there is not area section change. According to that condition there are defined two non dimensional Riemann invariants (α and β) along the characteristics lines C^+ and C^-:

$$\beta=A_i^{n+1}-\frac{\kappa-1}{2}*U_i^{n+1}=A_R^n-\frac{\kappa-1}{2}U_R^n, \tag{5}$$

$$\beta=A_i^{n+1}-\frac{\kappa-1}{2}*U_i^{n+1}=A_L^n+\frac{\kappa-1}{2}U_L^n\ . \tag{6}$$

For the homentropic flow there can be defined contraction of the gas between S and mesh node $(i,n+1)$:

$$\frac{p_S^n}{(\rho_S^n)^\kappa}=\frac{p_i^{n+1}}{(\rho_i^{n+1})^\kappa}\,. \tag{7}$$

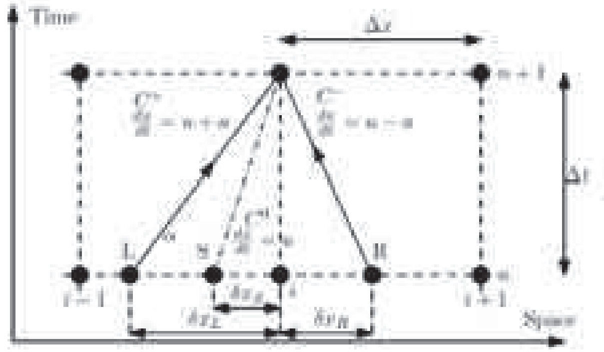

Fig. 1. MOC on space time diagram [10]

Having found δx_L, δx_R, δx_S there can be determined thermodynamic states at nodes L, R and S by linear interpolation. After that, there should be calculated: ρ_i^{n+1}, u_i^{n+1}, and p_i^{n+1}.

3. Appropriate Boundary (and Initial) Conditions – generally [1]

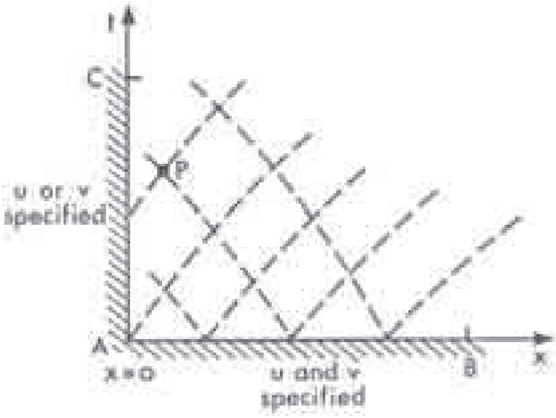

Fig. 2. Boundary conditions for the unsteady pipe flow [1]

A consideration of the computational domain x ≥ 0 and t ≥ 0 indicates that the point P close to the boundary x = 0 is partly determined by boundary conditions on AC and partly by initial conditions an AB, assuming that the governing PDEs are hyperbolic. Appropriate auxiliary conditions for this case are u and v specified on AB and u or v specified on AC [1]. The causes that number of auxiliary conditions is equal to the number of characteristics pointing into domain.

The direction along the characteristics needs to be chosen consistently. For time-dependent problems the positive direction will be in the direction of increasing time.

Generally [12], the number of boundary conditions should be equal to the number of characteristics pointing to the region. For subsonic outflow, there is needed one boundary condition, which can be density, the pressure or the velocity.

3.1. Two particular cases of 1-D pulsating flow in pipes (Jungowski) [2]

Jungowski proposed and showed two particular cases of the one dimensional pipe flow as below. There are presented gas parameters at two planes: space t = f(x) and stage a=f(u); where: a – speed of sound [m/s] and u velocity of flow at x direction [m/s]. We can find at Fig. 1, three areas at space plane (1, 2, 3) which are also represented at stage plane, according to the two basic phases of valve at A section. Upper index ‘ – represents small opening of valve and “ – represents small closing of valve.

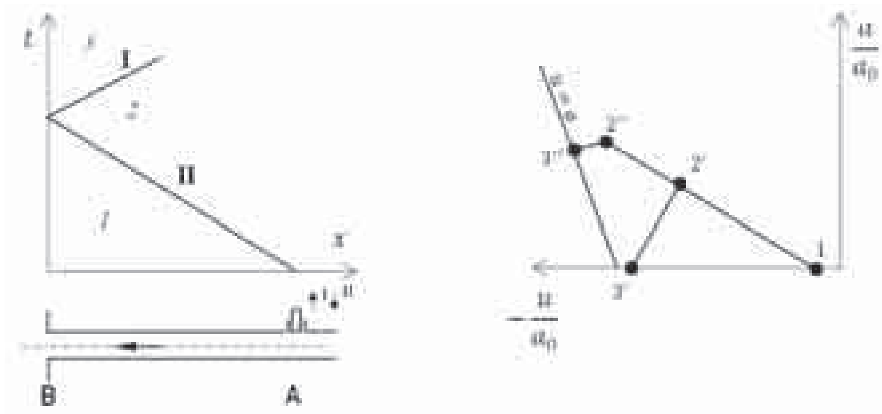

Fig. 3. Boundary conditions from Jungowski [2] for the unsteady flow at pipe with open end

For subsonic flows, the pressure at pipe should be equal to external one at initial while and after wave reflection. Deployment of upper case is shown below, Fig. 2. There is added a nozzle at the end of pipe. This nozzle can be described as cross section area change coefficient as a relation: $\varphi = \frac{F_n}{F_p}$ where $F_{n,p}$ –area of nozzle or pipe cross section [m^2].

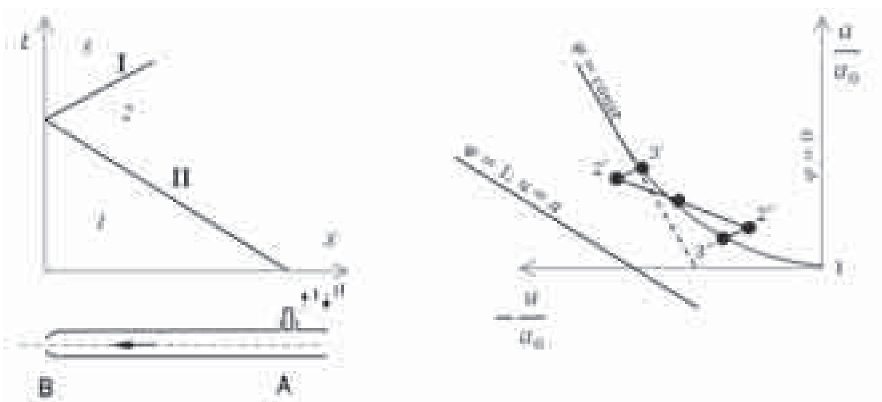

Fig. 4. Boundary conditions for the unsteady flow at pipe with partially closed end

According to boundary conditions and gas state for each value of mentioned coefficient the line is possible at state plane. Additionally, it can be proven, the $\varphi = const$ curve is straight line for sonic flows and their extension crosses centre of coordinate systems of state plane. Limiting values of φ corresponds to opened end ($\varphi = 1$ from Fig. 1) or completely closed ($\varphi = 0$). Solution: initial and reflected state must be on the $\varphi = const$ curve. It is worth to be mentioned, that abundance wave can be reflected as rarity, abundance or without reflection.

3.2. Boundary conditions for pressure and velocity at opened, closed and partially opened end [3, 6]

The acoustic phenomenon is very helpful at investigations of analysed cases. Pulsating pipe flow propagates along acoustic wave, it means that the elements move in parallel to wave direction. This way, it can be said that we consider pulsations at plane wave with time-like variable. According to this, the argument of acoustic wave is $(\omega t + kx)$ so called wave phase, where: ω - wave frequency, k - wave number. Mentioned variables are connected as follows:

$$a = \frac{\omega}{k} = \frac{\lambda}{T}, \tag{8}$$

where:
λ – wave length [m],
T – wave period [s].

Thanks to this equation, speed of sound is also called phase wave speed. As it was shown at Fig. 5 there are following rules for pressure and speed values according to the boundary conditions:

a) "*open end*" reflection of pressure wave is with change of sign. Speed and displacement reflect with phase change about $\lambda/2$. Arising standing wave has characteristic arrows and nodes at boundaries. For speed standing wave, there is "arrow" and there is node for pressure standing wave.
b) "*closed end*" reflected pressure wave has the same sign as falling ones. Reflection of speed and displacement waves is with change of sign. Arising standing wave has characteristic arrows and nodes at boundaries. For speed standing wave there is node and there is arrow for pressure standing wave.

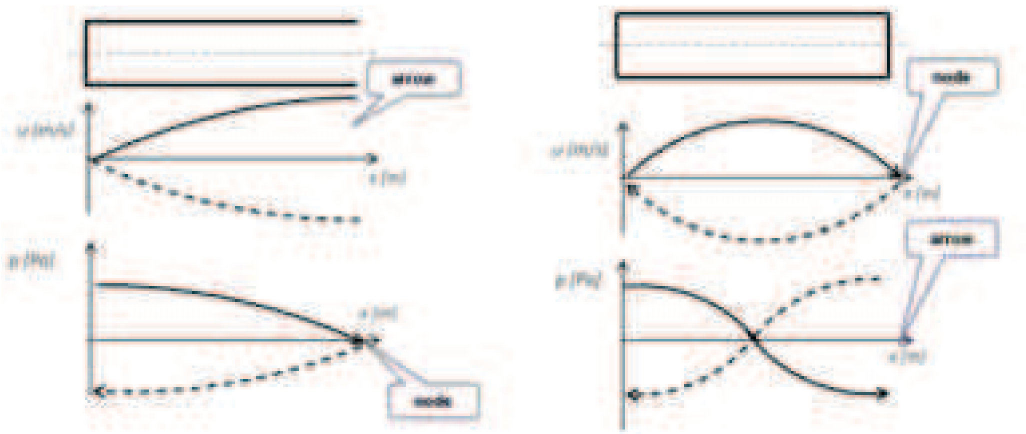

Fig. 5. Wave Pressure and speed diagram of gas elements at open and closed pipe[6]

At both references [3, 6], authors tries to estimate boundary conditions for partially opened end of pipe. They propose that there is division of propagating wave into two parts proportionally to cross section area change coefficient defined at 4.1. Reflected part of propagating wave will have boundary conditions from closed end type. Rest of falling wave will propagate outside of nozzle.

3.3. Constant pressure model valve boundary condition [11]

According to Fig. 4, there are six unknown quantities: density, speed and pressure at the boundary (i subscript) and throat (t subscript). There are not C^- characteristics, what causes the modification of previous principles (Fig. 1) of MOC. There are gases outflowing from pipe. It makes that, there is not needed entropy correction.

Because of that there should be written six equations to determine unknown quantities:

a) Non dimensional Riemann invariants along C^+ characteristic line:

$$\lambda_{in} = A_i^{n+1} - \frac{\kappa - 1}{2} * U_i^{n+1} = A_L^n + \frac{\kappa - 1}{2} U_L^n . \tag{9}$$

b) Mass conservation between pipe and the throat:

$$\rho_i^{n+1} * U_i^{n+1} * F_i^{n+1} = \rho_t^{n+1} * U_t^{n+1} * F_t^{n+1} . \tag{10}$$

c) Total energy conservation between the pipe and the throat:

$$\left(A_i^{n+1}\right)^2 + \frac{\kappa - 1}{2} * \left(U_i^{n+1}\right)^2 = \left(A_t^{n+1}\right)^2 + \frac{\kappa - 1}{2} * \left(U_t^{n+1}\right)^2 . \tag{11}$$

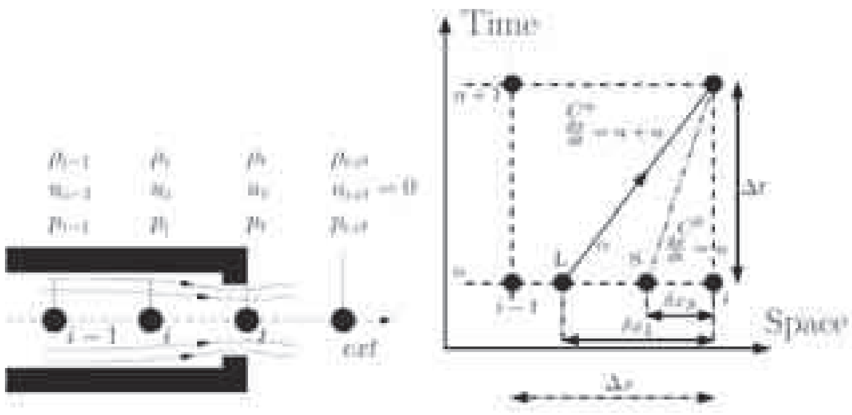

Fig. 6. Partially opened and boundary condition [11]

d) Homentropic contraction upstream the throat:

$$\frac{p_i^{n+1}}{p_t^{n+1}} = \left(\frac{A_i^{n+1}}{A_t^{n+1}} \right)^{\frac{\kappa-1}{2*\kappa}}, \tag{12}$$

e) For subsonic flow, the pressure in the throat equals the cylinder pressure:

$$p_t^{n+1} = p_{ext}\,, \tag{13}$$

f) Assuming a homentropic contraction downstream the throat:

$$\frac{p_t^{n+1}}{p_{ext}} = \left(A_t^{n+1} \right)^{\frac{\kappa-1}{2*\kappa}}\ . \tag{14}$$

4. Simulation model based on M.O.C [1]

According to presented main idea of MOC and their boundary conditions there was prepared simulation model of pipe with pulsating flow from test rig at Institute of Turbomachinery (Lodz University of Technology). The extensive research on dynamic phenomena in a straight pipe supplied with a pulsating flow of gas, executed in the Institute of Turbomachinery of Technical University of Lodz shows that the character of variation of flow parameters (pressure, temperature, velocity) depends on the nature of the pipe outlet. Basing on research of Olczyk [9] there could be implemented experimental data into simulation model elaborated at Matlab Simulink. The main idea of this model was presented at [13]. There were compared presented above conceptions of boundary conditions and results of synthesis of them is presented below.

The main parameters of simulated flow:

a) Range of desired values of frequency of pulse generator f = (20-200) [Hz],
b) Pipe diameter $D_p = 4210^{-3}$ [m],
c) Pipe length $L_p = 0{,}544$ [m], determined with resonance at 160 Hz,
d) Nozzle diameter $D_n = 1010^{-3}$ [m]. Nozzle is mounted at the end of pipe, at (3) cross section,
e) Desired flow temperature T = 313.5 [K],
f) Mean Flow speed $u = 20$ [m/s] (constant part),
g) Mean Pressure $p = 115000$ [Pa].

Measurement of transient and mean values of pressure, temperature and specific mass flow rate were conducted at control sections (0) and (3) Fig. 7. Additionally, transient pressure was measured at in section (K) paced in the pipe in the mi length.

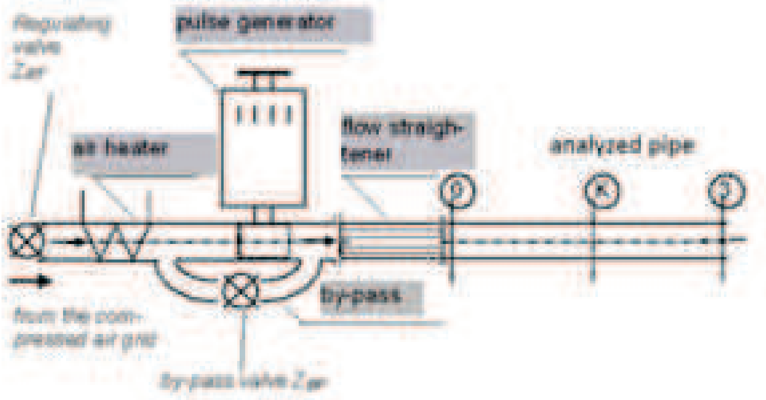

Fig. 7. The scheme of the test rig with testes pipe [9] (0,K,3 –cross sections with measurement equipment)

As far as measurement devices are concerned, the following equipment was used:

– piesoresistive transducers for pressure measurements: Endevco 8510C-15 and 8510C-50 (Endevco 2003),
– constant current thermometers (CCT) for temperature measurements,
– constant temperature anemometers (CTA) for specific mass flow rate measurements.

Both CCT and CTA probes incorporated a 5 lm tungsten wire.

There were compared presented above conceptions of boundary conditions and results of connection of them are presented below.

The simulation algorithm was prepared according to listed assumption:

a) there is not taken into consideration the heat transfer and friction phenomena. There is possible adding successive parameters to complete the solution with mentioned phenomena’s,
b) there is provided two direction iteration process, first at space field (*i* at Fig. 1), there are fifteenth steps, second at time (*n* at Fig. 1) 200 iterations. This is enforced by Courant Levy condition connected with demanded calculation resolution. This is needed because of the angle between space axis and the $u-a$ characteristic line.
When the maximum space iteration “*i*” is reached, the time “n” is switched to next value.
c) implemented algorithm enables easy switching between boundary conditions presented at 3 paragraph. Thanks to that there is possible acquisition of simulation data for clear comparison,
d) to make calculation more adequate with experiment, the initial conditions were implemented from measurement data. That data were processed by FFT,
e) simulation results – variable course are referenced with control section experimental data.

4.1. Simulation results

At Fig. 8, there were presented example a course of the pressure and the flow speed at the time-space plane. That diagrams were prepared for the pulse generator frequency 100 Hz. Horizontal axis represents space plane (t-x). X axis is represented by a pipe length at percentages and t axis is represented by time step (this is caused by changing time step in seconds) constant as a number of iterations. It is easy find values, which corresponds with referred boundary conditions at 3 paragraph. At the partially open end of pipe we can find the arrow (maximum value) of flow speed and node (minimum) at pressure course. It is significant, that the pressure does not reach constant value at all. It is caused by partially opened end of pipe. That values corresponds with cross section are change coefficient φ defined at 3.1. To sum up, at author opinion, both presented conceptions of boundary conditions for modelling 1-D pulsations at pipes according to the method of characteristics are acceptable.

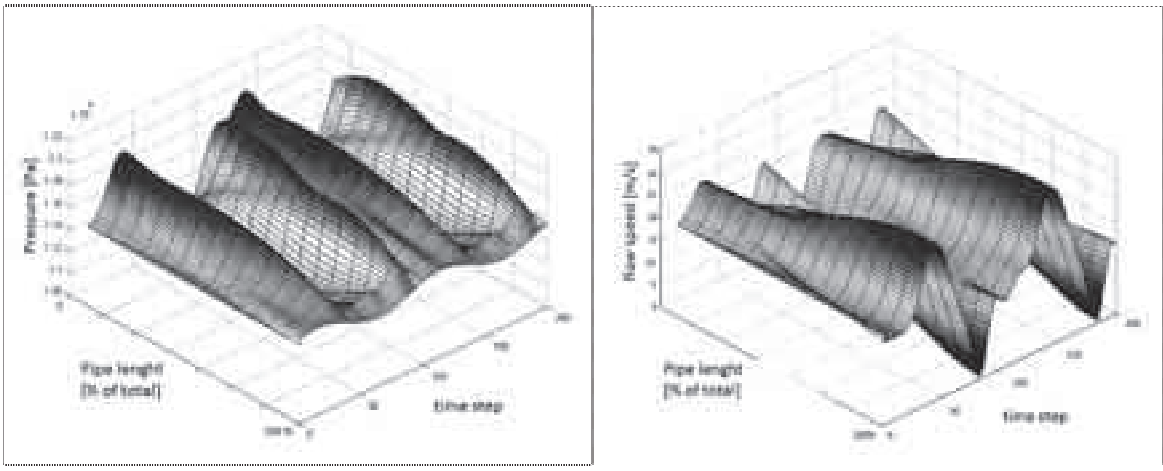

Fig. 8. The course of the pressure and flow speed changes at time space area.

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