

# THE FRICTION FORCE AND FRICTION COEFFICIENT IN THE JOURNAL SLIDING BEARING FERROFLUID LUBRICATED WITH DIFFERENT CONCENTRATIONS OF MAGNETIC PARTICLES

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## **Abstract**

*There is an important operational parameter in the case of sliding bearings are friction forces and coefficient of friction. It depends on how much proportions of heat in the gap of the oil film from the value of the friction force.*

*Ferrofluid lubricated sliding bearings have specific structure and are suitable only for use in specific cases. We may use them in the absence of gravity, vacuum, or in case of strong magnetic fields or radioactive. Maintenance of lubricant in the oil-gap as well as the viscosity change occurs through controlling of the external magnetic field. Change of the value of viscosity and mass forces (magnetic forces) in the equation of momentum depends on the concentration of magnetic particles and the intensity of external magnetic field.*

*The aim of this paper is to present the influence of concentration of magnetic particles on the friction force value and coefficient of friction.*

*The numerical calculations of friction forces and friction coefficient have been performed before setting the hydrodynamic pressure and a lift force from the Reynolds-type equation. Reynolds-type equation has been derived from basic equations, ie equations of momentum and equations of stream's continuity. There have been also used Maxwell's equations for the ferrofluid in the case of stationary magnetic field's existence. It has been assumed stationary and laminar flow of lubricant liquid and the isothermal model for lubrication of slide bearings. As the constitutive equation has been used Rivlin-Ericksen one. The cylindrical journal bearing of finite length with the smooth sleeve of whole angle of a belt has been taken into consideration.*

*In a thin layer of oil film has been assumed constancy of the oil density with temperature changes and the independence of the oil's thermal conductivity coefficient from thermal changes. The viscosity of the oil depends mainly on the magnetic field.*

**Keywords:** *hydrodynamic pressure, capacity, friction forces, friction coefficient, ferrofluid, numeric calculation*

## **1. Introduction**

There are friction forces and coefficient of friction an important operational parameter in the case of sliding bearings. It depends on how much proportions of heat in the gap of the oil film from the value of the friction force.

Ferrofluid lubricated sliding bearings have specific structure and are suitable only for use in specific cases. We may use them in the absence of gravity, vacuum, or in case of strong magnetic fields or radioactive. Maintenance of lubricant in the oil-gap as well as the viscosity change occurs through controlling of the external magnetic field. Change of the value of viscosity and mass forces (magnetic forces) in the equation of momentum depends on the concentration of magnetic particles and the intensity of external magnetic field.

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## 2. Basic equations

Analysis of magnetohydrodynamic lubrication of the cross sliding bearings for a stationary, laminar, isothermal flow involves the solution of fundamental equations, namely equations of conservation of momentum and continuity of the stream in the following form [1-4, 7]:

$$0 = \text{Div } \mathbf{S} + \mu_o (\mathbf{N} \cdot \nabla) \mathbf{H} + \frac{1}{2} \mu_o \text{rot}(\mathbf{N} \times \mathbf{H}), \quad (1)$$

$$\text{div}(\rho \mathbf{v}) = 0, \quad (2)$$

$$\text{rot } \mathbf{H} = 0, \text{div } \mathbf{B} = 0, \mathbf{B} = \mu_o (\mathbf{H} + \mathbf{N}), \mathbf{N} = \mathbf{H} \cdot \chi, \quad (3)$$

where:

$\mathbf{B}$  - vector of magnetic induction in ferrofluid [T],

$\mathbf{H}$  - vector of magnetic field strength in ferrofluid [ $\text{A} \cdot \text{m}^{-1}$ ],

$\mathbf{N}$  - vector of ferrofluid magnetization [ $\text{A} \cdot \text{m}^{-1}$ ],

$\mathbf{S}$  - ferrofluid's stress tensor with coordinates  $\tau_{ij}$  for  $i, j = \phi, r, z$  [Pa],

$\mathbf{v}$  - ferrofluid's velocity tensor [ $\text{m} \cdot \text{s}^{-1}$ ],

$\nabla$  - Nabla's operator,

$\mu_o$  - vacuum magnetic permeability [ $\text{H} \cdot \text{m}^{-1}$ ],

$\rho$  - density of ferrofluid [ $\text{kg} \cdot \text{m}^{-3}$ ],

$\chi$  - magnetic susceptibility factor of ferrofluid,

Rivlin-Ericksen formula describing the relationship between the coordinates of the stress tensor  $\mathbf{S} \equiv \|\tau_{ij}\|$  and the strain rate tensor coordinates of ferrofluid can be presented in the following form [3, 6-8]:

$$\mathbf{S} = -p \mathbf{I} + \eta \mathbf{A}_1 + \alpha \mathbf{A}_1 \mathbf{A}_1 + \beta \mathbf{A}_2. \quad (4)$$

Strain rate tensors can be defined by the following relation [3], [6], [7]:

$$\mathbf{A}_1 \equiv \mathbf{L} + \mathbf{L}^T, \mathbf{A}_2 \equiv \text{grad } \mathbf{a} + (\text{grad } \mathbf{a})^T + 2\mathbf{L}^T \cdot \mathbf{L}, \quad (5)$$

where the acceleration vector is given by:

$$\mathbf{a} \equiv \mathbf{L} \cdot \mathbf{v}, \mathbf{L} \equiv \text{grad } \mathbf{v}, \quad (6)$$

where:

$\mathbf{A}_1$  - the first one strain rate tensor [ $\text{s}^{-1}$ ],

$\mathbf{A}_2$  - the second one strain rate tensor [ $\text{s}^{-2}$ ],

$\mathbf{I}$  - elementary tensor,

$\mathbf{L}$  - tensor of gradient taken from the velocity tensor [ $\text{s}^{-1}$ ],

$\mathbf{a}$  - accelerating vector [ $\text{m} \cdot \text{s}^{-2}$ ],

$p$  - hydrodynamic pressure [Pa],

$\alpha, \beta$  - the experimental coefficients determining the viscoelastic properties of ferrofluid [ $\text{Pa} \cdot \text{s}^2$ ],

$\eta$  - dynamic viscosity coefficient [ $\text{Pa} \cdot \text{s}$ ].

The materials coefficients  $\alpha, \beta$  of lubricant liquid multiplied by the property rate of strain tensors take into account the additional stresses arising from the viscoelastic, non-Newtonian ferrofluid nature. In case of acceptance of material's coefficients  $\alpha, \beta$  equal to zero can be obtained the classical Newtonian relationship between stress tensor and strain rate tensor.

Ferrofluid's dynamic viscosity depends mainly on the magnetic induction  $\eta=\eta(B)$  and the material's coefficients  $\alpha, \beta$  were taken as constants.

The amount of base of oil gap  $h_p$  depends on the relative eccentricity  $\lambda$  and nonparallelity of shaft and sleeve axis with an angle of  $\gamma$  see Fig. 1.

Dimensionless height of the oil gap as a function of circumferential and longitudinal variable is as follows [3]:

$$h_{p1} = [1 + \lambda \cdot \cos\phi + a_\gamma z_1 \cdot \cos(\phi)], \quad a_\gamma = \frac{L_1}{\psi} \tan(\gamma) \quad (7)$$

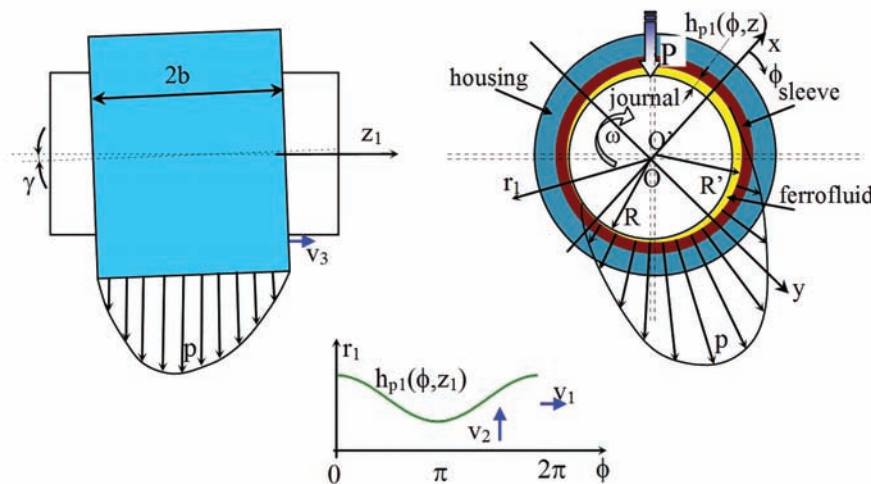


Fig. 1. Construction of cross sliding bearing, together with the characteristic dimensions for steady flow

The constitutive relations (6) between the coordinates of the stress tensor  $\tau_{\phi\phi}, \tau_{rr}, \tau_{zz}, \tau_{\phi r}, \tau_{\phi z}, \tau_{rz}$  and the coordinates of the strain rate tensor are substituted for the equations of motion (1) - (2). So non-stationary units as the forces of inertia in the equations of momentum shall be ignored. This kind of ignorance is justified in the slow and medium speed bearings. The whole set of equations of motion for classical steady flow of lubricating oil can be obtained in this procedure.

To estimate the magnitude of units in the system of equations and ignore small units of a higher order the making dimensionless and estimating the magnitude of units in the system of equations of conservation of momentum and continuity of the stream have been done. For this the following dimensional and dimensionless marks and numbers have been assumed [3, 6, 7].

The system of equations in dimensionless form contains the units with order of elementary magnitude are visible as units of the negligible order like the relative radial clearance  $\psi \approx 10^{-3}$ . Omitting units of the order of the relative radial clearance that means about a thousand times smaller than the value of other units, a new simplified set of equations has been obtained [3].

For further analysis of the basic equations it has been assumed that the dimensionless density  $\rho_1=1$  of lubricant is constant and not depend on temperature and pressure [3, 6, 7].

In order to solve the problem of hydrodynamic lubrication, which determine the size of the functions sought, such as: components of velocity, hydrodynamic pressure, load-bearing forces and friction forces the classical method of small parameter has been used. This method uncouples of nonlinear partial differential equations, forming two linear equations. The first set of equations allows to determine the flow parameters for the classical isothermal, Newtonian lubrication with the magnetic field effects on the change in viscosity. The second one allows to determine the so-called velocity components corrections, hydrodynamic pressure follow from the consideration of non-Newtonian properties.

Integrating twice after the radial variable corresponding momentum equation and applying boundary conditions circumferential and longitudinal component of the velocity vector have been obtained.

Boundary conditions for components of velocity vector of oil with Newtonian properties are as follows [3, 6, 7]:

$$\begin{aligned} v_1^{(0)} = 0, \quad v_2^{(0)} = 0, \quad v_3^{(0)} = 0, \quad \text{on sleeve } r_1=h_{p1}, \\ v_1^{(0)} = 1, \quad v_2^{(0)} = 0, \quad v_3^{(0)} = 0, \quad \text{on journal } r_1=0. \end{aligned} \quad (8)$$

These conditions indicate that the peripheral speed of the oil in contact with the journal assumes a value of peripheral speed of the journal and zero on the stationary sleeve, because the liquid lubricant is a viscous liquid, and it does not take into account the vibration of the shaft and the sleeve, or slips. For these reasons, the longitudinal velocity component of the oil is zero. The radial velocity component of the oil on the journal and sleeve is zero because the material is porous and it's assumed that the journal and sleeve do not perform transverse vibration.

For the distribution of hydrodynamic pressure in the oil with Newtonian properties Reynolds boundary conditions have been adopted in the following form [3, 6, 7]:

$$p_1^{(0)} = 0 \text{ for } \phi=\phi_p, \quad p_1^{(0)} = 0 \text{ for } \phi \geq \phi_k, \quad \frac{\partial p_1^{(0)}}{\partial \phi} = 0 \text{ for } \phi=\phi_k, \quad p_1^{(0)} = 0 \text{ for } z_1=+1 \text{ and } z_1=-1. \quad (9)$$

These conditions mean that the value of hydrodynamic pressure is equal to the ambient pressure (atmospheric pressure) equal to zero compared with the developed pressure in the bearing. Adoption a value of zero applies the site  $\phi=\phi_p$ , ie the initial coordinate equal to approximately  $4^\circ$  in the direction of the journal movement on the front end of the line which connecting centers of journal and sleeve is usually the place to bring the oil into the gap and at the site  $\phi=\phi_k$ , ie coordinate the end of the oil film. This value is unknown in terms of Reynolds, but it is known that it lies outside the rear end of the line which connecting centers of journal and sleeve. Using the continuity equation and the previously evaluated components: the longitudinal and circumferential and after integrating equation and imposing the appropriate boundary conditions we obtain the radial component of velocity and Reynolds-type equation which has the form [3]:

$$\frac{\partial}{\partial \phi} \left[ \frac{h_{p1}^3}{\eta_{IB}} \left( \frac{\partial p_1^{(0)}}{\partial \phi} - M_1 \right) \right] + \frac{1}{L_1^2} \frac{\partial}{\partial z_1} \left[ \frac{h_{p1}^3}{\eta_{IB}} \left( \frac{\partial p_1^{(0)}}{\partial z_1} - M_3 \right) \right] = 6 \frac{\partial h_{p1}}{\partial \phi}, \quad (10)$$

where:

$$M_1 = R_f \chi \left[ H_1 \frac{\partial H_1}{\partial \phi} + \frac{1}{L_1} H_3 \frac{\partial H_1}{\partial z_1} \right], \quad M_3 = R_f L_1 \chi \left( H_1 \frac{\partial H_3}{\partial \phi} + \frac{1}{L_1} H_3 \frac{\partial H_3}{\partial z_1} \right).$$

Dimensional value of the lift force  $C_\Sigma$  in the cross slide bearing is determined from the known formula [3, 7]:

$$C_\Sigma = C_1^{(0)} \cdot bR\eta_o\omega/\psi^2. \quad (11)$$

Dimensionless value of the lift force  $C_1^{(0)}$  of cross sliding bearing lubricated by ferromagnetic fluid with viscoelastic properties is calculated from the relation [3, 7]:

$$C_1^{(0)} = \sqrt{\left( \int_{-1}^{+1} \left( \int_0^{\phi_k} p_1^{(0)} \cos \gamma \sin \phi \, d\phi \right) dz_1 \right)^2 + \left( \int_{-1}^{+1} \left( \int_0^{\phi_k} p_1^{(0)} \cos \gamma \cos \phi \, d\phi \right) dz_1 \right)^2}, \quad (12)$$

where the symbol  $\gamma$  is the misalignment angle.

The total dimensional friction in the gap the cross slide bearing shows the following relationship [3, 7]:

$$Fr_{\Sigma} = Fr_1^{(0)} \cdot bR\eta_o\omega/\psi. \quad (13)$$

Dimensionless value of friction force for the classical Newtonian oil including the influence of the magnetic field to change the dynamic viscosity is calculated from the following relationship [3, 7]:

$$Fr_1^{(0)} = \int_{-1}^{+1} \left[ \int_0^{\phi} \left( \eta_{1B} \frac{\partial v_1^{(0)}}{\partial r_1} \right)_{r_1=h_{p1}} d\phi \right] dz_1 = \int_{-1}^{+1} \left[ \int_0^{2\pi} \left( \eta_{1B} \frac{\partial v_{1s}^{(0)}}{\partial r_1} \right)_{r_1=h_{p1}} d\phi \right] dz_1 + \int_{-1}^{+1} \left[ \int_0^{\phi_k} \left( \eta_{1B} \frac{\partial v_{1p}^{(0)}}{\partial r_1} \right)_{r_1=h_{p1}} d\phi \right] dz_1. \quad (14)$$

The function of peripheral velocity consists of a velocity caused by gradient of pressure and the velocity caused by peripheral movement of the journal (shear flow) and the magnetic field [3, 7]:

$$v_{1p}^{(0)} \equiv \frac{1}{2\eta_{1B}} \frac{\partial p_1^{(0)}}{\partial \phi} (r_1^2 - r_1 h_{p1}), \quad v_{1s}^{(0)} \equiv 1 - \frac{r_1}{h_{p1}} - \frac{1}{2\eta_{1B}} M_1 (r_1^2 - r_1 h_{p1}). \quad (15)$$

Total apparent coefficient of friction for the classical Newtonian oil including the influence of the magnetic field to change the dynamic viscosity is determined from the following formula:

$$\left( \frac{\mu}{\psi} \right)_{\Sigma} = \frac{Fr_{\Sigma}}{\psi C_{\Sigma}} = \frac{Fr_1^{(0)} b R \eta_o \omega / \psi}{C_1^{(0)} b R \eta_o \omega / \psi} = \left( \frac{\mu}{\psi} \right)_1 = \frac{Fr_1^{(0)}}{C_1^{(0)}}. \quad (16)$$

### 3. Numerical calculations

Numerical calculations of the pressure distributions and load carrying capacities and friction force and friction coefficient are performed in Mathcad 14 Professional Program by virtue of the equation (10), (12), (14), (16) by means of the finite difference method (see Fig. 2-5). On the ground of pressure distributions (Fig. 2) are calculated the load carrying capacities (see Fig. 3). The friction force is shown in Fig. 4 and the apparent friction coefficient in Fig. 5.

The numerical calculations of hydrodynamic pressure distribution, load carrying capacities, friction force and friction coefficient have been performed for the relative eccentricity:  $\lambda=0.1$ ,  $\lambda=0.2$ ,  $\lambda=0.3$ ,  $\lambda=0.4$ ,  $\lambda=0.5$ ,  $\lambda=0.6$ ,  $\lambda=0.7$ ,  $\lambda=0.8$ ,  $\lambda=0.9$  and the dimensionless length of bearing  $L_1=1/2$  at four concentrations of magnetic particles of ferrofluid: 0% (classic lubricating oil), 1%, 3% and 6%.

The components of the magnetic field strength have been determined by analytical-numerical solution of Maxwell's equations [4].

For all calculations, the following dimensional and dimensionless parameters have been assumed: angle of misalignment  $\gamma=0,000^\circ$ ; magnetic susceptibility corresponding to different concentrations of magnetic particles  $\chi=2.0$ ,  $\chi=2.5$ ,  $\chi=3.0$ ; the number of magnetic pressure  $R_f=0,5$ ; dimensionless coefficient describing the effect of magnetic induction on the dynamic viscosity suitable for different concentrations of magnetic particles  $\delta_{B1} = 0.100$ ;  $\delta_{B1} = 0.175$ ,  $\delta_{B1} = 0.225$ . For determining the distribution of hydrodynamic pressure boundary conditions of Reynolds have been adopted.

### 4. Conclusion

Increase in the number of magnetic particles causes an increase in hydrodynamic pressure and lift force with the existence of the same value of external magnetic field. This increase is in the range: 17-80% according the lubricating oil without the magnetic particles. As the relative eccentricity is also increase in lift force but to a lesser extent - from 1-5%.

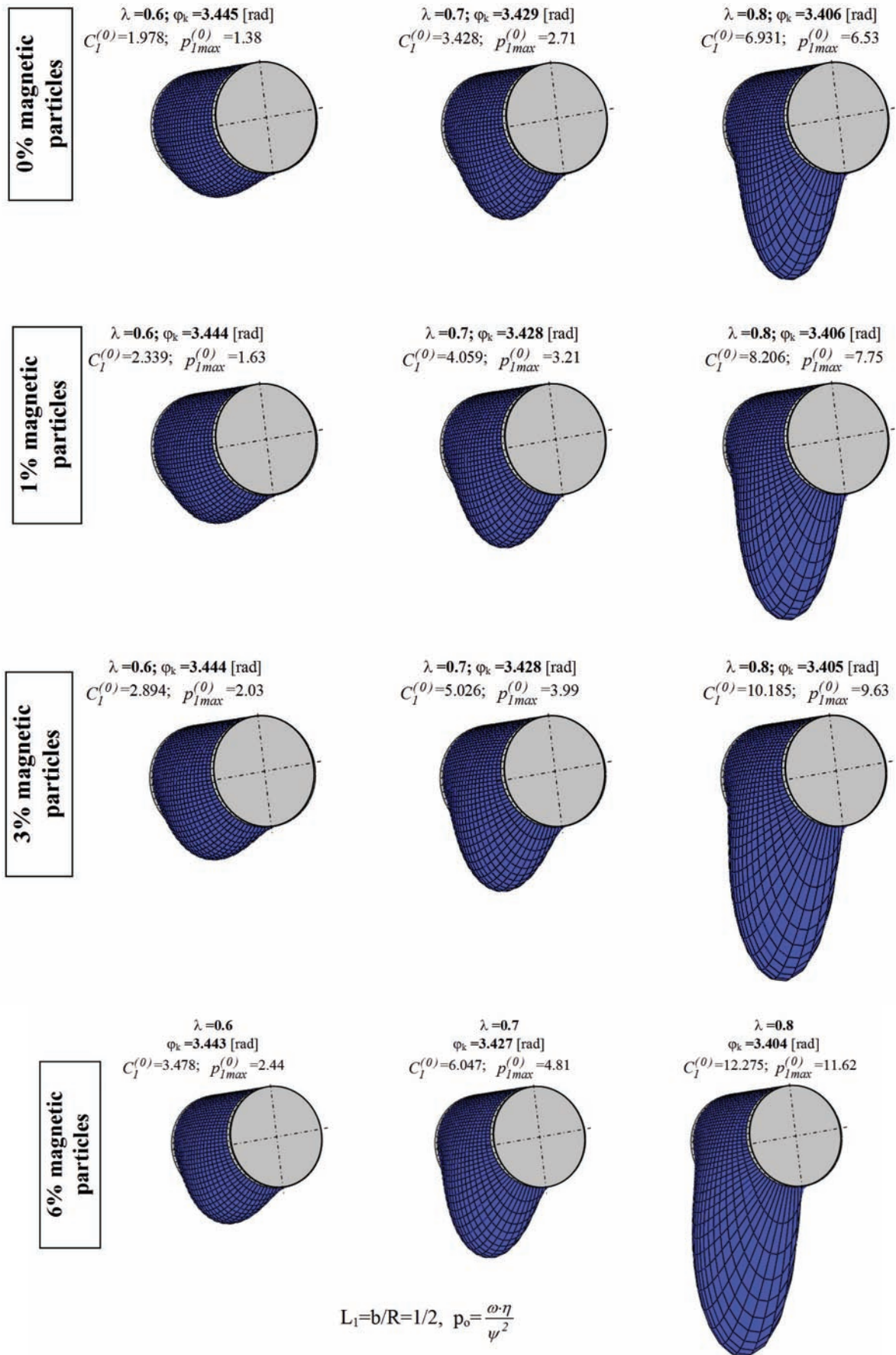


Fig. 2. The dimensionless pressure distributions  $p_1^{(0)}$  in cylindrical sliding journal bearings

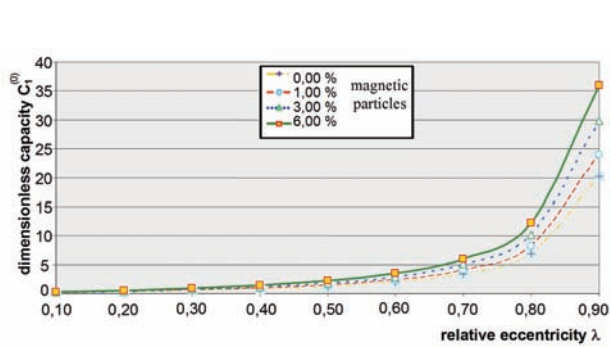


Fig. 3. The dimensionless load carrying capacities  $C_1^{(0)}$  in cylindrical sliding journal bearings

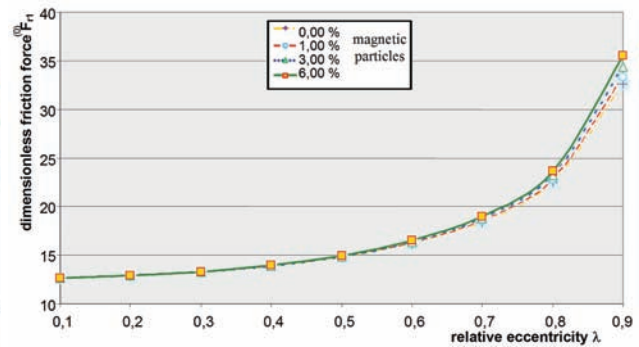


Fig. 4. The dimensionless friction force  $F_{r1}^{(0)}$  in cylindrical sliding journal bearings

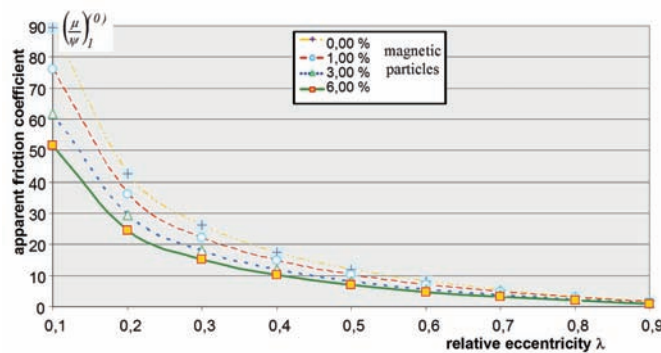


Fig. 5. The apparent friction coefficient  $\left(\frac{\mu}{\psi}\right)_1^{(0)}$  in cylindrical sliding journal bearings

Increase in the number of magnetic particles results in a slight increase in friction. This increase is in the range 0.1-9%.

With a large increase in lift force and a slight growth in friction force conventional friction coefficient decreases with increasing concentration of magnetic particles in ferrofluid. This decrease is in the order of 14-42%.

It should be clear that the quoted values are the result of computer simulation. The actual value of changes of hydrodynamic pressure and lift force will depend on the type of magnetic particles, the type of base fluid, the concentration of magnetic particles, the value of an external magnetic field, temperature and value of ferrofluid's hydrodynamic pressure which depends inter alia on the load bearing, rotational speed, radial clearance, and the geometric dimensions of the bearing.

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