

## **INFLUENCE OF FREEPLAY AND FRICTION IN STEERING SYSTEM ON DOUBLE LANE CHANGE MANOEUVRE – MODELLING AND SIMULATION STUDIES**

**Dariusz Więckowski**

*Automotive Industry Institute (PIMOT)  
Jagiellońska Street 55, 03-301 Warszawa, Poland  
phone: +48 22 811 18 92, fax: +48 22 811 60 28  
e-mail: dwieckowski@pimot.org.pl*

**Dariusz Żardecki**

*Military University of Technology (WAT)  
Faculty of Mechanical Engineering  
Gen Sylwestra Kaliskiego Street 2, 00-908 Warszawa, Poland  
phone: +48 22 683 75 24, fax: +48 22 683 73 66  
e-mail: dzardecki@wat.edu.pl*

### ***Abstract***

*Modelling and simulation investigations of a car non-linear dynamics with regard to freeplay (backlash in steering gear) and friction (kinetic and static friction in king-pins) is always a challenge for researchers. Fortunately, by special luz(...) and tar(...) projections and their mathematical apparatus the elaborated models and simulation procedures of stick-slip phenomena in a steering system are managed in efficient forms (without any entangle constraints) even for optimization and sensitivity studies. Exactly, the presented studies deal with problems of sensitivity of optimized maneuvers for vehicle models considering freeplay / friction effects. Double lane change maneuvers (overtaking and avoiding) are discussed. A criterion function refers to several evaluations (precision of maneuver, calm steering, feeling of comfortable travel). The optimization of steering wheel angle signal is done on a reference model with „nominal” freeplay and friction parameters. Such input signal is applied to a „real” vehicle having „real” freeplay and friction parameters. The differences of output signals as well as differences between criterion function values are a subject of sensitivity analysis. The sensitivity of reference models on a variation of freeplay and friction parameters appears as important from theoretical as well as practical points of view. The paper takes up the problems of sensitivity analysis of steering system model applied for synthesis steering algorithms for a double lane change maneuver.*

**Keywords:** vehicle dynamics, steering system, freeplay, friction, mathematical models, luz(...) and tar(...) projections, simulation studies, sensitivity analysis, optimization of maneuver, double lane change maneuver

### **1. Introduction**

Modern control systems basing on microprocessors and mechatronic devices change a car. Nowadays cars are equipped by standards with many automatic stabilization systems (e.g. ABS, ASR, ESP etc.). We can notice also a big progress of prototype systems designed for automation of maneuvers (e.g. so called assistance steering systems). Such systems use reference models of a car dynamics, which are processed in real time (“on-line” simulation), or are based on ready-to-use steering procedures elaborated with “off-line” simulation mapping, but activated automatically after detection and identification of road events.

Problem of efficient reference models seems to be the most important for assistant systems design. Of course, we would like to use enough accurate models which are not very complicated and have not many unknown parameters and variables. Looking on the subject bibliography one notices the

reference models based on very simple (static and linear) descriptions of steering system mechanisms with neglecting many important attributes, especially with neglecting their freeplay (backlash in steering gear) and friction (kinetic and static in king pins) non-linearities. Whereas stand tests of many passenger cars, busses, and trucks show (example results on Fig. 1) that resistance force (friction) and angular dead-zone (freeplay) parameters might be very diverse and high even in new cars. Note, freeplay and friction attributes are strictly tested when cars undergo seasonal diagnostic examinations. After all the freeplay and the friction influent directly on a steering signal, and can cause singular non-linear phenomena, e.g. stick-slip and shimmy vibrations These problems are described in monograph [8], and state-of-the-art paper [1]. So, the sensitivity of reference models on a variation of freeplay and friction parameters appears as important from theoretical as well as practical points of view.

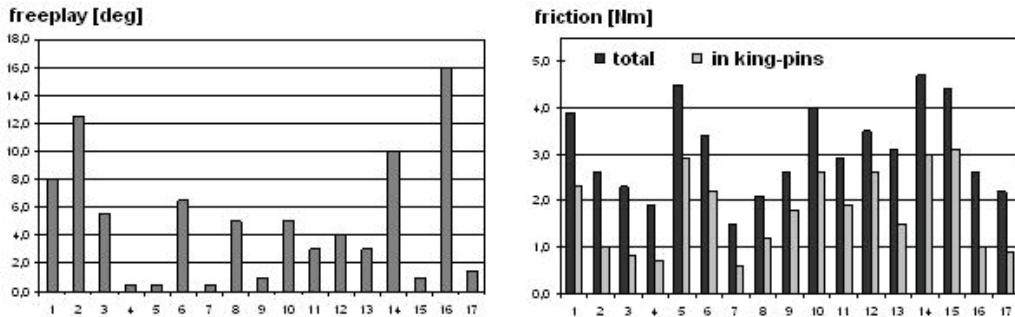


Fig. 1. Example results of PIMOT stand tests for 17 passenger cars' steering systems [1]

The paper takes up the problems of sensitivity analysis of steering system model applied for synthesis steering algorithms for a double lane change maneuver. Such maneuver is typical for avoiding and overtaking. This is continuation of authors' studies [4, 5].

The paper is organized as follows:

- Section 2 describes a method of modelling of steering system mechanism with inclusion of freeplay and friction effects (also stick-slip phenomena). The method bases on piecewise-linear  $\text{luz}(\dots)$  and  $\text{tar}(\dots)$  projections, so their definition and example formulas are given,
- Section 3 focuses on a sensitivity analysis of steering systems with freeplay and friction. A theoretical background and example previous results of simulation studies are presented,
- Section 4 contains information on modelling and optimization of typical lane change road maneuvers which are used in assistance system reference models,
- Section 5 presents results of sensitivity analysis done by of comparative simulations,
- Section 6 contains conclusions.

## 2. Modelling of steering systems with freeplay and friction

Modelling of multi-body systems with freeplay (backlash, clearance) and friction (kinetic and static) provide strong non-linear variable-structure differential equations with algebraic constrains. Generally, such models are very difficult for synthesis, analysis and simulation.

The method elaborated by Żardecki (all details in [7, 8]) simplifies this problem for large class of mechanisms. It bases on piecewise linear  $\text{luz}(\dots)$  and  $\text{tar}(\dots)$  projections (Fig. 2).

$$\text{luz}(x, a) = x + \frac{|x - a| - |x + a|}{2}, \quad a \geq 0 \quad \text{tar}(x, a) = \text{luz}^{-1}(x, a). \quad (1), (2)$$

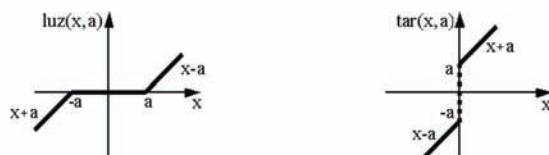


Fig. 2. Topology of  $\text{luz}(\dots)$  and  $\text{tar}(\dots)$  projections

The  $\text{luz}(\dots)$  and  $\text{tar}(\dots)$  projections have a lot of properties, which create some mathematical apparatus ([7], [8]). Example formulas for  $\text{luz}(\dots)$  are presented below ( $k, a, b > 0$ ):

$$\text{luz}(-x, a) = -\text{luz}(x, a), \quad k \cdot \text{luz}(x, a) = \text{luz}(k \cdot x, k \cdot a), \quad \text{luz}(\text{luz}(x, a), b) = \text{luz}(x, a + b). \quad (3), (4), (5)$$

If  $\text{luz}(y, b) = k \cdot \text{luz}(x - y, a)$  then:

$$\text{luz}(y, b) = \frac{k}{k+1} \cdot \text{luz}(x, a+b). \quad (6)$$

The method of modelling systems with freeplay and friction is explained by simple example (Fig. 3).

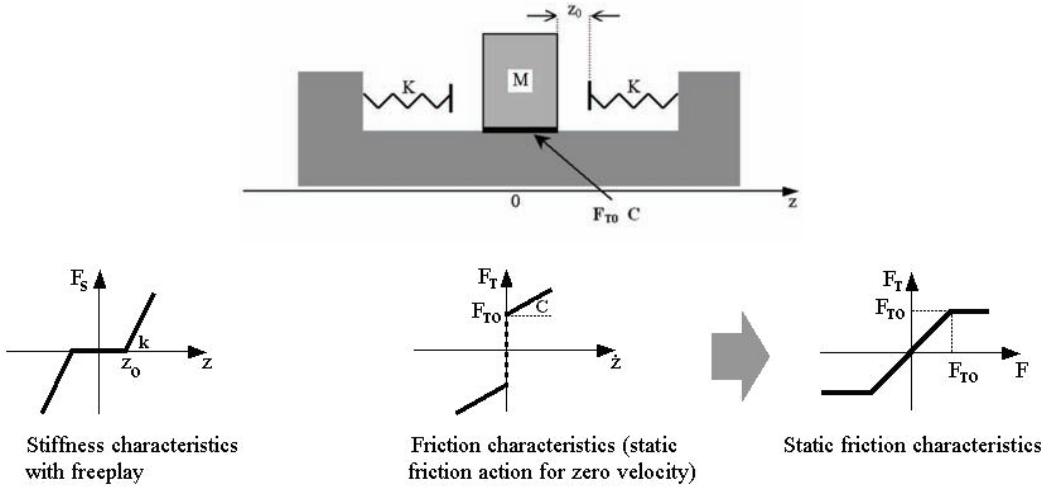


Fig. 3. Example single mass system, and his typical characteristics for freeplay and friction descriptions. Here is a version for kinetic and static friction force parameter  $F_{T0} = F_{TS0} = F_{T0}$ . Notation:  $k$  – stiffness coefficient,  $z_0$  – freeplay parameter,  $C$  – viscous friction coefficient,  $F_{T0}$  – dry friction parameter,  $F_S$  – stiffness force,  $F_T$  – friction force,  $z$  – relative displacement,  $\dot{z}$  – relative velocity,  $F$  – acting force

Analytical descriptions of freeplay / friction characteristics (Fig. 3) are following:

$$F_S = k \text{luz}(z, z_0), \quad (7)$$

$$F_T = \begin{cases} C \text{tar}\left(\dot{z}, \frac{F_{T0}}{C}\right) & \text{if } \dot{z} \neq 0, \\ F - \text{luz}(F, F_{T0}) & \text{if } \dot{z} = 0. \end{cases} \quad (8)$$

The balance of forces is:

$$M\ddot{z}(t) + F_T(\dots) + F_S(\dots) = 0. \quad (9)$$

The static friction force is balanced by the acting force, here:

$$F = -F_S(\dots). \quad (10)$$

So, the equation of motion has a variable structure form:

$$M\ddot{z}(t) = \begin{cases} -C \text{tar}\left(\dot{z}(t), \frac{F_{T0}}{C}\right) - K \text{luz}(z(t), z_0) & \text{if } \dot{z}(t) \neq 0, \\ -\text{luz}(K \text{luz}(z(t), z_0), F_{T0}) & \text{if } \dot{z}(t) = 0. \end{cases} \quad (11)$$

This model describes the stick-slip motion. When  $\dot{z}(t) = 0$  and  $|K \text{luz}(z(t), z_0)| \leq F_{T0}$ , then also  $\ddot{z}(t) = 0$  (stick state). For  $|K \text{luz}(z(t), z_0)| > 0$   $\ddot{z}(t) \neq 0$  (slip state).

Note:

$$\text{luz}(K \text{luz}(z(t), z_0), F_{T0}) = K \text{luz}\left(\text{luz}(z(t), z_0), \frac{F_{T0}}{K}\right) = K \text{luz}\left(z(t), z_0 + \frac{F_{T0}}{K}\right), \quad (12)$$

so thank to formulas (5), (6) the equation of motion is simplified again.

In the monograph [8] one can find more sophisticated examples. Among other things a problem of indeterminacy of friction forces (several friction pairs in a double mass system) is resolved. The elaborated mathematical apparatus is especially useful for simplifications and parametrically-made reductions of freeplay / friction models.

The  $\text{luz}(\dots)$  and  $\text{tar}(\dots)$  projections have been used for description of freeplay (gear backlash) and friction actions (with stick-slip effects) in steering system mechanisms.

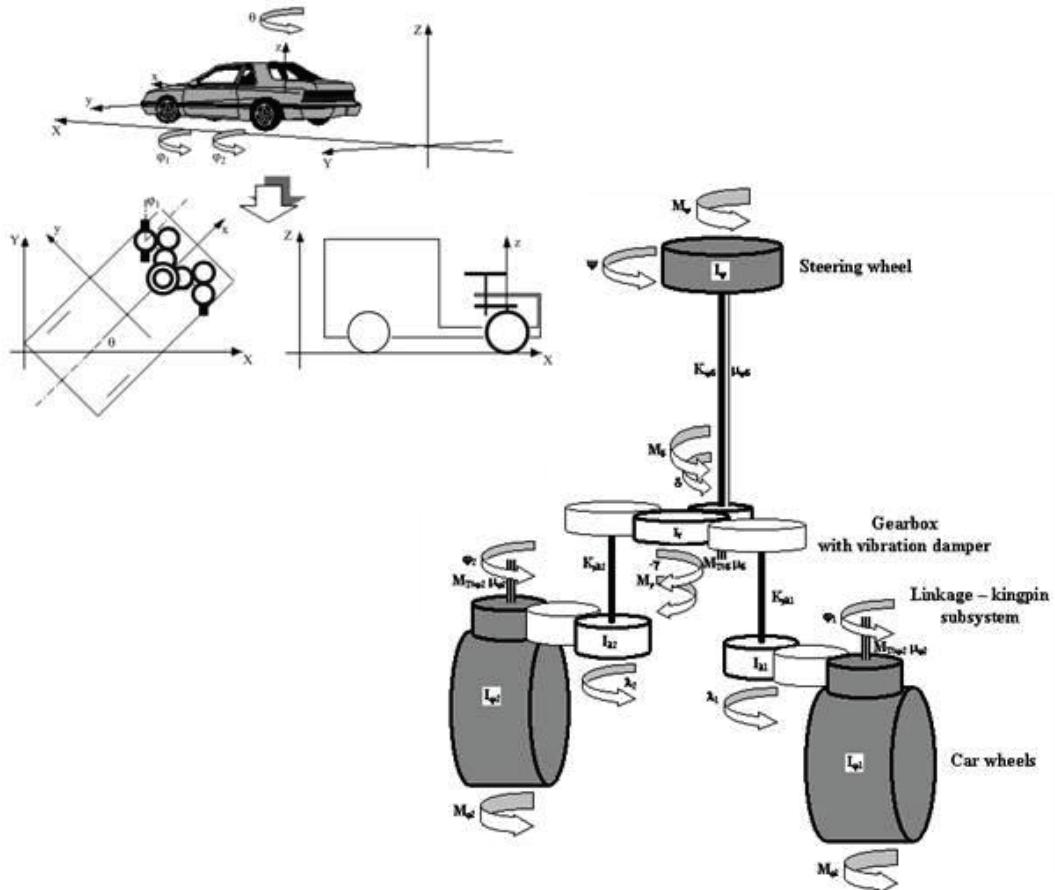


Fig. 4. Conception of MBS-type physical model of steering system mechanism

The monograph [8] contains several mathematical models – the most complex primary model (Fig. 4) and simplified ones (by parametrically-made reductions) suitable for additional assumptions. These steering system mechanism models are supplemented by components (transfer function and static blocks) which express dynamics of mechatronic elements in power assistance or 4WS systems.

The steering system simplest mathematical model presented below (13) (single-mass “bicycle” model basing on substitute parameters) is provided for symmetrical construction and excitations.

$$I_\phi \cdot \ddot{\phi}(t) = \begin{cases} -\mu_\phi \cdot \text{tar}\left(\dot{\phi}(t), \frac{M_{T0K\phi}}{\mu_\phi}\right) + M(t) & \text{if } \dot{\phi}(t) \neq 0, \\ \text{luz}(M(t), M_{T0S\phi}) & \text{if } \dot{\phi}(t) = 0, \end{cases} \quad (13)$$

where:

$$M(t) = p \cdot M_\psi(t) + M_\phi(t), \quad M_\psi(t) = K_{\psi\phi} \cdot \text{luz}((\psi(t) - p \cdot \phi(t)), (\delta - p\gamma)_0). \quad (14), (15)$$

Notation:

$\psi, \phi$  - angles of steering wheel and steered wheel,

$I_\phi$  - moment of inertia,

$\mu_\phi$  - viscous friction coefficient,  
 $M_{TOK\phi}, M_{TOS\phi}$  - kinetic and static dry friction parameters,  
 $K_{\psi\phi}$  - stiffness coefficient,  
 $(\delta - p\gamma)_0$  - freeplay parameter (one half of “dead zone”),  
 $p$  - gear ratio.

This model is enough to express the most important for our studies attributes of the steering system (its non-linear dynamics) and is applied here in a synthesis of steering signals for driver assistance systems.

### 3. Sensitivity analysis of steering systems models on account of freeplay and friction

Freeplay / friction influences on a car lateral dynamics can be studied on the model composed with partial sub-models: the model of steering system and the model of vehicle motion (Fig. 5).

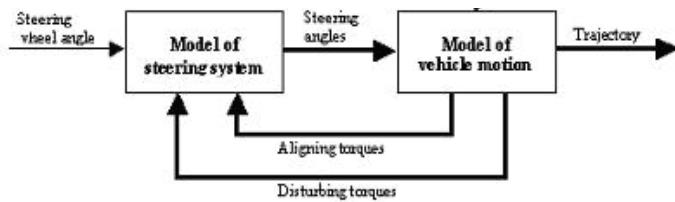


Fig. 5. Conception of decomposition of a car dynamics model into two partial sub-models

From mathematical point of view such studies can be treated as a parametrically-made sensitivity analysis of the car dynamics model (Fig. 6).

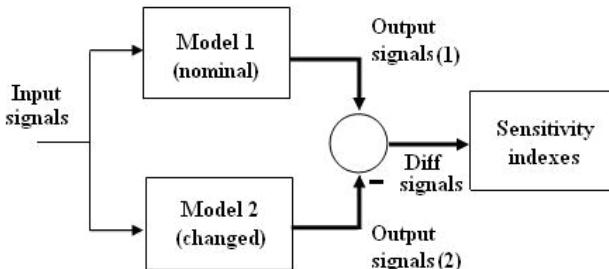


Fig. 6. General schematic diagram of sensitivity analysis

When model has a regular form and its changes are done by parametric perturbations, sensitivity indexes are continuous functions of these parameters. Classic sensitivity methods base on a variation analysis, so the continuity and differentiation of the model is demanded. In our case conditions for differentiation of equations are not fulfilled (non-smooth model). But the postulate of regularity is fulfilled and secured by the luz(...) and tar(...) apparatus (more details in [6, 9]). So difference signals are continue in relation to the freeplay or friction parameters, and sensitivity studies can be based on comparative simulations. According to this idea, the sensitivity analysis is provided by many simulations, for nominal and varied freeplay / friction parameters.

Such sensitivity / simulation investigations are presented in several authors' papers, e.g. [1], [8]. They concern open car tests according to ISO and ECE regulations and are conducted for different vehicle configurations (2WS and 4WS), different steering system structures (classic and with power assistance), different forms of kinetic friction characteristics (Coulomb- and Stribeck-type). Example results (Fig. 7, details in [8]) pertain strictly to the combined ISO road test of the 2WS car with the classic steering system.

In this paper the sensitivity / simulation analysis concerns optimized double lane change maneuvers (overtaking and avoiding).

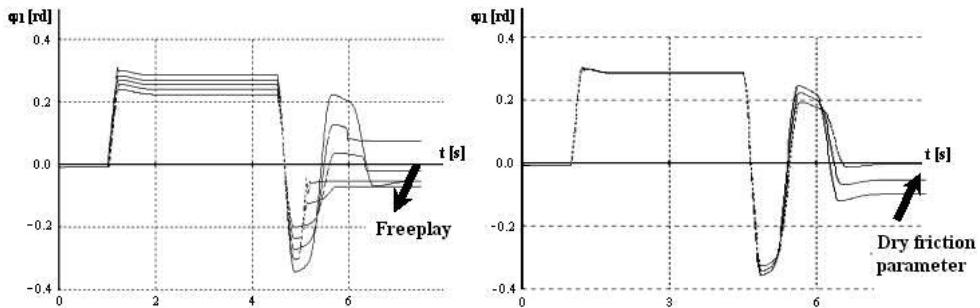


Fig. 7. Example simulation results (time history of left wheel angle) for combined ISO road test: ramp input on the steering wheel, then circular steady state motion, and finally steering wheel released

#### 4. Optimization of double lane change maneuver

Optimization of road maneuvers (for constant vehicle velocity by optimization of steering wheel angle) is a first step for synthesis a driver assistance system and then for auto-pilot system. An idea, a method and results of optimization of steering system input signals for typical road maneuvers have been presented with details in Wieckowski's dissertation [3] and several papers (e.g. [2]). An outline of this conception is shown below.

The optimal road maneuver means that the vehicle is controlled in the best way possible for given road situation. This is of course trivial and very general statement. After all, for concrete formulation of the optimization control problem (dynamic optimization) and then for its solution, one should have the object's mathematical model, the model of constraints and the criterion function.

The optimal control theory bases on the Belman's and Pontriagin's principles. According to this theory, the optimal control has a closed loop form. Of course when we know models of the object and his optimal controller, we can "extract" optimal steering signals and use them as set point signals for standard object's regulators. But direct application of this theory is practically limited only to very simple models. So, optimal control is frequently replaced by sub-optimal one with a set up structure and optimized parameters. Such method is used for synthesis sub-optimal classic regulators (e.g. PID controllers), where only several parameters are searched for a given controller model. Such method can be used also for synthesis sub-optimal steering functions as set point signals for standard controllers. In the synthesis of sub-optimal steering signals one can use combinations of standard time functions (linear, sinusoidal etc.). For setting the best shape of steering signal one should consider typical control signals which are present during manual control by an "expert". Having the best shape of the steering signal, the dynamic optimization is reduced to static optimization of parameters. The problem of qualification of criterion function seems to be the most difficult for such studies.

As regards to the optimization of double lane change maneuver the criterion function includes the following evaluations:

- precision of movement track performance – adherence to the defined lane,
- calm steering – avoiding violent, frequent movement of the steering wheel,
- feeling of comfortable travel for passengers.

It can be expressed by the criterion function formula (16)

$$J_w = w_1 \int_0^T \dot{\psi}(t)^2 dt + w_2 \kappa_{\max}^2 + w_3 a_{y\max}^2, \quad (16)$$

where:

$\kappa$  - inverse of distance between a car and an edge of traffic lane (measure of precision),

$\dot{\psi}$  - steering wheel angular velocity (measure of calm steering),

$a_{y\max}$  - maximal lateral acceleration value (measure of feeling of passenger comfort).

$w_1, w_2, w_3$  - weight" coefficients (they are speed-dependent factors – Tab. 1 [3]).

Tab. 1. Values of coefficients  $w_1$ ,  $w_2$  and  $w_3$  in travel speed function

Vehicle speed [km/h] ([m/s])	Coefficient $w_1$ [s <sup>2</sup> /rad <sup>2</sup> ]	Coefficient $w_2$ [1/m <sup>2</sup> ]	Coefficient $w_3$ [s <sup>4</sup> /m <sup>2</sup> ]
40 (11.11)	0.80	1.0	0.30
60 (16.67)	0.80	1.0	0.40
80 (22.22)	0.80	1.0	0.50
100 (27.78)	0.80	1.0	0.75
120 (33.33)	0.80	1.0	1.00

So, the task of optimization of double lane change manoeuvre is resolved in two stages:

- 1) Synthesis of a piecewise-sinusoidal / constant form of input signal with unknown values of amplitudes, frequencies and switch times. This is made by a detail analysis of signals registered during a real manoeuvre of the car and supported by simulation studies.
- 2) Optimization of unknown signal parameters. This includes: finding  $\min(J_w)$ , and fulfilling limiting conditions – vehicle adherence to a given movement track, and lateral acceleration values to the permissible extent ( $a_{y\max} \leq 4 \text{ m/s}^2$ ). If the limitations are not fulfilled  $\min(J_w) = \infty$ . The optimization task is made by series of simulations based on the reference car dynamics model. Exemplification of the method for double lane change manoeuvres (avoiding and overtaking) is showed on Fig. 8-10.

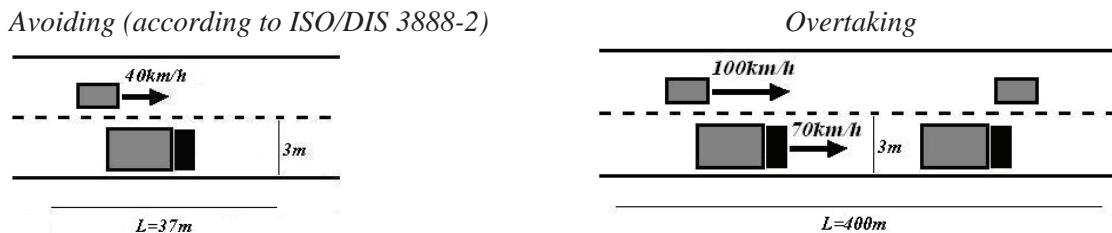


Fig. 8. Idea of a double change lane manoeuvre when avoiding or overtaking ( $L$  – length of “corridor”)

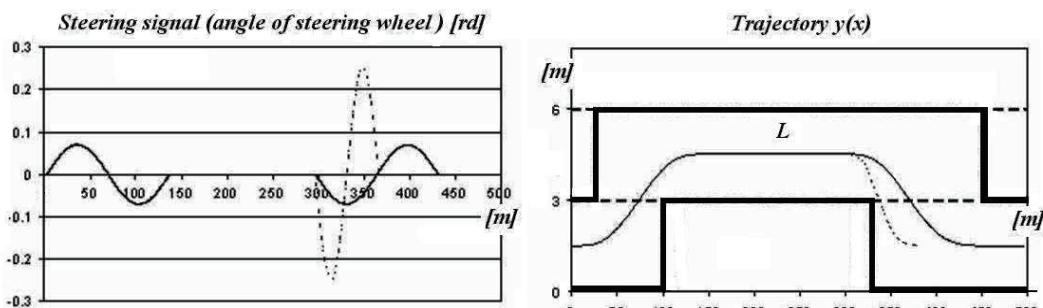


Fig. 9. Example results: optimal (continuous line), and another (dash line) – here for overtaking

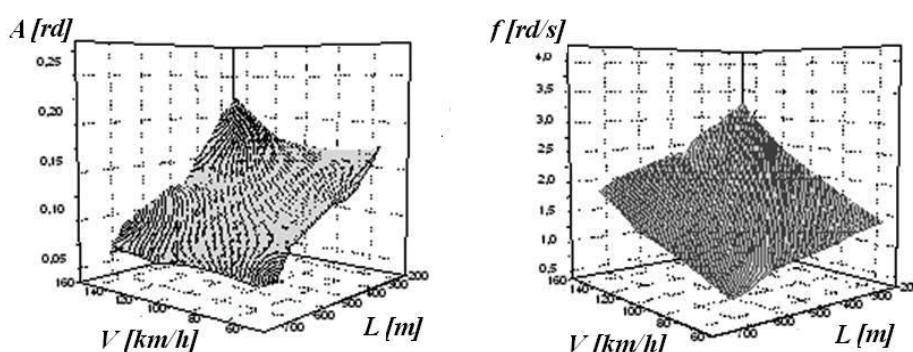


Fig. 10. Example dependences of input amplitude  $A$  and frequency  $f$  upon length of “corridor”  $L$  and speed  $V$

In these optimization studies the steering system model has been assumed as a simple one-mass linear system (no freeplay and friction effects). The vehicle motion has been described typically for constant speed manoeuvres. The full model describing the lateral and roll car dynamics has had 12 degrees of freedom [3].

The results of optimization give us an “optimal” steering signal  $\hat{\psi}(t)$  which is then applied to the real vehicle having the steering system with non-linearities (freeplay in steering gear, and friction with stiction in king-pins). It is very interesting and important to analyse the sensitivity of the results on simplifications of the reference model, especially checking of limiting conditions fulfilled. After all, an adherence to the given movement track is absolutely inadmissible! This can be tested by simulations of real manoeuvres with the non-linear steering system model.

## 5. Sensitivity analysis of reference model of double lane change manoeuvre

The studies have been organized as follow: Nominal and varied values of parameters  $(\delta - p\gamma)_0$  and  $M_{T0\phi}$  ( $M_{T0\phi} = M_{T0\phi} = M_{T0\phi}$ ) are assumed in two sets:  $\{(\delta - p\gamma)_0\}$  and  $\{M_{T0\phi}\}$ . For every one combination of the parameters an optimal steering signal  $\hat{\psi}_{i,j}(t)$  is calculated. So, the list of variants includes  $(i_{\max} \times j_{\max})$  functions having different amplitudes and frequencies and switching times of sinusoids. For the pair  $(\delta - p\gamma)_0 i$  and  $M_{T0\phi} j$  we receive the output signals (indexed by  $i, j$ ) and minimum value  $J_{wi,j}$  ( $J_{wi,j} = \infty$  when limiting are not fulfilled). Then the signals  $\hat{\psi}_{i,j}(t)$  are applied as steering signals to the model with varied parameters (from the sets). The signals and the values of  $J_w$  are compared to “optimal” ones. Thank to these sensitivity studies we can find pairs of freeplay / friction parameters  $(\delta - p\gamma)_0 i$  and  $M_{T0\phi} j$  for which the optimal steering is the most „robust”.

Because of publishing limitation only short sets have used here:  $(\delta - p\gamma)_0 \subset \{0, 0.05, 0.10\}$  rd,  $M_{T0\phi} \subset \{0, 4.05, 8.10\}$  Nm. The zero values should be treated as the „nominal” parameters.

The studies have concerned the avoiding as well as overtaking problems. The example results are showed on Fig. 11 and 12.

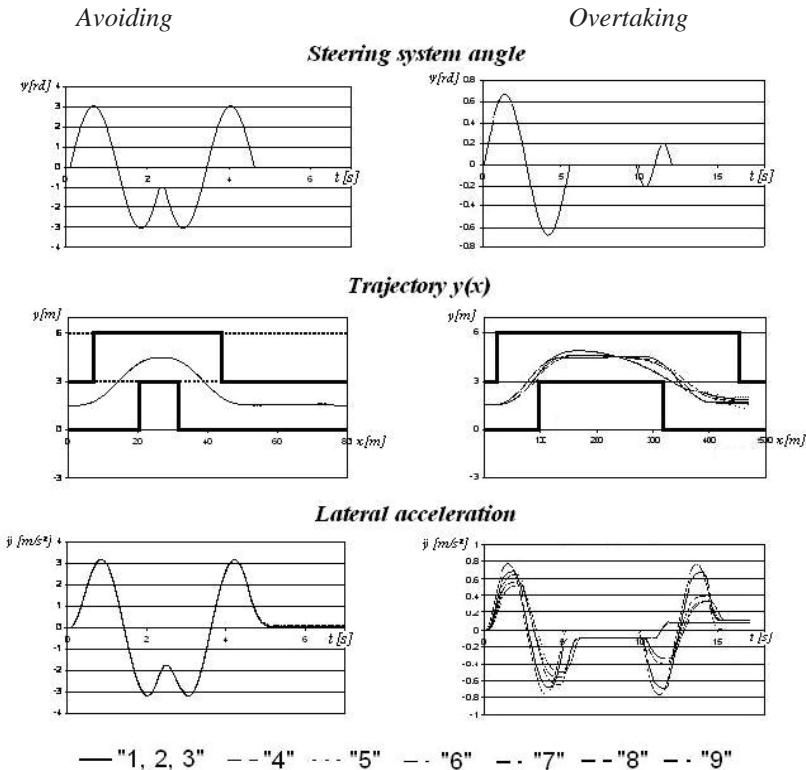
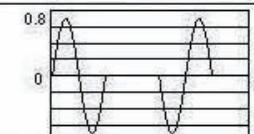
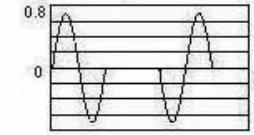
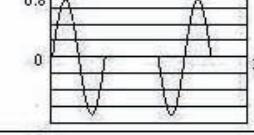
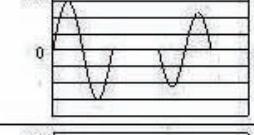
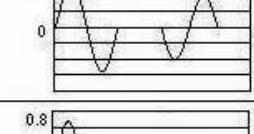
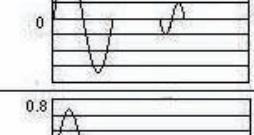
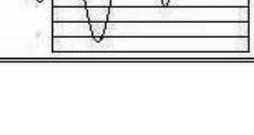


Fig. 11. Example simulation / sensitivity results for avoiding and overtaking manoeuvres. Numbering according to Fig. 12

## Overtaking

No	Freeplay [rad]	Friction [Nm]	Optimal steering signal [rad]	Value of criterion function $J_w$			
			20 [s]	Free Frict	0.00	0.05	0.10
1	0.00	0.00		Free	0.00	6.569	32.148
				Frict	4.05	6.017	35.671
					8.10	4.562	6.126
2	0.00	4.05		Free	0.00	6.569	32.148
				Frict	4.05	6.017	35.671
					8.10	4.5612	6.126
3	0.00	8.10		Free	0.00	6.569	32.148
				Frict	4.05	6.017	35.671
					8.10	4.5615	6.126
4	0.05	0.00		Free	0.00	0.05	0.10
				Frict	0.00	∞	6.569
					4.05	∞	5.861
5	0.05	4.05		Free	0.00	0.05	0.10
				Frict	0.00	∞	32.110
					4.05	∞	6.017
6	0.05	8.10		Free	0.00	0.05	0.10
				Frict	0.00	∞	6.688
					4.05	∞	6.129
7	0.10	0.00		Free	0.00	0.05	0.10
				Frict	0.00	∞	32.110
					4.05	∞	32.105
8	0.10	4.05		Free	0.00	0.05	0.10
				Frict	0.00	∞	35.676
					4.05	∞	35.671
9	0.10	8.10		Free	0.00	0.05	0.10
				Frict	0.00	∞	36.453
					4.05	∞	36.448
					8.10	4.562	6.126

## Avoiding

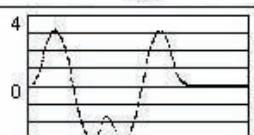
No	Freeplay [rad]	Friction [Nm]	Optimal steering signal [rad]	Value of criterion function $J_w$			
			7 [s]	Free Frict	0.00	0.05	0.10
1 - 9	0.00	0.00		Free	0.00	32.466	32.506
				Frict	4.05	32.466	32.513
					8.10	32.469	32.501

Fig. 12. Influence of freeplay / friction parameters on the  $J_w$  for different variants of optimized input signals

## 6. Conclusion

The results of studies are summed as follow:

- the optimal steering signal depends upon freeplay / friction parameters especially for longtime maneuvers (overtaking),
- an influence of the freeplay parameter is evident (for example, when the freeplay increases from 0° to 10°,  $J_w$  grows up from 4.859 to 32.148). An influence of the dry friction parameter seems to be passing over. This is rather clear because in these studies the input signal (steering wheel angle) has been a kinematic excitation. In case of dynamic excitation (steering torque) a situation seems to will be different,
- because of robustness it is recommended to made optimization on the model having zero value freeplay parameter,
- the studies should be continued (more extended excitation, different maneuvers, different structures of the steering system and the car).

## References

- [1] Lozia, Z., Więckowski D., Żardecki D., *Wpływ luzu i tarcia w układzie kierowniczym na kierowalność samochodu*, Zeszyty Naukowe Politechniki Świętokrzyskiej. Mechanika, Nr 8, pp. 89-120 (in Polish), 2008.
- [2] Stańczyk, T., Więckowski D., *Ocena poprawności wykonania typowych manewrów w ruchu samochodu*, Zeszyty Naukowe Politechniki Świętokrzyskiej, Mechanika 84, pp. 293-304 (in Polish), 2006.
- [3] Więckowski, D., *Optymalizacja wymuszenia działającego na kierownicę ze względu na kryterium poprawności wykonywania typowych manewrów w ruchu samochodu*, Rozprawa doktorska, Politechnika Świętokrzyska, Kielce 2006 (Dissertation in Polish).
- [4] Więckowski, D., Żardecki, D., *Simulation Studies of Optimized Manoeuvres for Steering System Freeplay and Friction Sensitivity Analysis*, Proc. of IAVSD'09, Sztokholm 2009.
- [5] Więckowski, D., Żardecki, D., *Badania wrażliwości optymalizowanego manewru wyprzedzania z uwagi na luz i tarcie w układzie kierowniczym*, Zeszyty Naukowe Instytutu Pojazdów (Politechnika Warszawska, Wydział SIMR), Nr 1(77), pp. 359-369 (in Polish), 2010.
- [6] Żardecki, D., *Structural Sensitivity Analysis of Non-Smooth Systems*, Proceedings of 9<sup>th</sup> DSTA Conference on Dynamical Systems - Theory and Applications, Vol. 1, pp. 489-496, Łódź 2007.
- [7] Żardecki, D., *Piecewise Linear luz(...) and tar(...) Projections. Part 1 – Theoretical Background. Part 2 – Application in Modelling of Dynamic Systems with Freeplay and Friction*, Journal of Theoretical and Applied Mechanics, Vol. 44, No. 1, pp. 163-202, 2006.
- [8] Żardecki, D., *Modelowanie luzu i tarcia oparte na odwzorowaniach luz(...) i tar(...) – podstawy teoretyczne i zastosowanie w symulacji drgań nieliniowych w układach kierowniczych samochodów*, (Rozprawa habilitacyjna), Wyd. WAT, Warszawa 2007 (habilitation monograph in Polish).
- [9] Żardecki, D., *Sensitivity Analysis of Steering System Model*, Journal of KONES Powertrain and Transport, Vol. 16, No. 1, pp. 571-581, 2009.