MODELLING OF CENTRE DIFFERENTIAL CONTROL

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Abstract

In this article we present a simplified model of centre differential control. At the beginning, the introduction highlights important role of center differential control and tests carried out using mathematical models. This work contains the full development of a model, consisting of: the equations which describe the vehicle, the structure of the model, important values and parameters used in the simulation. All these components allow you to expand an existing model, thus is not necessary to create it from the beginning. It presents detailed equations which were realized by a simulation model with a simplified block diagram illustrating the structure of the model and on the next the full structure. Then a concept of a simplified torque of split control system was described and which will be continue as the basis for the simulation model. The action of the control system is presented by the simulation model implemented in Matlab Simulink. Model refers to a linear motion in different road conditions and using the control of centre differential. In the following defines the parameters and data used in the model and the fifth chapter contains sample waveforms of model variables along with their brief description. The last two points are synthetic applications and supporting literature.

Keywords: *traffic modelling, 4WD, Matlab Simulink, dynamics, distribution of driving torque*

1. Introduction

Cars with four-wheel drive are an important segment of the automotive market. Their specificity is the drive system that allows to separate the driving torque on all wheels. 4WD transmission control, especially the steering torque of distribution, has a very significant impact on the behaviour of the vehicle and thus the safety of road users [1, 7]. Currently in produced 4WD vehicles are automatic control torque of distribution are more widely used, depending on traffic conditions of the vehicle [6]. The driving torque distribution to each wheel has fundamental importance to cooperation drive wheels with a road and thus for traction and vehicle dynamics. In the 2WD cars driving torque distribution is made in the mechanism of differential cross-axle (rear and front differential), while in 4WD vehicles also in an centre differential.

In modern solutions to control the friction of the differential are used in active systems based on electronics and mechatronics. The programmed controller is responsible for regulating and running the lock using the appropriate measuring signals. Control mechanism is realized by exerting pressure on the part of the plate assembly by the electro-hydraulic system or strong electro-magnet. Due to the fact that by using electro-hydraulic blockade produced significantly higher downforce, it is used primarily in the mechanism of differential rear axle. In centre differentials as blockades there are used both electro-hydraulic locks and electromagnetic locks. Using one of these solutions depends on the maximum transmitted torque, the manner of pressure distributions on the axles and the method torque distribution between the front and rear axle. In contrast to the mechanical hand locks, automatic control enables precise and smooth control of torque distribution from 0 to 100% [3]. Automatic control improves traction and improves the stability and steerability of car because distribution adjusts torque to the drive wheels to the wheel load conditions.

Both the modelling of components and the entire vehicle, is now a fundamental stage of designing. Higher demands for quality and correctness of the components in the era of the ruling downsizing is possible through the use of modern software supporting the design stage and the creation of mathematical models, reflecting their work in real conditions. The research, carried out by created in that way mathematical models allow to obtain the total holdings of the precise results received in the real tests, the same time being much less costly. Modelling of changes in objects enables the individual assessment of wear process in components, allows to describe the variable loads and their influence on the operation process, and also to determine its duration [8].

2. Vehicle model

The model of the car has been simplified because of limited access to detailed technical data. Figure 1 shows a flat forces system and moments acting on the car, moving linear motion on the road at α angle to consider its dynamics movement. It is a model with three degrees of freedom, namely the longitudinal centre $x(t)$ of mass displacement, angles of rotation of the wheels front $\varphi_1(t)$ and rear $\varphi_2(t)$ [5].

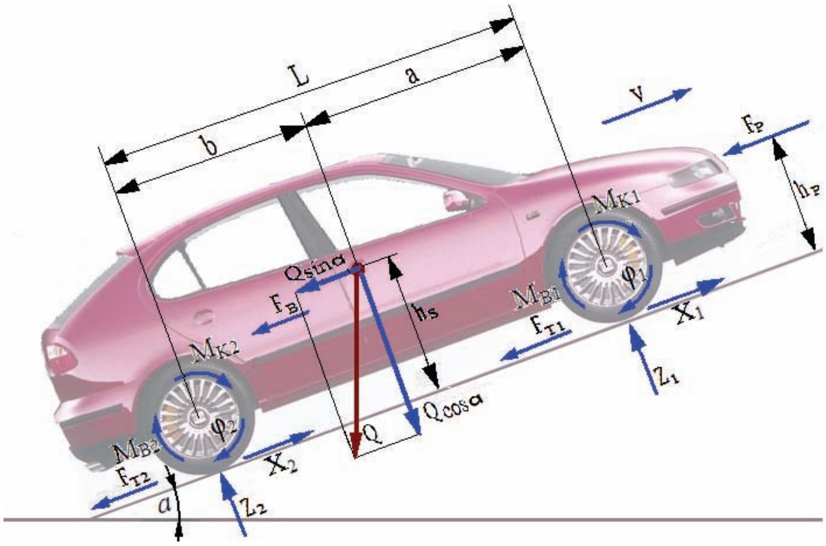

Fig. 1. System of forces and moments acting on the car: Q – vehicle weight, X_1, X_2 – longitudinal forces, F_T – rolling resistance, F_B – inertial force, F_P – aerodynamic drag force, Z_1, Z_2 – vertical forces, M_{K1}, M_{K2} – external torque wheel (driving torque or braking), M_{B1}, M_{B2} – moment of inertia wheel and related items, L – wheelbase, a, b – axis distance from the center of mass, v – vehicle velocity, h_{op} – pressure centre distance above the road surface , h_s – high of center mass location, α– angle of the road, φ_1, φ_2 – angles of wheels' rotation

Simulation model

In order to numerical simulation, state equations were formulated, based on equations describing the linear motion of the car, contained in [4]. In these equations were introduced signs:

$$v = \dot{x},\ \omega_1 = \dot{\varphi}_1,\ \omega_2 = \dot{\varphi}_2,\ J = 2J_K. \tag{1}$$

Equation 2 describes linear acceleration of the car, where the first two members are the relations of the tangential reaction force versus the slip (β is the coefficient of pressure distribution on the axles) and the third member describes power of the rolling resistance and air. The equations 3 and 4 describe the angular acceleration of the front and rear wheels respectively. The first member in both equations shows the torque on the wheel, while γ is the ratio of torque distribution. The second part of the equation is the relation tangential reaction force and the slip. Another two equations describe the formation of engine torque and rotational speed of the crankshaft.

$$\dot{v}(t) = \frac{1}{m}\left(mg\beta\mu_1(s_1(v(t),\omega(t)),t) + mg(1-\beta)\mu_2(s_2(v(t),\omega(t)),t) - mgf(t)0{,}5A\rho c_x v^2(t)\right), \qquad (2)$$

$$\dot{\omega}_1(t) = \frac{r_D}{J_{K1}}\left(\gamma\left(s_1(v(t),\omega_1(t))\frac{M_S(\omega_1(t),\omega_2(t))i_{UN}\eta_{UN}}{r_D} - mg\beta\mu_1(s_1(v(t),\omega_2(t)),t\right)\right), \qquad (3)$$

$$\dot{\omega}_2(t) = \frac{r_D}{J_{K2}}\left(\left(1-\gamma\left(s_2(v(t),\omega_2(t))\frac{M_S(\omega_1(t),\omega_2(t))i_{UN}\eta_{UN}}{r_D} - mg(1-\beta)\mu_2(s_2(v(t),\omega_2(t)),t\right)\right)\right), \qquad (4)$$

where: $M_S(\omega_1(t),\omega_2(t)) = M_S(\omega_S(\omega_1(t),\omega_2(t)))$ and $\omega_S(t) = 0{,}5(\omega_1(t),\omega_2(t))$.

Changes f and μ_1, μ_2 (resulting from the change of road surface) are modeled with the use of explicit functions of jump type time (used in a pseudo-function 1(t)). These changes are described by the following equations:

$$f(t) = f_0 + \Delta f(t) \qquad (5)$$

where:

$$\Delta f(t) = \Delta f * 1(t - t_0), \qquad (6)$$

$$\mu_1(s_1(v(t),\omega_1(t)),t) = \mu_{10}(s_1(v(t),\omega_1(t)),t) * w(t), \qquad (7)$$

$$\mu_2(s_2(v(t),\omega_2(t)),t) = \mu_{20}(s_2(v(t),\omega_2(t)),t) * w(t), \qquad (8)$$

$$w(t) = w_0 * 1(t - t_0), \qquad (9)$$

Equations describe the operation of the controller:

$$\gamma(s_1(v(t),\omega_1(t))) = 1 - \Delta\gamma(s_1(v(t),\omega_1(t))), \qquad (10)$$

where: $\Delta\gamma(s_1(v(t),\omega_1(t))) = 0{,}5(1 + sign(s_1(t) - s_{dop}))$,

where: s_{dop}– critical value, beyond which the second drive axle is turned on.

The simplified structure of the model is shown on Figure 2, the detailed structure is given in Figure 3.

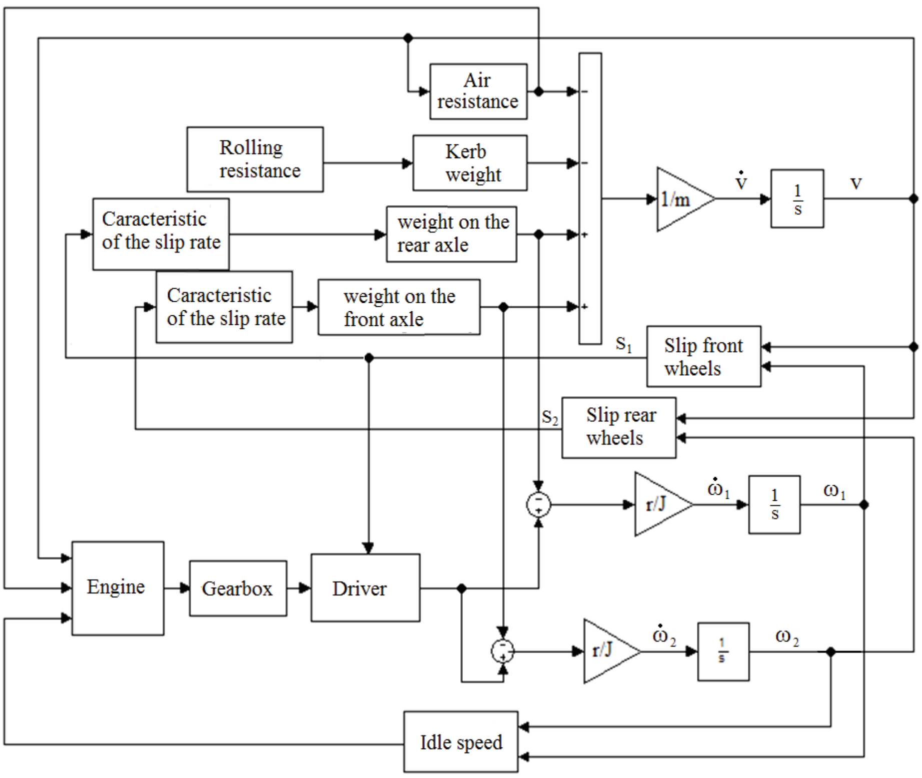

Fig. 2. Block diagram illustrating the structure of the model

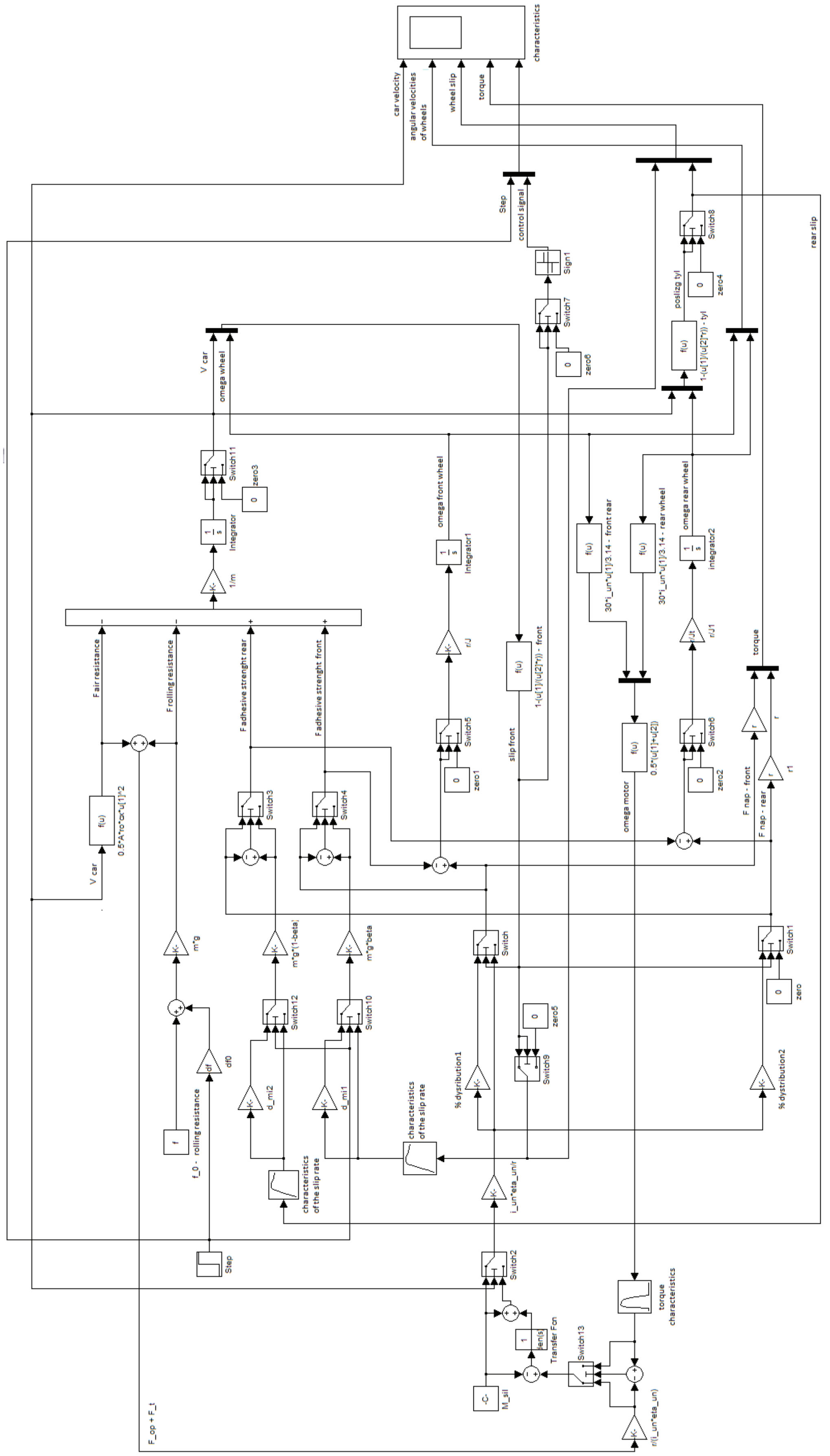

Fig. 3. The detailed structure of the model implemented in Matlab/Simulink

3. The concept of the control system

The presented driver conception explains the idea of controlling torque distribution in the mechanism of differential centre differential for exemplary Seat Leon Cupra. Under the conditions of having a good coefficient of adhesion (eg. asphalt) on the road, the drive is moved to the front wheels (drive 2WD). When the car drives on a surface with a reduced adhesion (eg. sand), the drive controller switches on also a drive of rear axle (4WD) (Fig. 4). Controlling of the second turning on the drive axle will also be implemented in case of starting from standstill and acceleration and also braking (Fig. 5).

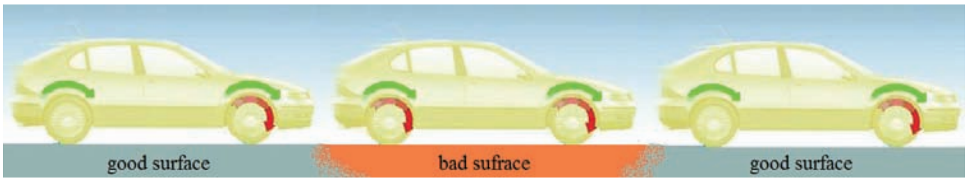

Fig. 4. The effect of the controller under varying conditions of adhesion: Green arrow – direction of rotation of the wheel, red arrow – wheel drive, bad road surface – a degraded surface properties, good surface - surface with good properties

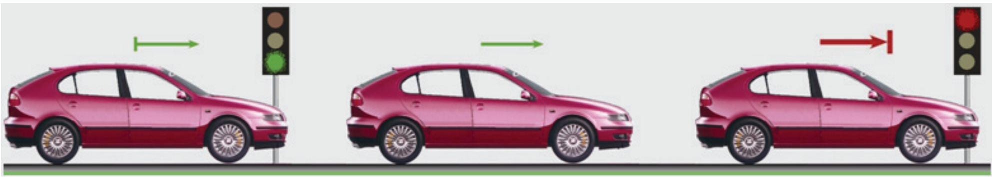

Fig. 5. Movement phase car: starting and acceleration, b) driving at a constant speed, c) inhibition

During starting and acceleration 4WD is turned on due to the tuning weight of the rear axle. When the car moves with established speed, 4WD is disconnected, and all the torque is directed to the front axle (FWD). During braking, the rear axle is unloaded and it is driven by only the front axle (FWD).

The proposed controller, working with the ASR system and on its basis, determines for the slipping wheels. When the front wheels slip, it will be greater than the slip boundary 0,2 (adopted on the basis of data from the literature [5]), the controller sends a signal to the centre differential clutch and turns on the second axle drive. To prevent the coupling pulse of clutch, such as during the passing through small puddles, changed conditions of sliding adhesion must maintain for minimum 1 second.

Turning off the 4WD will be implemented through an increase of sliding adhesion. Information about this driver will read off from decrease of the temporary slip below the slip limit. In this case the value of sliding adhesion is the average value of wheel slip front and rear axles.

Torque for 4WD is allocated for the axes at a fixed ratio of 63,7/36,3 (adopted on the basis of Seat Leon Cupra technical data).

4. Parameters for the simulation model

Model Car-controlled 4WD was developed for one of the states of motion described in section 3. The car goes at an established speed. The automation will be modelled, which - after the detection of an increasing slip over the limit value - turns on the drive to the rear axle.

The point of simulation is to model the distribution of the driver, not to detail the whole car. The simplifications that were adopted for the model are as follows:

- the car is treated as a two-wheeled vehicle with substitute wheels, having front and rear wheels,

– vehicle speed is matched in the way that it is not necessary to change the transmission ratio in increasing the resistance of the motion,
– open throttle and ratio drive system are constant (thanks to that the integrating system 4WD will be the only controlling one)
– it wasn't taken into account the dynamically changing the mass distribution on the axles;
– the vehicle moves on a flat surface with a zero angle of inclination,
– ruled out the possibility of pulsing operation,
– while in motion radius of the circle does not change

Data for the simulation model were developed on the basis of technical data SEAT Leon Cupra 1.9 TDi [9], [10] and the literature [5] (Table 1).

To determine the initial vehicle speed and the gear number, where it will be possible to maintain it at changing resistances of motion, it was necessary to develop a traction performance (Fig. 7) for Seat Leon Cupra 1.9 TDi. The wheels slip (s = 0,07) were included in determining the characteristics.

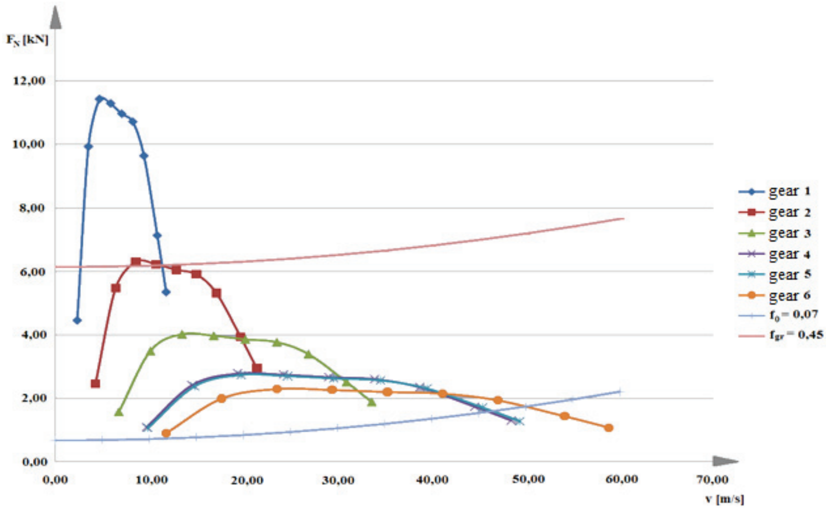

Fig. 7. Characteristics of vehicle traction Seat Leon Cupra 1,9 TDi

Tab. 1. Parameters for the simulation model based on literature [5]

Parameters		Symbol	Value	Unit
Efficiency of propulsion system		η_{UN}	0,95	
Air density		ρ	1,2	kg/m^3
Acceleration of gravity		g	9,81	m/s^2
Engine speed*		n_{sil}	1209	rpm
Front wheel slip*		s	0,05	
Unsurfaced road*	Rolling resistance	f_0	0,55	
	Adhesive strength	μ_0	0,07	
Wet sandy road	Rolling resistance	f	0,22	
	Adhesive strength	μ	0,14	
Dry sandy road	Rolling resistance	f	0,25	
	Adhesive strength	μ	0,20	
Car velocity*		v	8,47	m/s
Dynamic radius circle		r_D	0,3	m
Moment of inertia of wheel		J_K	0,5	
Scaling factor μ_0 for the wet sandy road		w	0,29	
Scaling factor μ_0 for the dry sandy road		w	0,42	

* *The value of the initial state*

5. Simulation results

The research was carried out using Matlab/Simulink – by the procedure the integral equations, ODE 4 (Runge-Kutta IV). The graphs were prepared using the Scope block. The block diagram of the model is given on Figure 2.

In initial stages of each of the simulations (by the first 2 seconds) a vehilce moves with a established motion, where there is a balance of strength and resistance movement forces which is needed to maintain the constant speed. When a car goes on a worse surface, the rolling resistance coefficient *f* and the coefficient of adhesion and μ are changed which causes the imbalance. The power unit react to the aggravated traffic conditions, increasing the swing speed of the crankshaft and it re-balances the forces. As a result of increasing of the wheels swing speed the limit values of slipping are exceed and then the driver works and turns on the 4WD. The research for: $\Delta f=0{,}07$; $\Delta\mu=0{,}33$, $m=1390$ kg, $\gamma=63{,}7\%$.

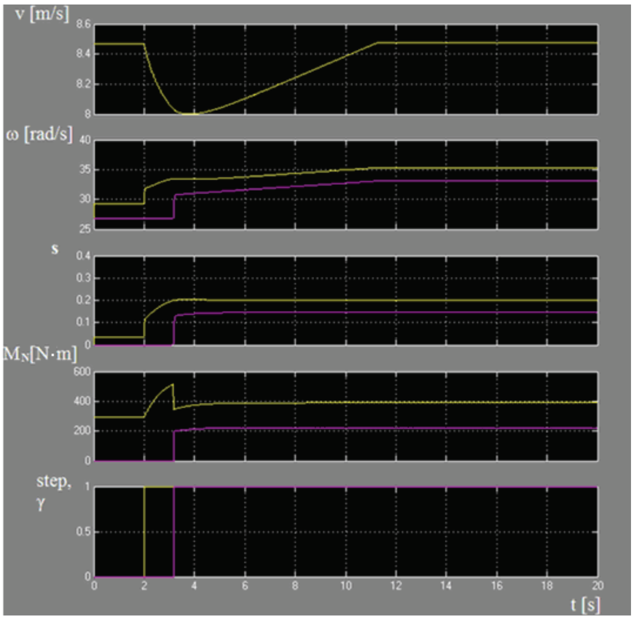

Fig. 8. Waveforms for the model variables $\Delta f=0{,}07$; $\Delta\mu=0{,}33$, $m=1390$ kg, $\gamma=63{,}7\%$

The car maintains a constant speed for the first two seconds. Wheels rotate at a constant angular speed, but the front wheel (yellow) has a greater angular speed than the rear wheel (purple).

After two seconds as a result of changes of rolling resistance coefficient (z $f = 0{,}07$ to $f = 0{,}14$) and sliding adhesion (z $\mu= 0{,}55$ to $\mu= 0{,}22$), the speed of the car begins to decline, while the angular speed of the front wheel begins to increase caused by the growth of the slip. Also the torque begins to increase to make a car maintain the constant speed. Further propel is not possible because of significant slippage of the wheels.

When the front wheel slip reaches s_{dop}, there is a turning on the second axle drive (Graph 5, purple). Angular speed of front and rear wheels have different values, even though both are driven. This is because of uneven loading axle and different proportions of torque distribution. After switching on the second axle the driving torque is distributed (in proportion 63,7:36,3) and as a result of decline of wheel speed and its stabilization it is possible to gather speed of the car to the initial velocity.

The research for: $\Delta f=0{,}13$; $\Delta\mu=0{,}30$, $m=1390$ kg, $\gamma=50$;

Even distribution of torque caused a slightly greater decrease of car speed than in previous tests. A greater slip, and thus the greater the angular velocity of the rear wheels is caused by a smaller rear axle downforce at the same time driving. It takes effect by slightly higher required output factor to speed up and maintain a constant velocity of the car (Fig. 9).

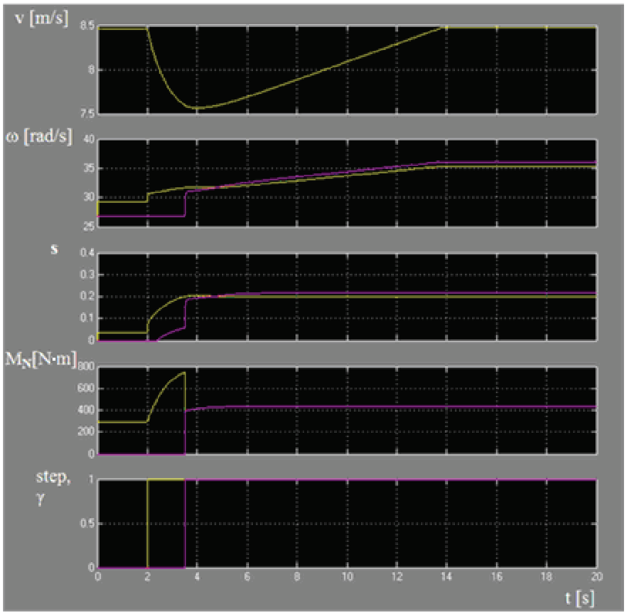

Fig. 9. Waveforms for the model variables Δf=0,13; $\Delta\mu$=0,30, m=1390 kg, γ=50

6. Conclusions

On the basis of these results we can be concluded that:

– at a total loss of traction of one of the wheels it is not possible to move the car without even a minimum differential lock (in case of the drive 2WD).

– the distribution driver of propulsion is working correctly, because the second axle of drive is switched on in accordance with the previous assumptions, thanks to that the vehicle is able to maintain a constant speed.

– the 4WD limits an influence of the slip factor and also the rolling resistance on the car speed. In conditions where a car powered by the 2WD slows down and maintains a lower speed than the original, the 4WD vehicle is able to accelerate.

Literature

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