DIFFERENCES METHOD TOPOLOGY OF HYDRODYNAMIC PRESSURE CALCULATIONS FROM REYNOLDS EQUATION

Krzysztof Wierzcholski

Pomeranian Academy Słupsk
Institute of Applied Mathematics
Arciszewskiego 22, 76-200 Słupsk, Poland
tel.: +48 59 840 53 28, fax: +48 59 840 54 75
e-mail: krzysztof.wierzcholski@wp.pl

Andrzej Miszczak

Maritime University Gdynia, Faculty of Marine Engineering
Morska Street 83, 81 225 Gdynia, Poland
tel.: +48 58 6901348, fax: +48 58 6901399
e-mail: miszczak@am.gdynia.pl

Abstract

The main topic of the presented paper concerns the method of determination and calculations of the general and particular numerical solutions of modified Reynolds equation in curvilinear orthogonal coordinates for curvilinear boundary conditions for hydrodynamic pressure distributions in bearing gap. A general solution matrix was derived and determined for various orthogonal coordinates of two cooperating bearing surfaces separated by the oil existing in slide journal bearings. From mathematical point presented method of solution of modified Reynolds equation leads this problem to resolving the partial recurrence non homogeneous, linear equation of second order with variable coefficients. As an example the numerical hydrodynamic pressure calculation for slide bearing two cylindrical surfaces limited bearing gap with various eccentricities was in this paper performed. In numerical calculations are applied the formulae and calculation algorithm derived in this paper. Moreover, presented numerical topology of calculations in this paper enables to find the optimum way of determination of pressure values for an applied difference method regarding to the stability of solutions, convergences of the obtained values with various orthogonal shapes of cooperating surfaces. An adaptation of the known numerical difference method to the various curvilinear orthogonal boundary conditions applied during the hydrodynamic pressure determination on the two various curvilinear cooperating surfaces, to be decisive after author knowledge about a new achievement consisted in presented paper.

***Keywords**: Reynolds equation, new numerical solution, curvilinear boundary conditions*

1. Introduction

The main topic of the presented problem concerns the method of determination and calculations of the general and particular numerical solutions of modified Reynolds equation in curvilinear orthogonal coordinates for curvilinear boundary conditions for hydrodynamic pressure distributions in bearing gap. A general solution matrix was derived and determined for various orthogonal coordinates of two cooperating bearing surfaces separated by the oil existing in slide journal bearings. From mathematical point presented method of solution of modified Reynolds equation leads this problem to resolving the partial recurrence non homogeneous, linear equation of second order with variable coefficients. In numerical calculations are applied the formulae and calculation algorithm derived in this section. Moreover presented numerical topology of calculations in this section enables to find the optimum way of determination of pressure values for an applied difference method regarding to the stability of solutions, convergences of the obtained values with various orthogonal shapes of cooperating surfaces. An adaptation of the known numerical difference method to the various curvilinear orthogonal boundary conditions applied during the hydrodynamic pressure determination on the two various curvilinear cooperating surfaces, to be decisive after author knowledge about a new achievement consisted in presented paper. Presented method is to be adapted to the various geometries of cooperating surfaces in curvilinear orthogonal co-ordinates.

2. The method of simulation for the arbitrary surfaces in curvilinear orthogonal coordinates

In curvilinear coordinates (α_1, α_2, α_3,) the modified Reynolds equations determines hydrodynamic pressure p(α_1,α_3,) between two surfaces with curvilinear sections. Such pressure is an unknown function of the following partial differential equation of second order:

$$C(\alpha_1,\alpha_3)\frac{\partial}{\partial\alpha_1}\left[A(\alpha_1,\alpha_3)\frac{\partial p}{\partial\alpha_1}\right]+F(\alpha_1,\alpha_3)\frac{\partial}{\partial\alpha_3}\left[B(\alpha_1,\alpha_3)\frac{\partial p}{\partial\alpha_3}\right]=$$

$$=G(\alpha_1,\alpha_3)\frac{\partial H(\alpha_1,\alpha_3)}{\partial\alpha_1}. \tag{1}$$

Above mentioned surfaces in curvilinear orthogonal coordinates are presented in Fig. 1.

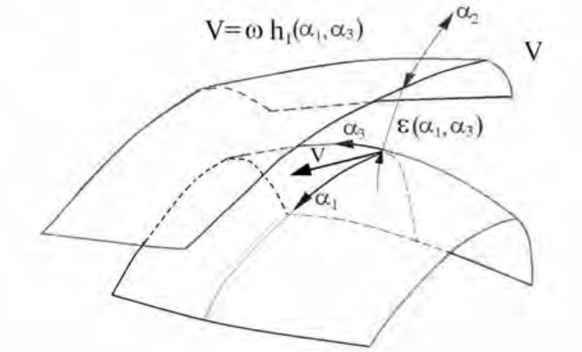

Fig. 1. The gap between two surfaces described in curvilinear orthogonal coordinates

In particular case if dynamic viscosity function η (α_1,α_3) is constant in gap height direction α_2, then we have:

$$A(\alpha_1,\alpha_3)\equiv-\frac{\varepsilon^3(\alpha_1,\alpha_3)}{12\eta(\alpha_1,\alpha_3)},\quad H(\alpha_1,\alpha_3)\equiv-\frac{1}{2}\varepsilon(\alpha_1,\alpha_3). \tag{2}$$

Now we will be transform equation (1) to the partial recurrence equation. After differentiating equations (1) has the following form:

$$C\frac{\partial A}{\partial\alpha_1}\frac{\partial p}{\partial\alpha_1}+CA\frac{\partial^2 p}{\partial\alpha_1^2}+F\frac{\partial B}{\partial\alpha_3}\frac{\partial p}{\partial\alpha_3}+FB\frac{\partial^2 p}{\partial\alpha_3^2}=G\frac{\partial H}{\partial\alpha_1}. \tag{3}$$

We seek the unknown dimensionless pressure solution p from equation (2), taking into account progressive differences in difference method and recurrence equations in lubrication region domain Ω(α_1,α_3). Subintervals in α_1 direction are described by the index i and subintervals in α_3 directions are denoted by index j. The division of region domain is defined as follows:

$$\Omega(\alpha_{1i},\alpha_{3j}):\begin{cases} i=1,2,3,.....,M \\ j=1,2,3,......., N. \end{cases} \tag{3}$$

The steps of division of region domain Ω have the following form:

$$h \equiv \alpha_{1i+1} - \alpha_{1i}, \quad k \equiv \alpha_{3j+1} - \alpha_{3j}, \quad dla \quad i = 1,2,...,M;\ j = 1,2,...,N. \tag{4}$$

Curvilinear region Ω contains NM nods where are 2M+2N–4 varies nods on the boundary of the region and NM–2(N+M)+4 internal nods. The Fig. 2 shows the region Ω and one nod.
The values of the functions: A, B, C,F, G, in the nods $w_{ij}=(\alpha_1=\alpha_{1i}, \alpha_3=\alpha_{3j})$ of divided region Ω for i=1,2,…,M–1,M; j=1,2,…,N–1,N we denote by the following formulae:

$$\begin{gathered} A(w_{ij}) \equiv A_{i,j}, \quad B(w_{ij}) \equiv B_{i,j}, C(w_{ij}) \equiv C_{i,j}, F(w_{ij}) \equiv F_{i,j}, G(w_{ij}) \equiv G_{i,j} \\ p(\alpha_1 = \alpha_{1i}, \alpha_3 = \alpha_{3j}) = p(w_{ij}) \equiv p_{i,j}. \end{gathered} \tag{5}$$

Taking into account steps of division (4) and progressive differences, then we obtain:

$$\left(\frac{\partial A}{\partial \alpha_1}\right)_i \left(\frac{\partial p}{\partial \alpha_1}\right)_i \approx p_{i+1,j}\frac{A_{i+1,j} - A_{i,j}}{h^2} - p_{i,j}\frac{A_{i+1,j} - A_{i,j}}{h^2}, \tag{6}$$

$$\left(\frac{\partial \mathbf{B}}{\partial \alpha_3}\right)_j \left(\frac{\partial p}{\partial \alpha_3}\right)_j \approx p_{i,j+1}\frac{B_{i,j+1} - B_{i,j}}{k^2} - p_{i,j}\frac{B_{i,j+1} - B_{i,j}}{k^2}. \tag{7}$$

a) b)

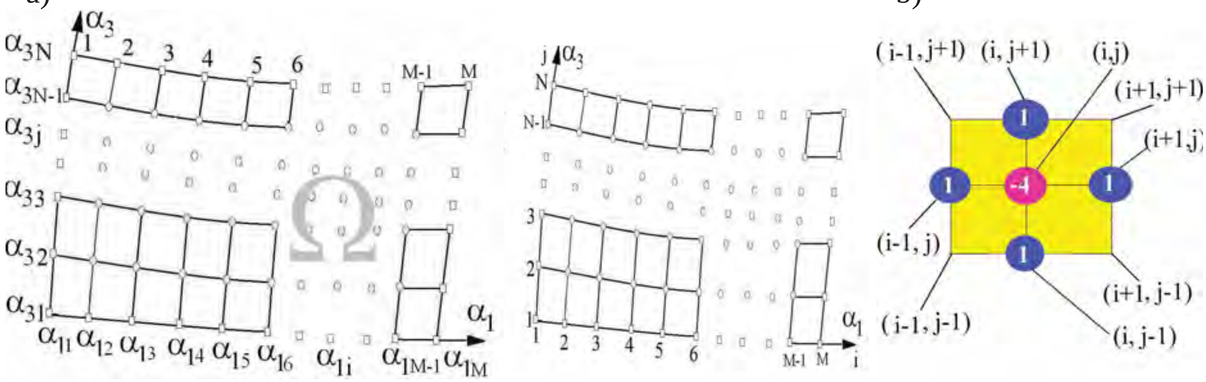

Fig. 2. Curvilinear orthogonal coordinate system: a) region Ω, b) calculation nod

The pressure function p(x,z) for $\alpha_1 \equiv x$, $\alpha_3 \equiv z$ we expand in Taylor series in neighborhood of the point (x+h,z+k) in following form:

$$\begin{gathered} p(x+h, z+k) = p(x,z) + \left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial z}\right)p(x,z) + \frac{1}{2!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial z}\right)^2 p(x,z) + ... + \\ + \frac{1}{n!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial z}\right)^n p(x + \Theta_x h, z + \Theta_z k), \quad 0 < \Theta_x, \Theta_z < 1. \end{gathered} \tag{8}$$

Taking into account index differentiation of the function p(x,z) in neighborhood of the point (i,j) then from (8) follows:

$$0.5\ \{h2\ pxx + k2\ pzz\}ij = pi+h,j+k - pij - h(px)ij - k(py)ij - hk(pxy)ij + O(h3) + O(k3). \tag{9}$$

We put h=1, k=0 and h= –1, k=0 into formula (9) and we multiply both hands of equation by CA. If we sum up obtained equations, then we obtain:

$$CA\frac{\partial^2 p}{\partial \alpha_1^2} = \frac{C_{i,j}A_{i,j}}{h^2}F_{i+1,j} + \frac{C_{i,j}A_{i,j}}{h^2}F_{i-1,j} - 2\frac{C_{i,j}A_{i,j}}{h^2}F_{i,j}, \quad \alpha_1 \equiv x. \tag{10}$$

We put h=0, k=1 and h=0, k=–1 into formula (9) and we multiply both hands of equation by FB. If we sum up obtained equations, then we obtain:

$$FB\frac{\partial^2 p}{\partial \alpha_3^2}=\frac{F_{i,j}B_{i,j}}{k^2}F_{i,j+1}+\frac{F_{i,j}B_{i,j}}{k^2}F_{i,j-1}-2\frac{F_{i,j}B_{i,j}}{k^2}F_{i,j},\quad \alpha_3\equiv z. \tag{11}$$

If we put dependencies (6), (7), (10), (11) into partial differential equation (2), and taking into account formulas (5), then for each nod from Fig. 2b we obtain following partial recurrence equations of second order with variable coefficients:

$$S_{i+1,j}\,p_{i+1,j}+S_{i,j+1}p_{i,j+1}+S_{i,j-1}p_{i,j-1}+S_{i-1,j}\,p_{i-1,j}-Z_{i,j}\,p_{i,j}=Q_{i,j}, \tag{12}$$

where for i=1,2,…, M–1, M; j=1,2,…,N–1, N we have the following coefficients:

$$S_{i+1,j}\equiv\frac{C_{i,j}A_{i+1,j}}{h^2},\quad S_{i,j+1}\equiv\frac{F_{i,j}B_{i,j+1}}{k^2},\quad S_{i-1,j}\equiv\frac{C_{i,j}A_{i,j}}{h^2},\quad S_{i,j-1}\equiv\frac{F_{i,j}B_{i,j}}{k^2}, \tag{13}$$

$$\mathrm{Z}_{i,j}\equiv \mathrm{S}_{i+1,j}+\mathrm{S}_{i,j+1}+\mathrm{S}_{i,j-1}+\mathrm{S}_{i-1,j},\quad Q_{i,j}\equiv G_{i,j}\frac{H_{i+1,j}-H_{i,j}}{h}. \tag{14}$$

The sequence of pressure values $\{p_{i,j}\}$ with unknown elements presenting the pressure values in nods of the region Ω is the solution is the solution of the partial recurrence equation (12) however the coefficients $S_{i,j}$ and $Q_{i,j}$ are known. If we take the same number of steps in α_1 and α_3 directions i.e. M=N, then the region $\Omega(\alpha_1,\alpha_3)$ presented in Fig. 2a has $(N–2)^2$ internal nods (i,j) for i=2,…,N–1; j=2,…,N–1.If we put index values i and j for the each internal nod into equation (12), then we obtain algebraic system of $(N–2)^2$ linear, non homogeneous equations with $(N–2)^2$ unknown. We assume known values Φ in 4N–4 external nods of the region $\Omega(\alpha_1,\alpha_3)$ in the form of the following boundary conditions:

pk,j=Φk,j, pi,k=Φi,k, for k=1,…, N; i,j=1,…, N–1, N; Φi,j=Φ (αi,αj). (15)

If the region Ω will be divided in N=100 steps in both directions α_1,α_3 ,then we obtain linear algebraic system of equations with 98^2=9604 equations and 9604 unknown. The solution of such system of equations requires the computer with the large memory.

3. Matrices simulation

We substitute into recurrence equation (12) the coordinates (i,j) or $(\alpha_{1i},\alpha_{3j})$ where i=2,…,99; j=2,…,99 for 98×98=9604 internal nods of the region $\Omega(\alpha_{1i},\alpha_{3j})$ presented on the Fig. 2a for N=M=100, and we respect the boundary conditions (15). We obtain the system of 9604 algebraic linear equations determining 9604 unknown pressure values p_{ij}.

Quadratic matrix **U** with 9604 rows and columns is presented as the matrix with 98 rows and 98 columns where each element denotes minor where each minor has 98 rows and 98 columns and is defined in following form:

$$\mathbf{U}=\begin{bmatrix} \mathbf{M}_2 & \mathbf{E} & \mathbf{O} & \mathbf{O} & \dots & . & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{E} & \mathbf{M}_3 & \mathbf{E} & \mathbf{O} & \dots & . & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{E} & \mathbf{M}_4 & \mathbf{E} & . & . & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{E} & \mathbf{M}_5 & . & . & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} & . & . & \mathbf{M}_{97} & \mathbf{E} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} & . & . & \mathbf{E} & \mathbf{M}_{98} & \mathbf{E} \\ \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} & . & . & \mathbf{O} & \mathbf{E} & \mathbf{M}_{99} \end{bmatrix}\quad \mathbf{E}=\begin{bmatrix} 1 & 0 & 0 & 0 & \dots & . & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & . & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & . & . & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & . & . & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & . & . & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & . & . & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & . & . & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

Matrix **U** consists of: minors **O,** where each element is zero, unit minors **E,** and minors $\mathbf{M_j}$.
In presented case for N=M=100 matrix U consists of:
① $N^2-7N+12=9312$ zero matrices **O**: $(N-2)\times(N-2)=(98\times98)$,
② $2N-6=194$ unit matrices **E**: (98×98),
③ $N-2=98$ three-diagonal matrices, with $N-2=98$ rows and 98 columns with the following form:

$$\mathbf{M}_{\mathbf{j}}=\begin{bmatrix} \mu_{2j} & 1 & 0 & 0 & . & , & 0 & 0 & 0 \\ 1 & \mu_{3j} & 1 & 0 & . & . & 0 & 0 & 0 \\ 0 & 1 & \mu_{4j} & 1 & . & . & 0 & 0 & 0 \\ 0 & 0 & 1 & \mu_{5j} & . & . & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0. & 0 & 0 & 0 & . & . & \mu_{97j} & 1 & 0 \\ 0 & 0 & 0 & 0 & . & . & 1 & \mu_{98j} & 1 \\ 0 & 0 & 0 & 0 & . & . & 0 & 1 & \mu_{99j} \end{bmatrix}, \tag{17}$$

for j=2, 3, 4, 5, 6, 7, 8,…….,N−1; i.e. j=2,3,…,98,99. Coefficients $\mu_{i,j}$ are calculated from the formula:

$$\mu_{ij} \equiv -\frac{Z_{i,j}}{S_{i,j}} = -\frac{S_{i+1,j}+S_{i,j+1}+S_{i,j-1}+S_{i-1,j}}{S_{i,j}}. \tag{18}$$

Taking into account Cramer law we calculate 9604 unknown pressure values from the formula:

$$p_{i,j} = \frac{\det \mathbf{U_{ij}}}{S_{i,j}\det \mathbf{U}} \text{ for i,j=2,3,3,5,….,98,99.} \tag{19}$$

If we replace in matrix **U** the column with elements μ_{ij} by the column with Q_{ij} elements, then we obtain matrix $\mathbf{U}_{ij}$. For example in matrix $\mathbf{U_{ij}=U_{53}}$ we replace the column with elements $\mu_{i,j}=\mu_{5,3}$ in matrix **U** by the column $[Q_{ij}]$. In this case it is: $i-1+(N-2)\cdot(j-2)=5-1+(100-2)(3-2)=102$ column in the matrix **U** with 9604 rows and columns.

4. The method of simulation for the cylindrical surfaces

Now on the Fig.3.we assume two cylindrical surfaces of the journal and sleeve.

a) b)

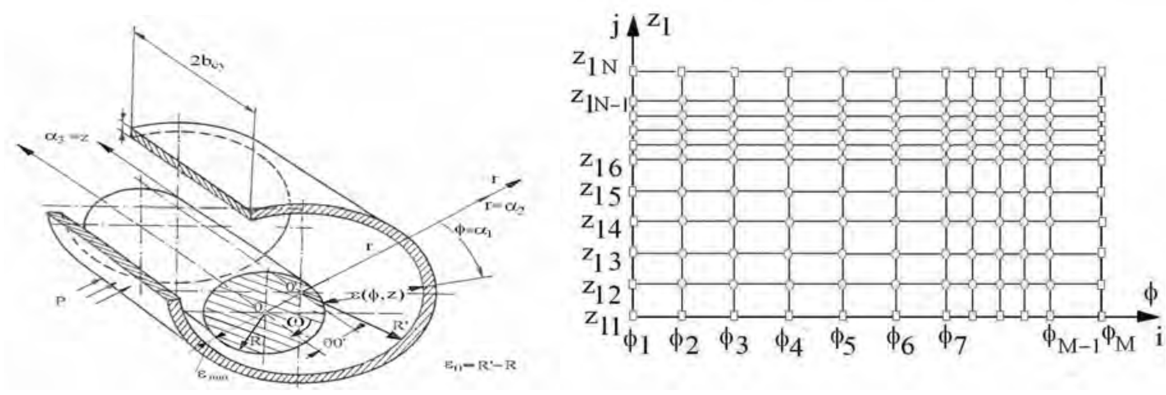

Fig. 3. Two cylindrical surfaces: a) two cylinders with radii R, R' separated with variable gap height $\varepsilon(\phi,z)$ filled with oil of viscosity η in circumferential ϕ and longitudinal z direction, b) region Ω lying on the cylindrical surface expanded to the plane

The height of the gap has the form:

$$\varepsilon(\phi,z)=\varepsilon 0(1+\lambda cy\cos\phi) + fcy(z), \tag{20}$$

where ε_0=R'–R denotes radial clearance, λ_{cy}=OO'/ε_0 - eccentricity ratio, R-radius of the journal, $2b_{cy}$-journal length. If the dynamic viscosity is independent of the gap height direction i.e. $\eta(\phi,z_1)$, and z_1 we denote as dimensionless coordinate, then the coefficients of Eq.(1) have the form:

$$A = B = \frac{\varepsilon^3}{\eta}, \quad C = \frac{1}{R^2}, \quad F = \frac{1}{4b_{cy}^2}, \quad G = 6\omega, \quad H = \varepsilon. \tag{21}$$

Now we will be transform partial differential equation (2) to the recurrence equations (12) into cylindrical region $\Omega(\phi,z_1)$ indicated in Fig. 3b. Subintervals in ϕ direction are described by the index i=1,2,…,M, and subintervals in z_1 directions are denoted by index j=1,2,…,N similarly as in formula (3).The steps of division of the cylindrical region Ω have the following form:

$$h \equiv \phi_{i+1} - \phi_i, \quad k \equiv z_{1j+1} - z_{1j}, \quad dla \quad i = 1,2,...,M;\, j = 1,2,...,N. \tag{22}$$

The coefficient (13), (14) in recurrence equations (12) are as follows:

$$S_{i+1,j} = \frac{1}{h^2R^2}\frac{\varepsilon_{i+1,j}^3}{\eta_{i+1,j}},\ S_{i,j+1} = \frac{1}{4k^2b_{cy}^2}\frac{\varepsilon_{i,j+1}^3}{\eta_{i,j+1}}, \quad S_{i-1,j} = \frac{1}{h^2R^2}\frac{\varepsilon_{i,j}^3}{\eta_{i,j}}, \quad S_{i,j-1} = \frac{1}{4k^2b_{cy}^2}\frac{\varepsilon_{i,j}^3}{\eta_{i,j}}, \tag{23}$$

$$Z_{i,j} = \frac{1}{h^2R^2}\left(\frac{\varepsilon_{i+1,j}^3}{\eta_{i+1,j}} + \frac{\varepsilon_{i,j}^3}{\eta_{i,j}}\right) + \frac{1}{4k^2b_{cy}^2}\left(\frac{\varepsilon_{i,j+1}^3}{\eta_{i,j+1}} + \frac{\varepsilon_{i,j}^3}{\eta_{i,j}}\right), \quad Q_{i,j} = 6\omega\left(\varepsilon_{i+1,j} - \varepsilon_{i,j}\right). \tag{24}$$

In 2M+2N–4 boundary nods of the region $\Omega(\phi,z)$ presented in Fig.3b, we impose known values in the form of the following boundary conditions:

$$\begin{aligned} &p_{1,j} = 0 \quad for \quad j = 2,3,...,N-1, \quad p_{M,j} = 0 \quad for \quad j = 2,3,...,N-1, \\ &p_{i,1} = 0 \quad for \quad i = 1,2,3,...,M, \quad p_{i,N} = 0 \quad for \quad i = 1,2,3,...,M. \end{aligned} \tag{25}$$

Pressure attains not atmospheric value on the boundary line determined by the coordinate $\varphi=\varphi_M$ i.e. i=M on the right side of the region $\Omega(\phi,z_1)$. Atmospheric values (≅zero values) are attained on the unknown curve of film end $\varphi_{i,j}^{(k)}$ which is restricted the region $\Omega(\phi,z_1)$ from the right side. Region $\Omega(\phi,z_1)$ in φ direction is restricted by the inequality:

$$0 \le \phi \le \varphi_{i,j}^{(k)}, \quad 0 \le z_1 \le L_{cy} \equiv b_{cy}/R, \quad \left.\frac{p_{i+1,j} - p_{i,j}}{h}\right|_{(\phi,z)\in\varphi_{i,j}^{(k)}} = 0. \tag{26}$$

5. The numerical calculations for the cylindrical surfaces

We determine hydrodynamic pressure distribution between two cylindrical surfaces of the journal and sleeve in slide journal bearing for the following data: R=0.026m, $L_{cy}=b_{cy}/R=1$, η_0=0.15Pas, $\varepsilon_0=26\times10^{-6}$ m, $\omega=1.0\ s^{-1}$. Gap height changes not in z direction hence from equation (20) follows that f(z)=0. We assume that dynamic viscosity is constant i.e. $\eta=\eta_0$ and we take following eccentricities: $\lambda_{cy}=0.7$; $\lambda_{cy}=0.8$; $\lambda_{cy}=0.9$.

We assume the hundred number of steps in each direction hence we have N=M=100 in dimensionless region $\Omega(\phi,z_1)$: $0\le\phi\le1.5\pi$; $0\le z_1\le1$. Thus we have the following length of the steps: k=1/100, h=1.5π/100. For f(z)=0 and for the constant viscosity, the coefficients (23), (24) are reduced.

Coefficients (i,j) or (ϕ_i,z_j) where i=2,…,99; j=2,…,99 for 98×98=9604 internal nods of the region Ω indicated on the Fig. 3b for N=M=100, are substituted to the partial recurrence equations (12) and after imposing boundary conditions (25), (26) we obtain 9604 algebraic equations with 9604 unknown $p_{i,j}$. The matrix of solutions is defined by the equations (16), (17), (18). Unknown pressure values are determined from the formula (19). Dimensional pressure values $p_{i,j}$ are marked on the cylindrical surface presented in Fig. 4 in two columns. Right column of picture shows the end coordinate line.

6. Conclusions

This paper describes the topology of differences method for Mathcad and Matlab Professional Program in pressure calculations performances from modified Reynolds equation occurring in hydrodynamic theory of micro-bearing design.

R = 0.026 [m], $L_{d1}=b_d/R=1$, η_o = 0.15 [Pas], ω=1.0 [1/s], p_o=0.15 [MPa],

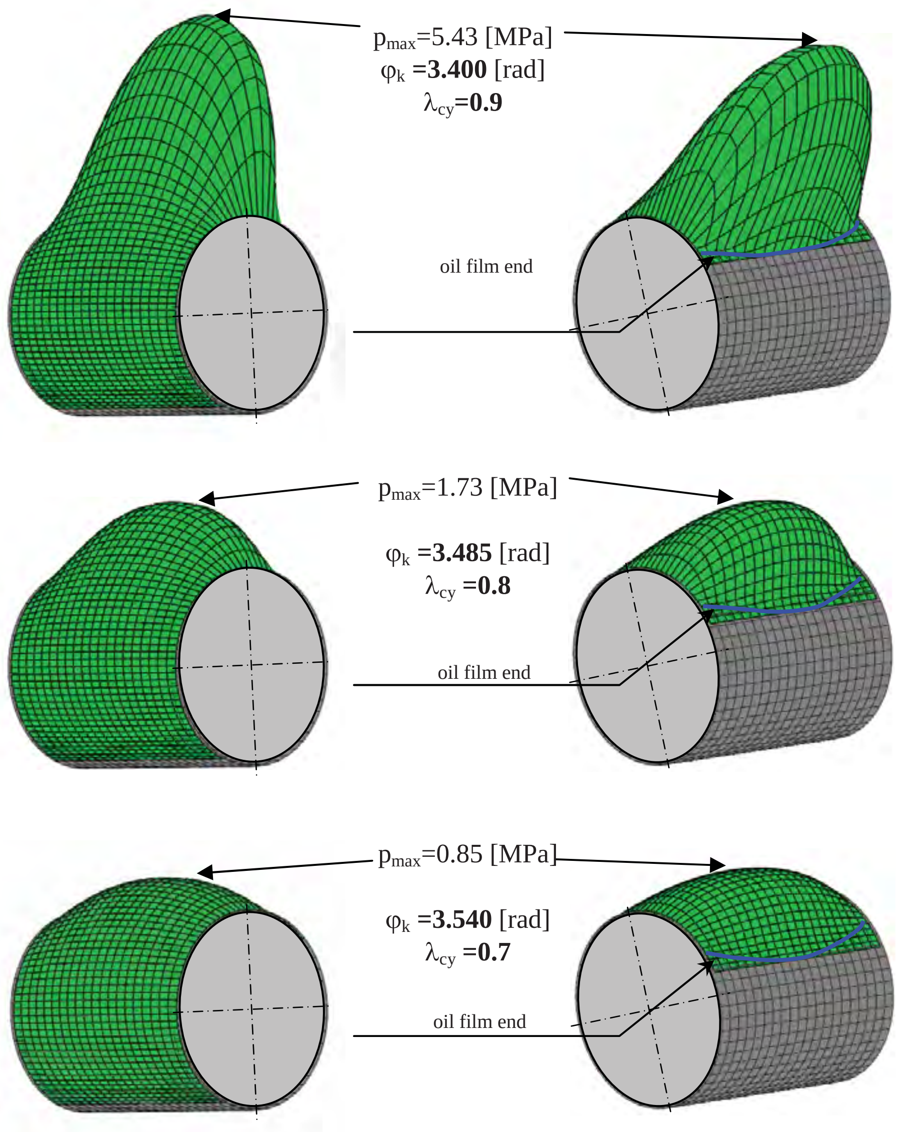

Fig. 4. Pressure distributions in the cylindrical sidle bearing gap

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