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ENGINE TORQUE FOR ACCELERATION OF ROTATIONAL MASS IN VEHICLES

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Abstract

This study presents analysis of errors of calculations for engine load during vehicle acceleration phase, resulting from some simplifications or errors in selection of mass moment of inertia. A varied impact of the assumed, on unknown basis, values of the mass moments of inertia on calculation error was shown.

According to the author, more accurate results of calculations of coefficient of rotating masses δ can be obtained by means of equations which make use of mass moments of inertia for rotating masses in vehicle I_s and I_k rather than values of reduction ratios in gearbox, for which the calculated values of δ are inflated. This study presents an overview of methods for calculation of inertia mass in vehicles which impact on inertial resistance. Different opinions in references on rotational mass (engine with clutch mechanism and the road wheels) provided the author an impulse to verify them with respect to currently manufactured vehicles.

In particular, profile of reduced mass moment of inertia of rotational masses in vehicle as a function of overall gear ratio, ranges of non-stationary engine working conditions from engine speed, characteristics of demand for engine torque needed for acceleration of rotational masses in the vehicle with acceleration of engine speed are shown in the paper.

Keywords: *rotational mass, torque of rotational mass acceleration, mass moment of inertia*

1. Introduction

In his work, the author proposed solution to the problem of dependence of engine efficiency in non-stationary working conditions on engine speed [1, 2]. The developed methodology of measurements and calculations can be used for non-stationary engine working conditions induced by the load torque selected through employing particular conditions of investigations [3]. Solving of first case of non-stationary conditions of engine work induced by engine speed allowed for initial analysis and some generalization concerning engine efficiency in second case, where, at particular intensity, only load torque per time unit dT_e/dt changes [3].

In order to solve first case, it is necessary to understand complex reasons for drop in engine efficiency in non-stationary working conditions induced by engine speed. They include adaptive (follow-up) control of the amount of injected dose. Engine speed, in terms of mean value, and its changes dn/dt, comprises leading parameters of adaptive control, whose task is to change mixture composition to stoichiometric value in possibly shortest time [2]. Moreover, in instantaneous point of engine work in general (static) performance map certain part of engine torque generated by the engine is used for acceleration of major rotational masses in vehicle: engine with clutch mechanism and road wheels [4]. Proper consideration of these data determines location of engine working point in general performance map as well as efficiency and the level of emissions of toxic compounds from the engine.

In the present study, with the example of two cars, the impact of inaccuracy of calculation of coefficient of rotational masses on accuracy of determination of engine's working points in general performance maps [5-7] was indicated.

2. Torque of rotational mass acceleration in vehicle – theory

Generalized equation for engine torque is given by:

$$T_o = \frac{P_e}{2\pi \cdot n}, \tag{1}$$

where:

P_e - engine power, W,
n - mean engine speed, s^{-1}.

After transformation of the equation of torque loss $T_{o,w}$ for non-stationary engine working conditions induced by engine speed, the following equation was obtained:

$$\Delta T_o = \frac{\Delta P_e}{2\pi \cdot n}, \tag{2}$$

where power lost for acceleration of rotational masses in vehicle is given by the following equation:

$$\Delta P_e = P_{e,\delta} - P_e \, . \tag{3}$$

If average v vehicle speed in a given moment of accelerating phase (t+dt/2) and power transmission efficiency η_p are given and driving force on F_n wheels can be calculated, engine power given by the equation of engine power:

$$P_e = F_n \cdot v \cdot \eta_m \, , \tag{4}$$

can be easily calculated:

$$\Delta P_{e,\delta} = (F_{n,\delta} \cdot v - F_n \cdot v) \cdot \eta_m \, , \tag{5}$$

where:

$F_{n,\delta}$ - driving force on wheels in consideration of coefficient of rotational masses in engine, N,
F_n - driving force on wheels without consideration of coefficient of rotational masses in engine, N.

After development of each driving force from equation (5), the following equation was obtained:

$$\Delta P_{e,\delta} = v \cdot \eta_m \cdot [F_t + F_p + F_{b,\delta} + F_w - (F_t + F_p + F_b + F_w)] \, , \tag{6}$$

Since only $F_{b,\delta} \neq F_b$ in equation (6), the other mononomial values of motion resistance are reduced, giving the following equation:

$$\Delta P_{e,\delta} = v \cdot \eta_m \cdot (F_{b,\delta} - F_b) \, , \tag{7}$$

Developed form of both components of inertia resistance forces from the equation (7):

$$F_{b,\delta} = m \cdot (1 + \delta_s \cdot i_b^2 + 4 \cdot \delta_k) \cdot \frac{dv}{dt}, \tag{8}$$

$$F_b = m \cdot \frac{dv}{dt}. \tag{9}$$

After substitution and reduction:

$$\Delta P_{e,\delta} = m \cdot v \cdot \eta_m \cdot (\delta_s \cdot i_b^2 + 4 \cdot \delta_k) \cdot \frac{dv}{dt}, \tag{10}$$

where:

dv/dt - vehicle acceleration, m/s^2,
δ_s - mass moment of inertia of engine with clutching mechanism, $kg \cdot m^2$,

δ_k - mass moment of inertia of road wheels, kg·m^2,
η_m - power transmission system efficiency,
i_b - reduction ratio in gearbox.

Generalized form of equations for mass moments of engine inertia:

$$\delta_s = \frac{I_s \cdot \eta_m \cdot i_g^2}{m \cdot r_d^2}, \tag{11}$$

for road wheels:

$$\delta_k = \frac{4 \cdot I_k}{m \cdot r_d^2}, \tag{12}$$

where:
i_g - axle ration,
r_d - dynamic radius of road wheels, m.

After substitution of (11) and (12) to (10):

$$\Delta P_{e,\delta} = m \cdot v \cdot \eta_m \cdot \left(\frac{I_s \cdot \eta_m \cdot i_g^2}{m \cdot r_d^2} \cdot i_b^2 + 4 \cdot \frac{I_k}{m \cdot r_d^2} \right) \cdot \frac{dv}{dt}. \tag{13}$$

Since the focus is on drop in engine torque from unstable working conditions induced by rotational speed n, the conversion from vehicle speed to engine speed should be made, through commonly known equation:

$$v = \frac{2\pi \cdot n \cdot r_d}{i_c}, \tag{14}$$

Hence:

$$\frac{dv}{dt} = \frac{2\pi \cdot r_d}{i_c} \cdot \frac{dn}{dt}. \tag{15}$$

After substitution of (14) and (15) to (13):

$$\Delta P_{e,\delta} = m \cdot \frac{2\pi \cdot n \cdot r_d}{i_c} \cdot \eta_m \cdot \left(\frac{I_s \cdot \eta_m \cdot i_g^2}{m \cdot r_d^2} \cdot i_b^2 + 4 \cdot \frac{I_k}{m \cdot r_d^2} \right) \cdot \frac{2\pi \cdot r_d}{i_c} \cdot \frac{dn}{dt}. \tag{16}$$

After reduction and dividing of power by ($2\pi \cdot n$) the following equation was obtained:

$$\Delta T_o = 2\pi \cdot \eta_m \cdot \left(I_s \cdot \eta_m + \frac{4 \cdot I_k}{i_c^2} \right) \cdot \frac{dn}{dt}, \quad \mathrm{N \cdot m}. \tag{17}$$

This expression resembles the equation of loss in engine torque used for acceleration on rotational masses in vehicle [5]:

$$\Delta T = I_m \cdot \frac{dn}{dt}, \quad \mathrm{N \cdot m}, \tag{18}$$

where:

$$I_m = 2\pi \cdot \eta_m \left(I_s \cdot \eta_m + \frac{4 \cdot I_k}{i_c^2} \right), \quad \mathrm{kg \cdot m^2}. \tag{19}$$

Value of I_m calculated in (19) can be treated as a reduced moment of rotational masses in vehicle. It comprises the total of two components: constant from mass moment of inertia for engine and variable, from road wheels. Values of both components as a function of total reduction ratio in power transmission system are presented in the form of characteristics in Fig. 1.

By means of I_m, having coefficient of non-stationary engine working conditions from engine speed v_n, demand for torque of rotational mass acceleration in vehicle ΔT can be calculated.

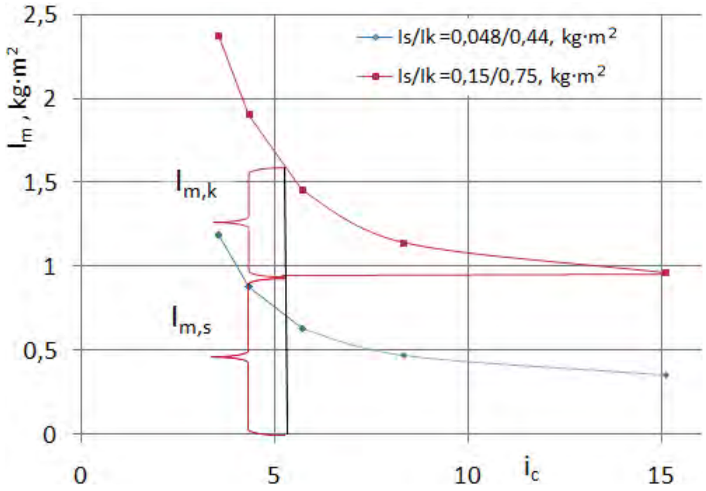

Fig. 1. Profile of reduced mass moment of inertia of rotational masses in vehicle as a function of overall gear ratio

3. Determination of v_n in 1.6 SI engine in B/K class vehicle

At initial point of functional investigations of a vehicle presented in Fig. 2, the most often used, in each ratio of gearbox, coefficient of non-stationary engine working conditions from engine speed $v_n = dn/dt$ (s^{-2}) were described. These values were registered for B/K class vehicle along the measurement distance of 400 m in variable driving cycle (Tab. 2) and for C car with its acceleration at individual gear ratios at full load (Tab. 3).

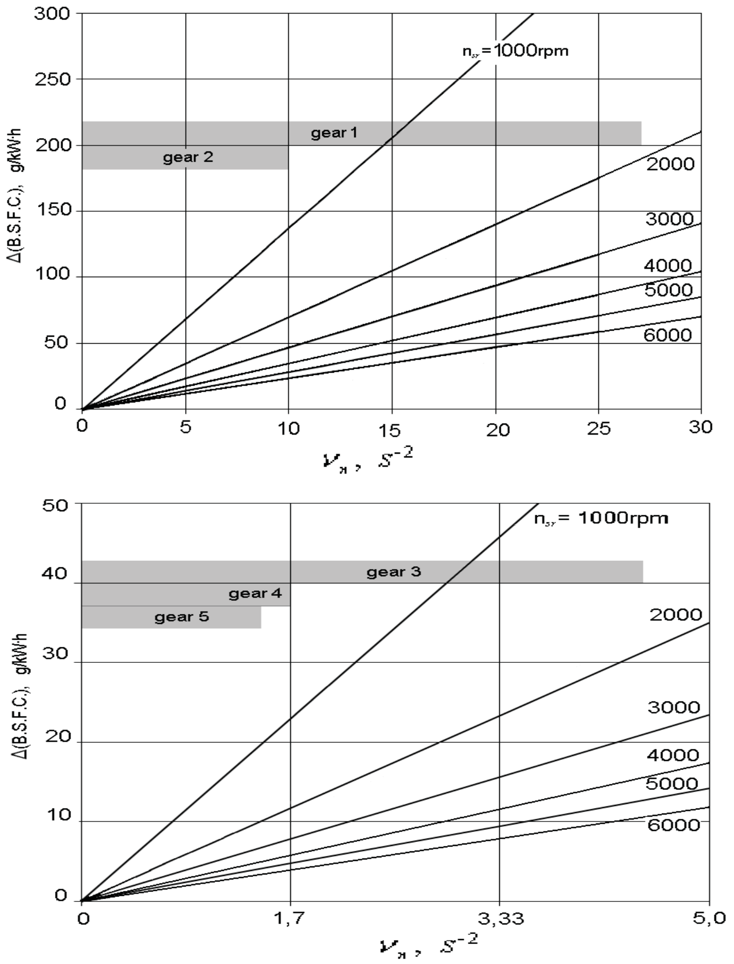

Fig. 2. Ranges of non-stationary engine working conditions from engine speed v_n in individual gears [2]

Tab. 1. Differences in coefficient of rotational masses δ calculated by means of different formulae for the vehicle with $m = 1400$ kg and axle ratio $i_g = 3.74$, $I_s = 0.14$ kg·m^2, $I_k = 0.77$ kg·m^2

No.	Source	Formula	Reduction ratio in gearbox i_b				
			$i_1 = 3.73$	$i_2 = 2.14$	$i_3 = 1.41$	$i_4 = 1.21$	$i_5 = 0.89$
1.	[6]	$\delta = (m + m_e)/m$ 1)	1.23	1.074	1.03	1.024	1.013
2.	[7]	$\delta = 1.04 + 0.05\, i_b^2$	1.735	1.27	1.14	1.11	1.080
3.	[8]	$\delta = 1 + 0.07\, i_b^2$	1.974	1.32	1.139	1.102	1.055
4.	[9]	$\delta = 1 + I_s\, i_b + I_k$	1.186	1.102	1.059	1.05	1.038
1) m_e is a reduced weight of rotational masses in vehicle to road wheels axle while m means vehicle weight [12]							

Tab. 2. Data for B/K car with SI engine 1.6 dm^3 ($T_e = 150$ N·m): $I_s = 0.14$ kg·m^2, $I_k = 0.77$ kg·m^2, $i_g = 3.74$, $\eta_m = 1.0$, $m = 1400$ kg, tire: 195/60 R15

No.	Feature	Units	Overall gear ratio $i_c = i_g \cdot i_b$				
			13.95	8.00	5.27	4.53	3.33
1.	δ 1)	–	1.186	1.102 4)	1.059 4)	1.05 4)	1.038 4)
2.	I_m 2)	kg·m^2	1.53	1.70	2.1	2.32	3.14
3.	v_n 3) $= dn/dt$	s^{-2}	7.0-25.0	1.7-10.0	0.8-5.0	0.3-2.5	0.16-1.3
4.	ΔT	N·m	11-38	3-17	1.7-10.5	0.7-5.8	0.5-4.2
1) scatter of coefficient of rotational masses in vehicle calculated by mean of four different formulas, 2) the parameter assumed in the study [5], 3) coefficient of non-stationary working conditions from engine speed (acceleration of engine speed for 2/3 of the used engine load), 4) coefficient of rotational masses in vehicle calculated on the basis of mass moments of inertia of the engine with clutch mechanism I_s and road wheels I_k on the basis of the equation of $\delta = 1 + \delta_s \cdot i_b + \delta_k$ [10]							

In order to determine influence of road slope in measurement distance, driving cycles were performed in two traffic directions while constant load in driving phases of consecutively performed driving cycles were determined on bumpers through changes in maximal inclination of acceleration pedal. Each of driving cycles at constant load at two mean engine speeds were carried out twice. Extreme values of load torque were adjusted so that to ensure, at highest gear, minimal vehicle acceleration during uphill movement and stabile engine work during acceleration in first gear (up to 80% of load).

Then, by means of four formulae popular in literature, coefficient of vehicle rotational masses was calculated for individual total reduction ratios in B/K class car with 1.6 SI engine (Tab. 1). Considerable contradictions between the values of coefficient of rotational masses δ calculated on the basis of reduction ratios in gearbox (line 2 and 3) and values calculated on the basis of mass moments of inertia (line 1 and 4) point to suitability of this method.

Table 2 presents calculations of the range of engine torque necessary to overcome resistance of mass moment of inertia of rotational masses for overall gear ratio and reduced mass moment of inertia I_m in B/K class vehicles. The ranges of demand for engine torque result from the registered scatter v_n in variable driving cycle.

As results from equation (16) and line (2) in Tab. 2, assumption of constant value of equivalent moment of rotational masses $I_m = 1.024$ kg·m^2 [5] for lower reduction ratios in gearbox leads to considerable errors of calculations, reaching as high as 150%.

4. Determination of decrease in torque as a function of v_n in "C" car

Much more measurement points for non-stationary engine working conditions induced by engine speed allow for determination, for any engine speed, of characteristics of dynamics of C car

Tab. 3. Calculations of I_m and ΔT_o during acceleration of C vehicle with weight of m = 1750 kg, 1.6 SI engine and 175/70R14 road wheels in individual gears (calculations on the basis of [11])

No.	Acceleration technique	i_b –	n_{av} rpm	I_s/I_k kg·m^2	I_m kg·m^2	v_n, s^{-2} *dn/dt*	ΔT_e N·m
1.	Using gears, full load	1	2000	0.048/0.44 1), 0.14/0.75	0.35/0.96	28.4	10/27.3
2.		2	3000	0.048/0.44, 0.14/0.75 2)	0.47/1.14	10.0	4.7/11.4
3.		3	3000	0.048/0.44, 0.14/0.75	0.63/1.45	5.1	3.2/7.4
4.		4	3000	0.048/0.44, 0.14/0.75	0.88/1.9	3.0	2.7/5.7
5.		5	3000	0.048/0.44, 0.14/0.75	1.19/2.37	1.53	1.8/3.6
6.		5	4630 1)	0.048/0.44, 0.14/0.75	1.19/2.37	0.43	0.5/1.0
7.	Full load fom v_{min} to v_{max}	3	2000	0.048/0.44, 0.14/0.75	0.63/1.45	3.3	2.1/4.8
8.			3000	0.048/0.44, 0.14/0.75	0.63/1.45	3.7	2.3/5.4
9.			4000	0.048/0.44, 0.14/0.75	0.63/1.45	3.3	2.1/5.0
10.			6750 1)	0.048/0.44, 0.14/0.75	0.63/1.45	0.95	0.6/1.4
11.	Full load from v_{min} to v_{max}	4	2000	0.048/0.44, 0.14/0.75	0.88/1.9	2.0	1.8/3.8
12.			3000	0.048/0.44, 0.14/0.75	0.88/1.9	2.8	2.5/5.3
13.			4000	0.048/0.44, 0.14/0.75	0.88/1.9	1.74	1.5/3.3
14.			5800 1)	0.048/0.44, 0.14/0.75	0.88/1.9	0.33	0.3/0.6
12.	Full load v_{pocz} = 80 kph	5	3000	0.048/0.44, 0.14/0.75	1.19/2.37	0.91	1.1/2.2
15.			4000	0.048/0.44, 0.14/0.75	1.19/2.37	0.69	0.8/1.6
16.			4660 1)	0.048/0.44, 0.14/0.75	1.19/2.37	0.38	0.45/0.9
17.	Full load $v_{initial}$ = 60 kph	5	2000	0.048/0.44, 0.14/0.75	1.19/2.37	1.2	1.4/2.9
18.			3000	0.048/0.44, 0.14/0.75	1.19/2.37	0.83	1.0/2.0
19.			4000	0.048/0.44, 0.14/0.75	1.19/2.37	0.70	0.8/1.7
20.			4400 1)	0.048/0.44, 0.14/0.75	1.19/2.37	0.54	0.64/1.3
21.	Full load $v_{initial}$ = 50 kph	5	2000	0.048/0.44, 0.14/0.75	1.19/2.37	1.125	1.35/2.7
22.			3000	0.048/0.44, 0.14/0.75	1.19/2.37	0.87	1.0/2.1
23.			4000	0.048/0.44, 0.14/0.75	1.19/2.37	0.63	0.8/1.5
24.			4416 1)	0.048/0.44, 0.14/0.75	1.19/2.37	0.48	0.6/1.1
25.	Full load $v_{initial}$ = 60 kph	4	3000	0.048/0.44, 0.14/0.75	0.88/1.9	1.6	1.4/3.0
26.			4000	0.048/0.44, 0.14/0.75	0.88/1.9	1.17	1.0/2.2
27.			4400 1)	0.048/0.44, 0.14/0.75	0.88/1.9	0.54	0.5/1.0
28.	Full load $v_{initial}$ = 40 kph	4	2000	0.048/0.44, 0.14/0.75	0.88/1.9	1.74	1.5/3.3
29.			3000	0.048/0.44, 0.14/0.75	0.88/1.9	1.71	1.5/3.3
30.			4000	0.048/0.44, 0.14/0.75	0.88/1.9	1.82	1.6/3.5
31.			5600 1)	0.048/0.44, 0.14/0.75	0.88/1.9	0.625	0.55/1.2
Average					0.90/1.91		
1) values assumed in [7], 2) values measured and calculated in [12].							

acceleration in individual gears [10]. The results of the calculations are compared in Tab. 3. In order to determine impact of mass moment of inertia of the engine and road wheels, calculations were carried out for two I_s/I_k data sets, by the author of the present study and by [7].

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$\Delta T_e = f(v_n)$ characteristics were determined on the basis of the data from Tab. 3 in logarithmic scale in Fig. 3. In double logarithmic scale, the impact of vehicle acceleration disappears in individual gears. However, considerable impact of mass moments of inertia of the engine and road wheels can be observed.

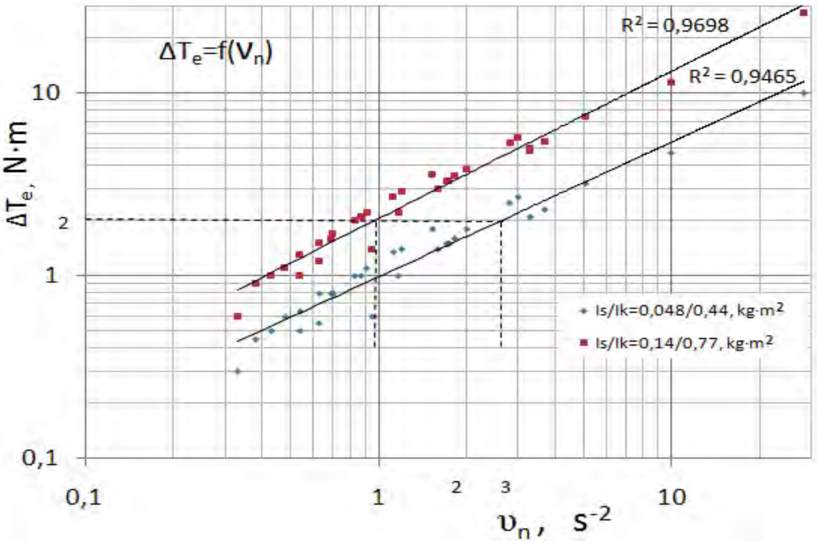

Fig. 3. Characteristics of demand for engine torque T needed for acceleration of rotational masses in the vehicle with acceleration of engine speed of v_n (s^{-2})

5. Conclusions

On the basis of the presented results of investigations, one can conclude that:

1. A varied method of calculations of the coefficient of rotational masses and mass moments of inertia in vehicles is a reason for considerable contradictions in calculations of component driving force concerning vehicle inertia.
2. Particularly critical error in calculation of engine load in working points in general performance map in non-stationary working conditions from engine speed v_n, resulting from first conclusion might be made for lowest reduction ratios in gearbox, for which coefficient $v_n = dn/dt$ is the highest.
3. According to the author, more accurate results of calculations of coefficient of rotating masses δ can be obtained by means of equations which make use of mass moments of inertia for rotating masses in vehicle I_s and I_k rather than values of reduction ratios in gearbox, for which the calculated values of δ are inflated.
4. The obtained characteristics of demand for engine torque in order to overcome the resistance of acceleration of rotational masses depend mainly on coefficient of non-stationary engine working conditions induced by engine speed v_n (and this coefficient, on the other hand, depends on overall gear ratio i_c) and the values of the assumed mass moments of inertia of the engine with clutch mechanism and road wheels (Fig. 3).

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