Journal of KONES Powertrain and Transport, Vol. 17, No. 4 2010

SOME ASPECT OF THEORY OF CUMULATED FUEL CONSUMPTION

Lech J. Sitnik

Wroclaw University of Technology
Institute of Machine Design and Operation
I. Łukaszewicza Street 7/9, 50-371 Wroclaw, Poland
tel./fax: +48713477918, tel.: +48601793142
e-mail: lech.sitnik@pwr.wroc.pl

Abstract

A theory of cumulated fuel consumption in process of vehicle operation is described. The method of creation the mathematical model and procedure of determination of model variables is shown. The main features of cumulated fuel consumption are painted attention. It is shown that the theory of cumulated fuel consumption can be use to description of the fuel consumption from beginning the exploitation of vehicles and also for the situation when the vehicle are in exploitation from any time and further fuel consumption are unknown. An example of utilisation of elaborated theory for assessment of operation of passenger car is also presented. I t has been shown that applying the theory of cumulated fuel consumption, the course of such consumption can be accurately described also in this case. This proposal can be formulated in spite of making some simplifications. This is because the unknown value of the accumulated fuel before start the time of observation operation will be determined using the average (from the average) fuel consumption. This simplification greatly facilitates the flow of the calculations and it seems does not offend errors in the calculations. Method of determining the average is discussed in the article. It appears that such treatment may be justified by the fact that the mistake is, in principle, systematic error. Very high coefficients of mathematical model to fit the measured data are very interesting. Such regularity was observed during researches in buses transport [1], intercity buses, and the fleets of various vehicles (the results of researches are not yet published).The introduction of the theory of cumulated fuel consumption as one of the important parameters to assess the operation of the vehicle or vehicles may also be useful in exploitation researches. At present times the researches in which the vehicles are already in service and the engine control maps are tuned (chip tuning), are made. A good performance of these changes is often observed on chassis dyno but that is not properly sanctioned in natural exploitation. Experimentation with such issue is underway and will be the subject of future analysis (and publications).

***Key words**: vehicle, engine, fuel consumption*

1. Introduction

Energy consumption and for the vast majority of modern vehicles, fuel consumption is a basic measure of fitness to perform operational tasks. Fuel consumption is associated with both non-toxic (carbon dioxide) and toxic components emission. Reducing emissions of these compounds is associated primarily with the limitations of fuel consumption. It is interesting that in over a century's development of the automotive on fuel consumption was paid, contrary to expectation, relatively little attention.

Fuel consumption is estimated, in principle, only by dyno research - both motor and chassis. These assessments are fairly accurate but insufficiently reflect the real conditions [1]. During exploitation in real conditions the short-term measurements carried out on the road during the specified test procedure. These measurements are burdened with a big mistake and do not reflect real operating conditions. Another way is to analyze the fuel consumption which is determined over a longer period of time based on the records of fuel consumption as well as mileage of vehicles. This procedure, although not without a number of disadvantages is used as conclusive in assessing the suitability of the vehicle, and often also in comparison of other factors such as: in transportation business the drivers work. Example results of this kind of analysis are shown in

Fig. 1. Measuring data relate to the real-condition exploitation of car with a spark-ignition engine and cylinder capacity of 3500 ccm.

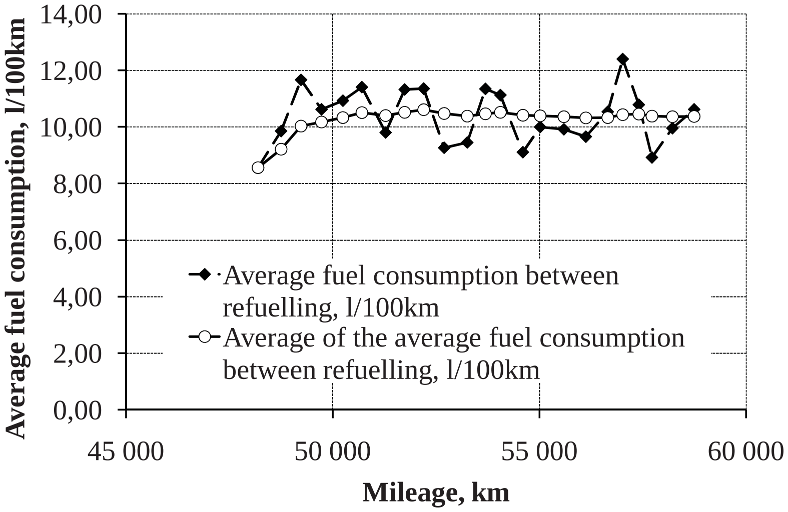

Fig. 1. An example of the assessment of vehicle fuel consumption in real condition exploitation

Under current practice, the average fuel consumption is designated between refuelling and related to the vehicle mileage. Sometimes, average fuel consumption is determined on the course and fuel consumption at a given time. For example, in public transport companies settlement period is one month and the average fuel consumption is determined by dividing the volume of fuel consumed by motor vehicles during the month by the travelled distance during this time. This method makes the designated average characterized by smaller fluctuations and is thus easier to analyze.

Figure 1 presents a different method (not presented in the literature) - the average value is calculated from the average which is fixed on the basis of fuel consumption and mileage between refuelling. Simply, this is summed average from the individual refuelling divided by the number of refuelling. This is cumulative calculation . For example, the average of three refuelling is the average of the first plus second plus third refuelling, divided by 3. This average is shown as a function of vehicle mileage. That average is smooth and its value as a function of vehicle mileage is stabilized. This curve is easier to assess, yet recognizes the individual refuelling.

Fuel consumption is widely known as an extremely important indicator, but its deeper theoretical description has yet not been done fully. This thesis addresses this subject. It also presents a theory of cumulative fuel consumption [1 Sitnik]. In previous work the author presents the theory and its applications to assess accumulated fuel consumption in vehicles. Analyses of cumulated fuel efficiency of city transport buses, intercity transport fleets as well as with various vehicles are made. All of this analysis confirmed the usefulness of the theory and showed good compliance theoretically derived models with reality. Also the possibility of applying the theory to predict fuel consumption as well as to clearly distinguish operational properties in cases where other methods fail was demonstrated. The common feature of all previous studies was that they related to vehicles since their introduction into service. This thesis deals with the case when the vehicle is already in operation at a specified time, but fuel consumption has not yet been known.

2. Theory [1]

Fuel is inducted into the combustion chamber of the engine in a “quantum”. Quanta are doses of fuel inducted in the duration of the intake stroke (in older engines the fuel/air mixture is prepared outside the combustion chamber), or at the end of the compression stroke (when it is injected into the combustion chamber). Quanta of fuel, also called as dose of fuel, have a random size. Randomness is in the sense intended, since results from the need to change the energy transmitted by the engine to cover the random demand in the system of exploitation such as the propulsion system of the car or is not intended when the dose values are determined by the processes implemented in the engine fuel system. Quanta of fuel can be characterized by the volume or weight. Traditionally and mostly the volume measurement is used.

The sum of the volume (or weight) quanta of fuel inducted to the engine for obtain energy is defined as a fuel consumption. Aggregation of the quantum of fuel leads to determine the cumulative fuel consumption. The volume of cumulated fuel consumption can be determined at any time of the engine exploitation. Hence, the accumulated fuel consumption can be related to generally understood life of the engine. Life of engine can be given in motohours of the engine or vehicle mileage. After each period of engine work the cumulative fuel consumption can be determined. It is the sum of fuel spent since the time of the engine is introduced to operate until the moment corresponding to the time for analysis.

Accumulated fuel consumption, related to the time of the engine operation is the intensity of the cumulative fuel consumption after given time of engine operation. It should be distinguished the cumulative from the instantaneous fuel consumption. Instantaneous fuel consumption is measured in a specified, relatively short time, at any time of the overall operation time of the engine. The easiest way to explain it when the time of the engine running is implicitly characterized as mileage, in which the engine is a source of propulsion. Popular quantity featuring the instantaneous fuel consumption is intensity of fuel consumption given as litters per 100 kilometres.

For simplicity, the term of fuel consumption instead of accumulated fuel consumption and term of fuel consumption intensity instead the intensity of cumulative fuel consumption will be used. Accordingly, fuel consumption resulting from the engine operating time *t* can be designated as

$$Q_{sp}(t) = \sum_{i=1}^{n(t)} \nu_i = n(t) \cdot \bar{\nu}(t)\,, \tag{1}$$

where:

ν_i - i-th quantum of spent fuel (e.g. fuel dose in one rotation of the engine),

$\bar{\nu}(t)$ - the average size of quantum of fuel consumed for the time *t*,

$n(t)$ - the number of quanta of fuel consumed for the time *t*,

$Q_{sp}(t)$ - the cumulative fuel consumption for the time *t*.

Thus, in order to know the cumulative fuel consumption in time *t*, the average size of the quantum of fuel (fuel doses per motor revolution) and the number of quanta of spent fuel by that time must be known. Therefore, if a quantum of fuel is the dose inducted per one motor revolution (this is a random variable size), and the motor in the time *t* made the number of revolutions, which are fuel (e.g. in four-stroke engine, fuel is inducted every second revolution of the engine) equal to $n(t)$ then, the cumulative fuel consumption is calculated by multiplying the average dose and the number of revolutions of the engine.

In [1] a mathematical argument leading to the designation of accumulated fuel consumption as a function of time is presented:

$$Q_{sp}(t) = c \cdot t^{a} \cdot t = ct^{a+1}. \tag{2}$$

It is a relatively simple correlation describing the accumulated fuel consumption as a function of time.

Intensity of cumulative fuel consumption, can be specified by:

$$\frac{dQ_{sp}}{dt} = c(a+1)t^{a} \tag{3}$$

The intensity of accumulated fuel consumption takes infinite values if $a < 0$ and $t \to 0$, and so almost immediately after the start of the engine. If the engine drives the vehicle, this phenomenon is observed on a onboard computer that if shows the instantaneous consumption, e.g. in litters per 100 km, in the first period (prior to the course of these 100 km) shows the intensity of the cumulative fuel consumption, which is often extremely high (but rapidly decreases with increasing length of distance travelled).

3. Method of determining the cumulative fuel consumption on the basis of experience-based data

The mathematical model of a cumulative consumption, is known if *c* and *a* factors are known. Their values are determined, for example by the mathematical study of the results of experiments. Equation (2) can be converted to a convenient form of first-degree polynomial, because after taking logs of both sides we obtained:

$$\ln Q_{sp}(t) = \ln (c\, t^{a+1}) = \ln c + (a+1) \ln t \tag{4}$$

After substituting

$$\ln Q_{sp}(t) = y; \quad \ln c = b_o; \qquad (a+1) = b_1; \qquad \ln t = x; \ , \tag{5}$$

(2) converts to

$$y = b_o + b_1 x\,. \tag{6}$$

This is the equation of a straight line and to set its rates b_o and b_1 is necessary and sufficient to know the location of two points through which line passes them. In studies of cumulative fuel consumption, this means that for designate of coefficients of a mathematical model, is needed to know the cumulative fuel consumption after at least two periods of the engine exploitation. In [1] it is also demonstrated how to determine the parameters *a* and *c*.

4. The results of calculations

This thesis presents the calculation of the cumulative fuel consumption in the case where its values are not known since the introduction of a vehicle exploitation. The mileage of vehicle is known. To illustrate the phenomenon data collection of performance and results of relevant calculations in Tab. 1.

In column 1 of table the number of the next refuelling of vehicles is shown; in column 2 mileage; in column 3 the volume of fuel filled up; in column 4 the cumulative fuel consumption; in column 5 the course after taking logs; in column 6 value of cumulative fuel consumption after taking logs; in column 7 an logarithmic effective-value of fuel consumption calculated on a set of mathematical model; in column 8 the percentage of deviation of the values in columns 7 and 8; in column 9 the relevant value of the deviation in litters (from values in column 4).

Table 2 presents the results of regression analysis to determine coefficients of model (2) - on the basis of columns 5 and 6 of Tab. 1.

From the above theoretical considerations it follows that:

$$a = exp(b1) - 1 = 1.71546448, \tag{7}$$

$$c = exp(b0) = 0.10488471. \tag{8}$$

It can therefore provide such a chart for e.g. the course of the cumulative fuel consumption.

Tab. 1. Data of exploitation and results of the calculation of accumulated fuel consumption

No of refuelling	Mileage, km	Amount of fuel filled up, litters	Cumulated fuel consumption, litters	X=ln(km)	Y=ln cumulated litters)	Y_{mod}	Deviation 100* (Y-Y_{mod})/Y,%	Deviation, litters
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
0	47 198.41	4 897.06		10.762115	8.496390	8.496062	0.003865	1.60765
1	47 658.58	48.15	4 945.21	10.771818	8.506174	8.505754	0.004938	2.07682
2	48 192.77	45.74	4 990.95	10.782964	8.515381	8.516889	-0.017707	-7.53120
3	48 754.31	55.36	5 046.31	10.794549	8.526412	8.528462	-0.024036	-10.35261
4	49 233.79	55.95	5 102.26	10.804335	8.537438	8.538238	-0.009366	-4.08132
5	49 730.97	52.83	5 155.09	10.814383	8.547739	8.548275	-0.006269	-2.76328
6	50 242.63	55.93	5 211.02	10.824619	8.558531	8.558501	0.000347	0.15497
7	50 706.03	52.86	5 263.88	10.833800	8.568623	8.567672	0.011101	5.00481
8	51 277.22	55.97	5 319.85	10.845002	8.579200	8.578862	0.003936	1.79625
9	51 737.40	52.10	5 371.95	10.853936	8.588946	8.587787	0.01349	6.22049
10	52 200.79	52.64	5 424.59	10.862853	8.598697	8.596695	0.023288	10.85179
11	52 694.75	45.77	5 470.36	10.872271	8.607099	8.606103	0.011573	5.44633
12	53 253.07	52.77	5 523.13	10.882811	8.616700	8.616632	0.000785	0.37366
13	53 692.33	49.82	5 572.95	10.891025	8.625679	8.624838	0.009754	4.68675
14	54 052.75	40.12	5 613.07	10.897716	8.632853	8.631521	0.015422	7.46804
15	54 601.42	49.97	5 663.04	10.907815	8.641716	8.64161	0.00122	0.59701
16	55 021.36	42.00	5 705.04	10.915477	8.649105	8.649264	-0.001841	-0.90847
17	55 590.95	56.49	5 761.53	10.925776	8.658958	8.659552	-0.006864	-3.42537
18	56 123.53	51.42	5 812.95	10.935310	8.667843	8.669077	-0.014237	-7.17791
19	56 652.89	55.80	5 868.75	10.944698	8.677397	8.678455	-0.012201	-6.21655
20	57 013.31	44.70	5 913.45	10.951040	8.684984	8.68479	0.002233	1.14681
21	57 405.90	42.33	5 955.78	10.957902	8.692117	8.691646	0.005423	2.80682
22	57 724.48	28.44	5 984.22	10.963437	8.696881	8.697174	-0.003373	-1.75566
23	58 216.84	49.04	6 033.26	10.971930	8.705042	8.705659	-0.007079	-3.71913
24	58 746.20	56.20	6 089.46	10.980982	8.714314	8.714701	-0.004438	-2.35569

Tab. 2. Results of regression analysis of factors determining the model (2)

<table>
<tr><td colspan="9">Regression statistics</td></tr>
<tr><td>Multiple of R</td><td colspan="8">0.999901</td></tr>
<tr><td>R squared</td><td colspan="8">0.999802</td></tr>
<tr><td>Fitted R squared</td><td colspan="8">0.999793</td></tr>
<tr><td>Standard error</td><td colspan="8">0.000966</td></tr>
<tr><td>No of observations</td><td colspan="8">25</td></tr>
<tr><td colspan="9">Analysis of variance</td></tr>
<tr><td></td><td><i>df</i></td><td><i>SS</i></td><td><i>MS</i></td><td><i>F</i></td><td colspan="4"><i>Significance F</i></td></tr>
<tr><td>Regression</td><td>1</td><td>0.10802</td><td>0.10802</td><td>115865.9584</td><td>4.36E-44</td><td></td><td></td><td></td></tr>
<tr><td>Tail</td><td>23</td><td>2.14E-05</td><td>9.32E-07</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Total</td><td>24</td><td>0.108041</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td><i>Factor</i></td><td><i>Standard error</i></td><td><i>t Stat</i></td><td><i>p value</i></td><td><i>Bottom 95%</i></td><td><i>Top 95%</i></td><td><i>Bottom 95.0%</i></td><td><i>Top 95.0%</i></td></tr>
<tr><td>Intersection (<i>b0</i>)</td><td>2.254894</td><td>0.031925</td><td>70.63173</td><td>2.12411E-28</td><td>2.320935</td><td>2.188852</td><td>-2.320935</td><td>2.1888521</td></tr>
<tr><td>Variable X 1 (<i>b1</i>)</td><td>0.998963</td><td>0.002935</td><td>340.3909</td><td>4.36298E-44</td><td>0.992892</td><td>1.005034</td><td>0.992892</td><td>1.0050348</td></tr>
</table>

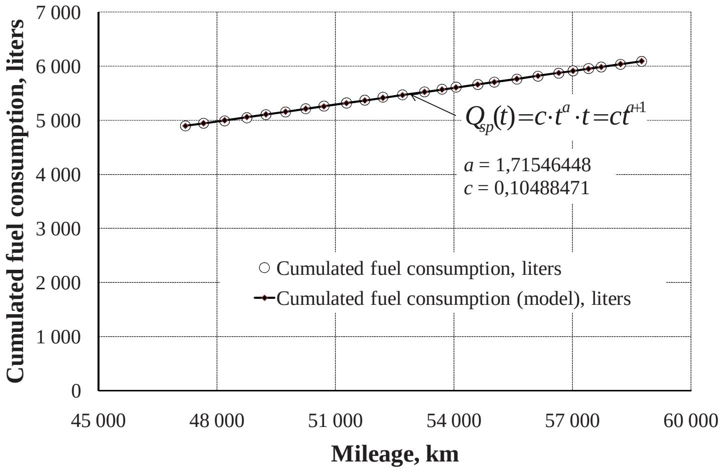

Fig. 2. Process of accumulated fuel on the basis of the designated operational data

5. Conclusion

For vehicles which mileage is known and the fuel consumption since its entry into service is not known. In this first work devoted to this problem applying the theory can accurately describe the process of such fuel consumption is shown. This conclusion can be formulated in spite of making some simplifications. This is because the unknown value of the accumulated fuel before start the time of observation operation will be determined using the average (from the average) fuel consumption. This simplification greatly helps the calculations and it seems does not make glaring errors in the calculations. Method of determining the average is discussed in the introduction to this article. It appears that such treatment may be justified by the fact that the mistake is, in principle, systematic error.

Interesting is very high coefficient of mathematical model to fit the measured data. Such regularity was observed during researches in buses transport [1], intercity buses, and the fleets of various vehicles (the results of researches are not yet published).

The introduction of the theory of accumulated fuel consumption as one of the important parameters to assess the operation of the vehicle or vehicles may also be useful in exploitation researches. At present times the researches in which the vehicles are already in service and the engine control maps are tuned (chip tuning), are made. A good performance of these changes is often observed on chassis dyno but that is not properly sanctioned in natural exploitation. Experimentation with such issue is underway and will be the subject of future analysis (and publications).

References

[1] Sitnik, L. J., *Cumulated fuel consumption*, The Archives of Automotive Engineering, No. 3, Vol. 7, pp. 225-254, 2004.