THE INNOVATIONS IN DESCRIPTION OF PROPRIETY OF STRUCTURES THE HYPER-ELASTIC MATERIALS

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Abstract

The hyper-elastic materials e.g.: t he elastomers, frothed structures PUR about open and closed pores, the materials using the matrix of natural rubber materials w hether as well as synthetic the gels be practical used in different engineer applic ations from over 30 years. The y really several su mmers and intensive w orks on field of material engineering made possi ble extension the spectres of uses as well as the utilization of elastomers compositions.

In this paper, will be presented built methodology of elasticity propriet y materials with large def ormation. In article be become presented method of identification and description of hyper-elastic materials from special return on numeric models the attention. Quantitative analysis be becomes exec uted in support about execute d experimental investigations. The numeric permissive on comparison of received conception of description of hyper-elastic material worked out in support about theoretical foundations with results of experimental investigations analyses' be become executed.

Keywords: *plastics, hyper-elastic materials, numerical simulation, identification of propriety elastomers materials*

1. Introduction

Engineers more and more often reach after hyper-elastic material. The question from which it comes to meet on stage of project of article then the selection of material and proper identification of propriety. Therefore it on preliminary necessary stage is already the meeting and proper description the work of materials - in case of elastomers materials of phenomenon setting during work of element they are much folded.

Every kind of material requires parties of different mathematical description - the theoretical model, from regard the considerable influence of temperature and speed of loading. In theory of the hyper-elastic materials be practical two ways of description: multinomial models special cases from which be well-known: Mooney-Rivlin's and Yeo's, and models in which non-linear modules step out already in first component - the most well-known models are here Ogden ones [1].

The material models of the hyper-elastics materials use function of thickness of energy deformation. The springy energy of body deformed to possibly record as catfish of deformations in the figure passed for the first time by Helmholtz, and M. T. Huber in Poland (1):

$$\phi = \frac{1}{2}K(\varepsilon_x + \varepsilon_y + \varepsilon_z)^2 + \frac{1}{3}G\left[(\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 + (\varepsilon_x - \varepsilon_y)^2 + \frac{2}{3}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{xz}^2)\right] + \\ + \frac{1}{3}G\left[(\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 + (\varepsilon_x - \varepsilon_y)^2 + \frac{2}{3}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{xz}^2)\right], \tag{1}$$

$$\phi = \frac{1}{2}K(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)^2 + \frac{1}{3}G\left[(\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2 + (\varepsilon_1 - \varepsilon_3)^2\right]. \tag{2}$$

According from practical in Strength of Materials approximation: the square of sum of deformations is even the increase of volume - first words of examples (1) and (2) they represent so the volumetric deformation, remaining figure deformation.

Comparing to notice the accessible models of hyper-elastics materials we can possible considerable variety of the individual authors' approach. The practical mathematical description of materials does not contain well-known and the practical kinds of materials there now the state the profile of definite group of structures, formed on the ground the experimental investigations the given author [2, 3].

Comparing it is possible in every model to show phrase describer deformations: figure and volumetric. It linear dependences were described was volumetric deformations: all models, Mooney's and Rivlin's, non-linear description steps out in models: Blatz's and Ko's, Ogden's, Yeoh's and multinomial. The descriptions of figure deformations considerably more be diverse.

Comparing to deal out the way of building it's possible to dependence two groups: the group of models the using the invariant the function of thickness of energy the deformation and group using the eigenvalue of tensor of deformation figure.

Models from first group was it been possible was to notice, from only Mooney's models, Mooney's-Rivlin's and multinomial they use from all three invariants of tensor of deformation. Characteristic it is for all models that the function of thickness of energy deformation accepts for all models the same figure near suitable solid values. In case when: n=1 and c_{01}=0 model multinomial is equivalent two- parameters model Mooney's - Ryvlin's. If n = 2 is equivalent penta- parameters, and for N = 3 nona-parameters model Mooney's - Ryvlin's. Ogden's model to possibly it balances new model Hooke's, if n = 1 and . If n = 2 and model Ogden's it stands up oneself suitable from two-parameters model Mooney's - Ryvlin's. In case of model Arrud's and Boyce, as well as if the model Gent's if λ_L (I_m) it will aim to infinity we will receive the new model Hooke's. In case of Yeoh's model the equivalence from new Hooke's model it receives oneself across substitution the n = 1.

The characteristic of propriety of hyper-elastics materials requires the comparisons of curves of expansion, the grip and change of volume with experimental investigations and the calculations of FEM. It the example of identification of propriety soft foam coming from with work [3] it was introduced was on Fig. 1 and Fig. 2 (the point - the experimental investigations, squares - the calculation from Ogden's model).

2. The model of internal dumping in elastomers materials

To description of elastic proprieties it's possible PUR elastomers materials to use the worked out theories well in which the most important they are the multinomial models and them special coincidence: Mooney's – Ryvlin's, Yeoh's and Ogden's. A deficiency of description of elasticity propriety in questions dissipation the energy, dumping of vibration is a problems.

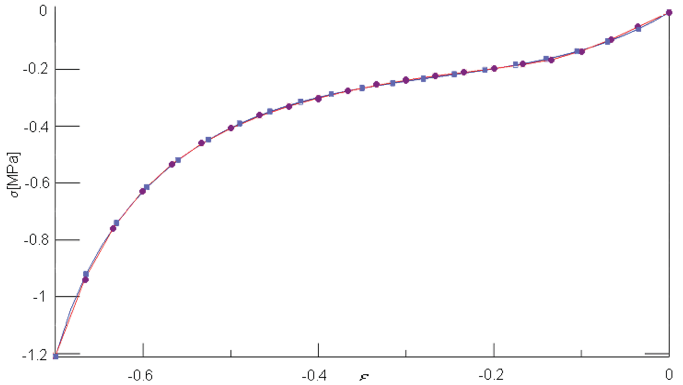

Fig. 1. The compatibility of profile of tension - the deformation of curve with test with curve for model Ogdena n=3

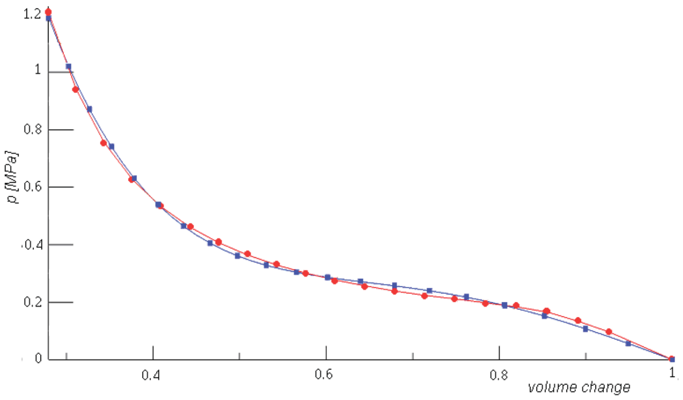

Fig. 2. The compatibility of profile the pressure - the change of volume of curve with test with curve for model Ogdena n=3

In paper a description of model of internal damping was proposed with the proof the influence of component part of polynomial: figure and volumetric. It the opinions of external factors, in work were executed was having on elasticity propriety - the level of dumping.

A polynomial model reduced of the sixth order was used for describing the elastic features peaceably from experience with work [4]. Above mentioned model is formulated with elastic functional as followed (3):

$$W = \sum_{i=1}^{6} C_{i0}(\bar{I}_1 - 3)^i + \sum_{i=1}^{6} \frac{1}{D_i}(J^{el} - 1)^{2i}, \tag{3}$$

the first element of this formula describing a non-dilatational deformations – first invariant of deviator part of deformation is:

$$\bar{I}_1 = \bar{\lambda}_1^2 + \bar{\lambda}_2^2 + \bar{\lambda}_3^2. \tag{4}$$

A elasticity deviator proper parts of state of deformation to description were used, peaceably with dependence:

$$\bar{\lambda}_i = \frac{\lambda_i}{\sqrt[3]{J^{el}}}.$$

Second module of dependence (5) it describes the volumetric deformations - third invariant of state of deformation (being with measure of change of volume) the figure has:

$$J^{el} = \lambda_1 \lambda_2 \lambda_3. \tag{5}$$

Dissipation function defines as (6):

$$H = \sum_{i=1}^{N} C_i(\dot{I}_1 - 3). \tag{6}$$

Was accepted main direction and introduced in figure with strength of suppression (7) to characterize vibration of one degree discretion:

$$F_t = \sum_{i=1}^{N} h_i * \dot{\lambda}_i^i. \tag{7}$$

The viscosity proprieties - the damping level dependent of many parameters: the composition of, structure, of building and composition segments, additions of modifiers, and the technology of production. The quantitative estimate could be execute only by use the results of experimental investigations. Arrangement was has given an examination described with equation (8):

$$P(t) = m\ddot{x} + \sum_{i=1}^{N} h_i * \dot{\lambda}_i^i + \sum_{i=1}^{N} k_i * \dot{\lambda}_i^i \, . \tag{8}$$

A coefficients k_i was accepted with results of investigations [4, 5]. The different possibilities of selection damping coefficients h_i were considered, in peculiarity proportional principle described dependence (9):

$$h_i = w_{pr} * k_i \, , \tag{9}$$

as well as selection separately every coefficient.

The coefficients k_i and w_i to equation $h_i = w_i k_i$ were accepted:

- the coefficients k_i on the ground experimental results:
  $k_1 = 0$,
  $k_3 = 0$,
  $k_5 = 0$,
- the coefficients w_i with comparison of hysteresis loop:
  $w_1 = 0.002$,
  $w_3 = 0.002$,
  $w_5 = 0.002$.

Arrangement was has given an examination described with equation (10):

$$m\ddot{x} + \sum_{i=1}^{N} h_i * \dot{\lambda}_i^i + \sum_{i=1}^{N} k_i * \dot{\lambda}_i^i = P(t) \, . \tag{10}$$

It the hysteresis loops in aim of comparison were executed was experimentally and computationally (by burden and relief one may with strength) and dynamic (in arrangement serf harmonic burdens). It the row of variants was considered: the materials differing with arrangement samples (the proportion of soft segments and stiff) in different temperatures (this essential meaning with regard on propriety of material has). It on drawing 3 and 4 the hysteresis appointed loops were introduced was near different values executed for material Epunit2 deflection experimentally L_{max}, near solid speed V_1 (near different values speeds V and near solid value deflection L_{max}). It the hysteresis static loop for comparison on drawing 5 was introduced was in temperature 25°C material EPUNIT2 executed near use numeric simulation.

According with theory of hyper-elastic materials the largest *n* value was accepted n=6. The results of executed tests and numerical calculation proved that the only odd components of explication had meaning the calculations one - accepted so the reduction, the components about numbers in modules of equation took into account i = 1, 3, 5.

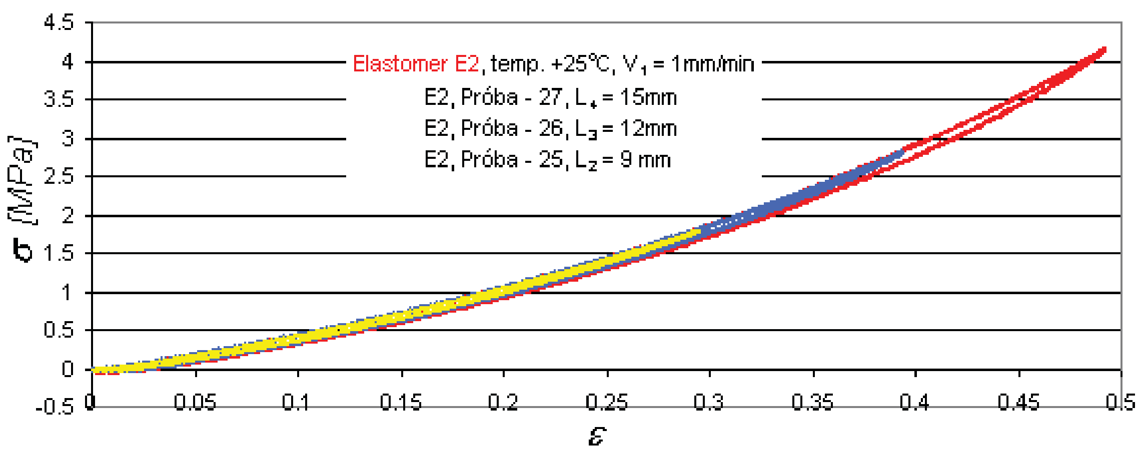

Fig. 3. Graph σ = f(ε) the uniaxial grip of sample E2 near different values deflection and near solid value deflector V_1

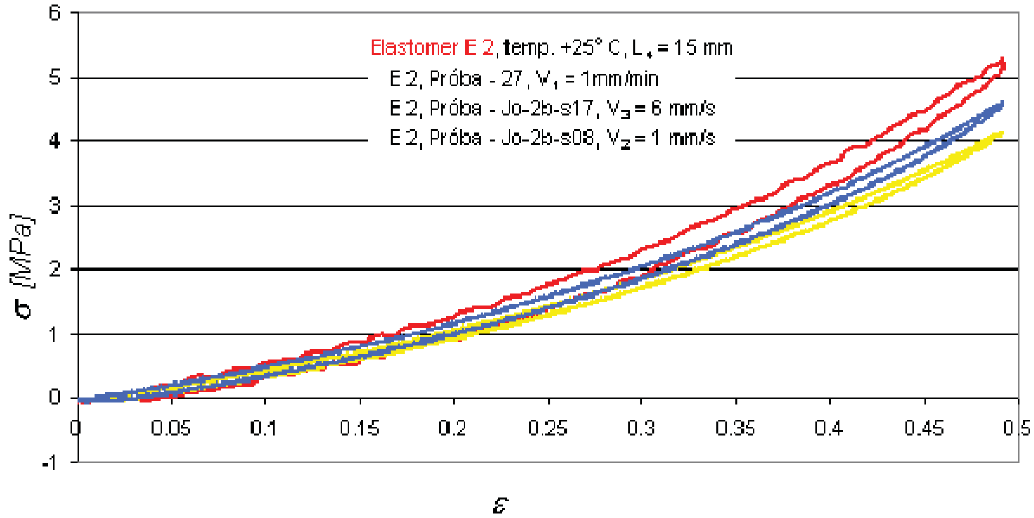

Fig. 4. Graph σ = f(ε) the uniaxial grip of sample E2 near different values speeds V and near solid value deflection L_{max}

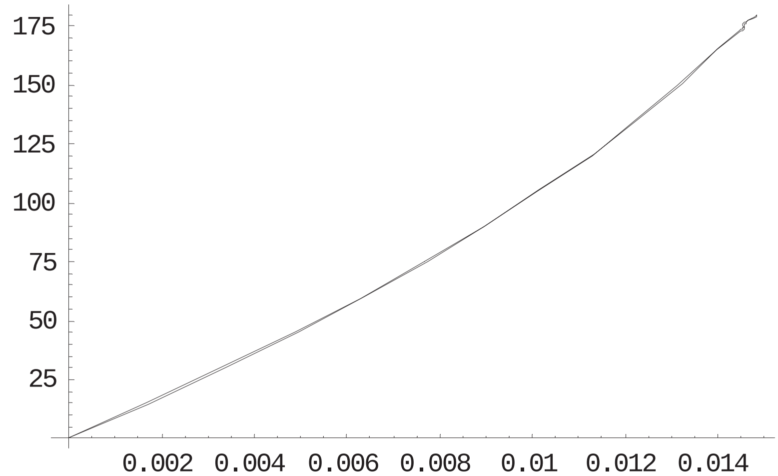

Fig. 5. Static hysteresis loop Epunit 2 in temperature 25°C

It the hysteresis loops in aim of comparison was executed: experimentally and used numerical calculation, by loading and unloading with slowly growing strength and dynamic in arrangement serf harmonic load.

It was any tests variants considered:

- the samples materials differing with arrangement: the proportion of soft and straight piece,
- with regard on propriety of material – in different temperatures.

It the hysteresis static loop on drawing 5 was introduced was in temperature 25 for example °C material EPUNIT 2.

The simulations were executed the free numeric oscillation with use of packet helping the calculation the mathematical MATHEMATICA. Estimations system with the elastomeric element, with free oscillations and forced taking into account linear and non-linear damping were performed. Non-linear oscillation the occurrence in results was observed. Results are convergent with theoretical base with this field [2, 6, 7].

It the comparison of course of oscillation on drawing 6 was introduced from linear dumping and non-linear dumping on drawing 7. We can see that the plot of oscillation from non-linear dumping, it is the largest at the beginning and grow less to value near dumping linear then.

The simulation of dumping in arrangement from element was executed Epunit2 considering the extortion harmonic. It was marked was the course of answer of arrangement as well as draw the hysteresis dynamic loop (Fig. 8)

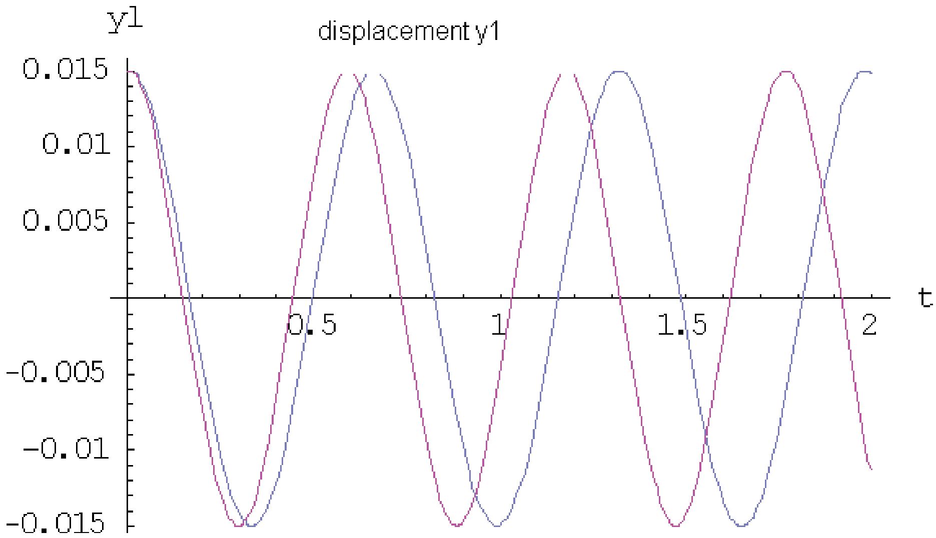

Fig 6. Non-linear dumping with linear suppression

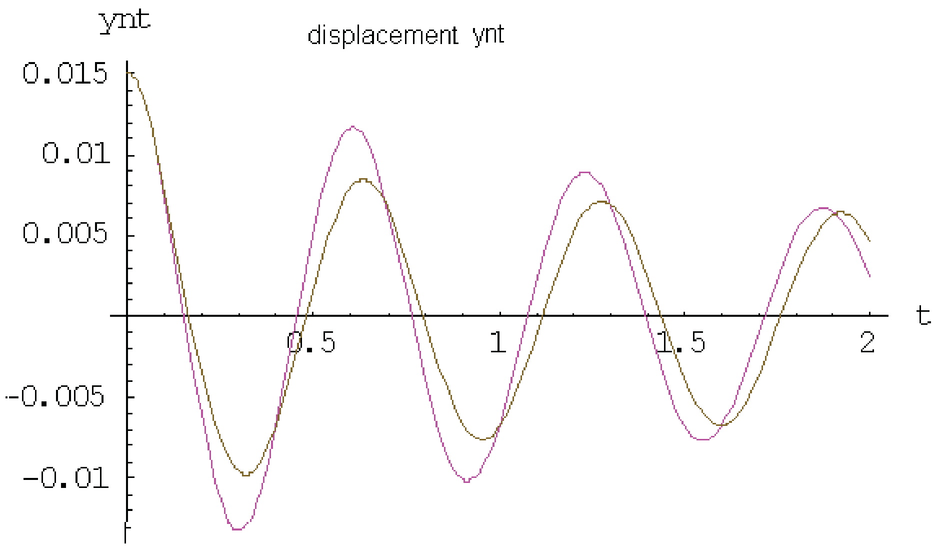

Fig. 7. Non-linear dumping with non-linear suppression

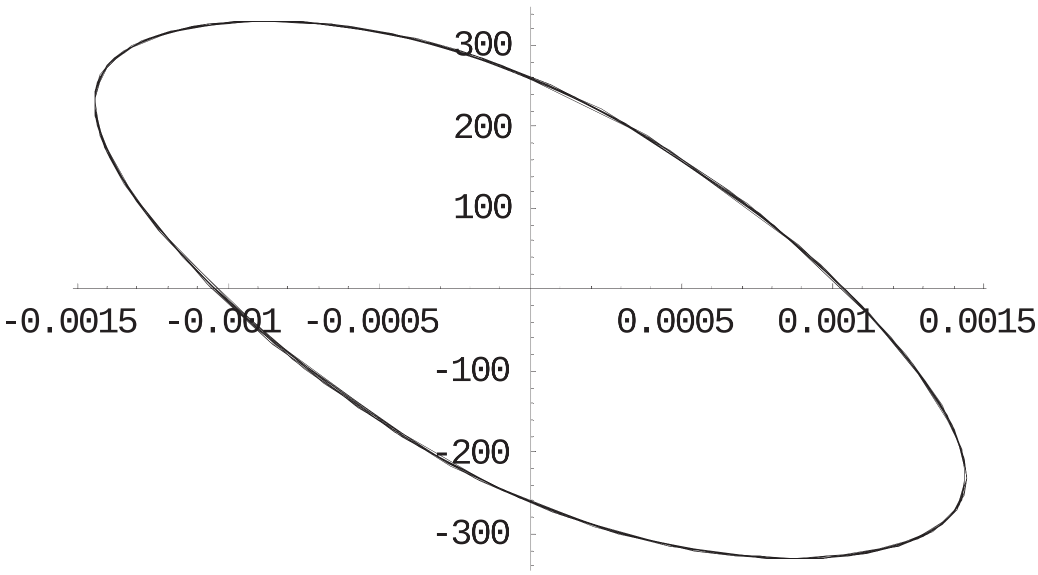

Fig. 8. Hysteresis dynamic loop

Visible it is that the typical features of non-linear dumping in arrangement from elastomers element step out: the shift of resonance frequency in side of higher frequencies and the violent drop of amplitude after passage of resonance. Visible it is also that non-linear suppression extinguishes factorials resonance in arrangement - low-end 0.002 caused that resonance does not it step out practically.

Qualitatively the character of vibration from non-linear dumping the type the x^3 was executed by method Kryłow-Bogelubow in [8, 9], solution this be introduced in monograph [6]. The received in experimental tests results with used elastomers confirm these dependences.

4. Conclusions

The present article is the composition of results led by author of research works having in view the identification of propriety of hyper-elastic materials. Present study exemplifies way identification of propriety of viscoelastic elastomers materials.

Although observed on world of tendency in development of elastomers materials, the continually lack of sufficient information is in building of elements of machines the basic problem with this structures with range of propriety the mechanical - strength. Therefore the basis to correct description of characteristic feature of hyper-elastic materials is realization of cycle folded experimental investigations of this group of materials.

The correlation of experimental results in composition from accessible models of materials the permitted on preliminary opinion of behaviour hyper-elastic materials the construction the subjected numeric analysis with FEM. To structural description could be complete necessary identifying and description behaviour were been precise these group materials in different variables in time the working conditions, in peculiarity in function: speed and temperature. The author proposes the construction the method of direct identification of kind material with used the direct method - date table or near used of system the data array on neural network.

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