APPLICATION OF MULTIPLE MOVING APPROXIMATION WITH POLYNOMIALS IN CURVE SMOOTHING

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Abstract

The paper has characterized the method of multiple moving least-square approximation with polynomials, known as the Savitzky-Golay filter. This method enables smoothing the measurement series, decomposition and separation of disturbances, generation of derivatives as well as approximate integration of measurement series. The smoothing properties of the method as well as the possibilities of separation and decomposition of disturbances are shown on the examples of treatment of the selected indicator graph. The attention was paid to the need for additional application of special filters in case of abnormal impulse deviations. The examples show that in case of the analyzed curve, the results of smoothing by means of several wave filters from Wavelet Explorer package, are worse than those received with the methods of moving approximation. On the derivatives of smoothened curve by means of wavelet filters, there are significant oscillations, what is typical of the whole-interval approximation with spline functions. One should highlight the high simplicity of algorithms of multiple moving approximation and, associated with this, high speed of operation, what particularly predisposes this method of data processing in an on-line mode. The authors presented their own proposals of programs for measurement data processing with the method of multiple moving approximation with polynomials, with the possibility of selection of an approximation polynomial to the fifth degree. The programs were developed in the Excel and Delphi environment.

***Keywords**: multiple moving least-square approximation, decomposition and separation of disturbances*

1. Introduction

Application of polynomial least-squares approximation on a moving interval for smoothing the time series [3] was a natural extension of the method of moving average, used earlier. Its establishment is connected with popularization of the application of computers for data processing of the measurements, the effect of which was the intense development of mathematic methods based on the application of computers in many domains of science and technology, which took place in the late 1950s and ’60s.

2. Least-square approximation with orthogonal polynomials

Wherever the form of a mathematical model is unknown, the measurement data is most often approximated with polynomials. The approximating polynomials of j degree can be written as follows:

$$\check{y}^{(j)} = \sum_{i=0}^{j} c_{ji} x^i. \tag{1}$$

where: c_{ji} coefficients determined with the least-squares method.

In particular, for approximation, one may use a multinomial function which is the sum of orthogonal polynomials in the measuring interval, in the form of:

$$\check{y} = C_0 P_0 + \dots + C_i P_i + \dots + C_I P_I, \tag{2}$$

where:

P_i - orthogonal polynomial of *i* degree $i = (0 \dots I)$,

C_i - constant, determined with the least-squares method,
P_i - polynomial of i degree.

Condition of polynomial orthogonality (2) on a measurement set $\{x_1, x_N\}$ is as follows:

$$\sum_{n=1}^{N} P_i P_j = 0, \tag{3}$$

for $i, j = (0 \ldots I)$, $i \neq j$.

In such case, the coefficients C_i of approximating equation are determined very easily:

$$C_i = \frac{\sum_{i=1}^{N} \tilde{y}_i P_i}{\sum_{i=1}^{N} P_i^2}, \tag{4}$$

where:
$\tilde{y}_i$ - measurement results (sample values),
N - number of samples in an approximation interval.

Building a set of orthogonal polynomials for the general case of an ordinate axis does not constitute a difficulty of substantive nature. However, nearly all measurements performed in the field of technology take place with constant frequency of sampling, that is with permanent period of sampling. These kinds of measurement sets are called sets with parallel knots. Any approximation interval for the above case of measurement (axis x) can always be brought to interval [-K, K], where: K = a natural number.

It's easy to check for a case of parallel knots, in any measurment interval with parallel nodes, the following polynomials are orthogonal:

$$\begin{gathered} P_0 = 1;\ \ P_1 = c_{10} + x;\ \ P_2 = c_{20} + x^2;\ \ P_3 = c_{31}x + x^3; \\ P_4 = c_{40} + c_{42}x^2 + x^4;\ \ P_5 = c_{51}x + c_{53}x^3 + x^5 \end{gathered} \tag{5}$$

Based on the structure of expressions (5), one can anticipate the form of orthogonal polynomials of subsequent degrees. Coefficients c_{ji} are determined from an orthogonality condition (3).

3. Multiple moving approximation

While a moving approximation can be considered as an obvious development of the idea of a moving average, the application of multiple (multipass) moving approximation for measurement data processing should be considered an achievement of great importance. Moving multiple approximation enables smoothing of the curves, generation of derivatives, and approximate integration, as well as decomposition and and filtration (separation) of disturbances [1, 2].

Moving approximation consists of setting the approximated value at one point of the approximation interval and moving along the measurement axis, or generating along with the inflow of measurement data in the on-line systems of analysis. The set that is subjected to approximation in a subsequent pass is the result of approximation from the previous pass of approximation. At each pass of approximation, new sets of coefficients of approximation expression (1) are received c_{ji} .

Calculation templates are subjected to special simplification for central symmetric points of approximation interval for which $x = 0$. The result of expression (1) states that the smoothed value of the function is equal to c_{j0}, the value of the first derivaive is equal to c_{j1}, and the value of the integral is equal to $\int_{-0,5}^{0,5} \breve{y}^{(j)}$. The expressions for the values of derivatives at higher grades are obvious.

For each pass of approximation, one may change the width of approximation interval as well as the type of the approximating function, and in the case of application of a polynomial for approximation, its exponent. In this way, one may shape the quality of the curve smoothing, namely the parameters of a smoothing filter.

The difference between the results of a one-time and a four-time approximation (smoothing) of the curve of pressure of combustion by a power multinomial of third degree, for the same (in total) approximation intervals is shown in Fig. 1. In both cases, the total number of points of approximation interval used to generate one point of the smoothed curve amounts to 129.

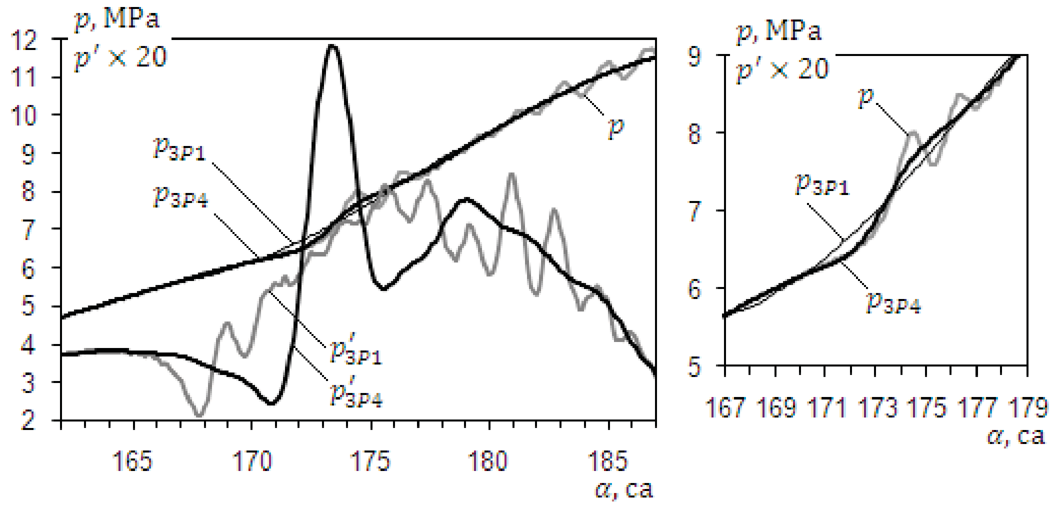

Fig. 1. Comparison of the results of moving approximation with a polynomial of third degree: p – smoothed fragment of the curve of pressure in a cylinder (engine 6AL20/24), p_{3P1}, p'_{3P1} – smoothed curve and the first derivative for one-pass approximation for $K = 64$, p_{3P4}, p'_{3P4} – smoothed curve and the first derivative for four-passe approximation for $K = 16$

As one can see in Fig. 1, as compared with multiple approximation, one time smoothing is characterized by distortion of the curve, and amplitude and phase errors of the derived of the first grade near the point of autoignition. The differences between the smoothed curves are noticeable only after relevant magnification of the figure.

4. Separation and decomposition of disturbances (deviations)

For each subsequent smoothing pass, one can assign smoothed deviations from the formula:

$$Dy_{l-1;s} = \check{y}_{l-1} - \check{y}_s, \tag{5}$$

where $l = 1(1)s$. For first pass: $\check{y}_0 = \tilde{y}$.

For cases of four-pass approximation, one can separate 8 different divisions (sets) of disturbances, which include sets: $\{Dy_{01}, Dy_{12}, Dy_{23}, Dy_{34}\}$, $\{Dy_{01}, Dy_{12}, Dy_{24}\}$, $\{Dy_{01}, Dy_{13}, Dy_{34}\}$, $Dy_{14}\}$, $\{Dy_{02}, Dy_{23}, Dy_{34}\}$, $\{Dy_{02}, Dy_{24}\}$, $\{Dy_{03}, Dy_{34}\}$, $\{Dy_{04}\}$.

In these sets there are 9 different forms of disturbances. For each set, the following occurs: $\sum_1^s Dy_{l-1;s} = Dy_{0s}$, which means an obvious observation that the total disturbance is equal to the sum of component disturbances.

Figure 2 shows set A of disturbances $\{Dp_{A01}, Dp_{A12}, Dp_{A23}, Dp_{A34}\}$, separated in four subsequent passes of approximation from curve p showed in Fig. 1.

Knowing the processes generating smoothed curve p, one can easily note that the designated disturbances (Fig. 2) are mainly the components of the deviations caused by gas oscillations in a short measurement channel between a combustion chamber and a sensor, and for this reason may be summed up or designated directly from the formula $Dp_{A04} = p - p_{3P4}$.

A decomposition of deviations can be made using many methods, none affecting or effecting to any insignificant degree the results of the course smoothing. The sense of decomposition (division) of deviations depends on the knowledge of physical and mathematical features of disturbances.

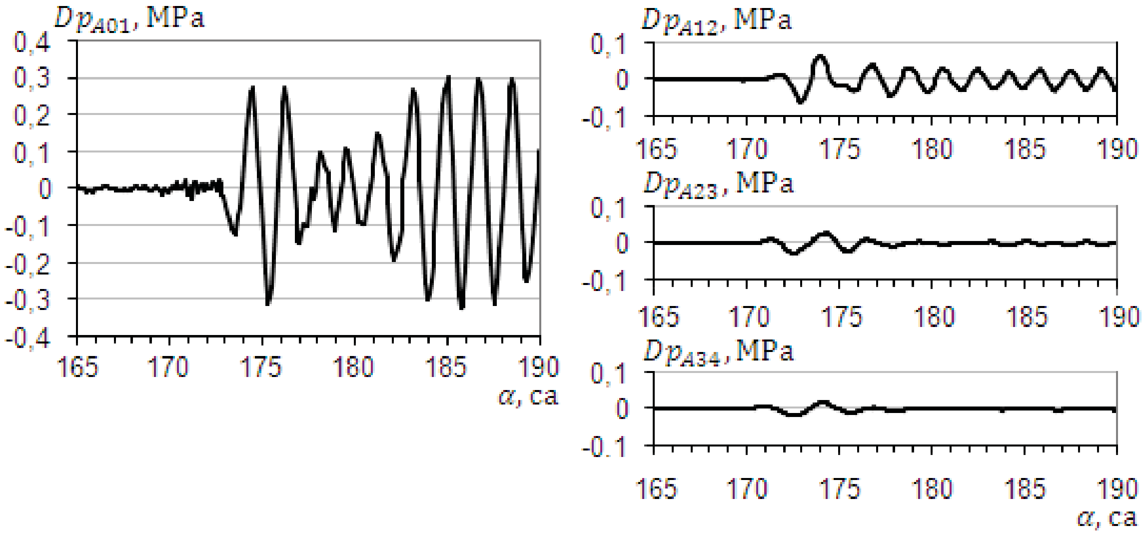

Fig. 2. Disturbances separated from curve p (Fig. 1) in each of four succesive passes of approximation with a polynomial of 3 degree for $K = 16$ – example A

In the case we're concerned with, the obvious physical sense has a separation of disturbances into measurement disturbances (high-frequency), arising in a measurement system, and into channel disturbances that are the result of gas oscillation in the measurement channel (Fig. 3).

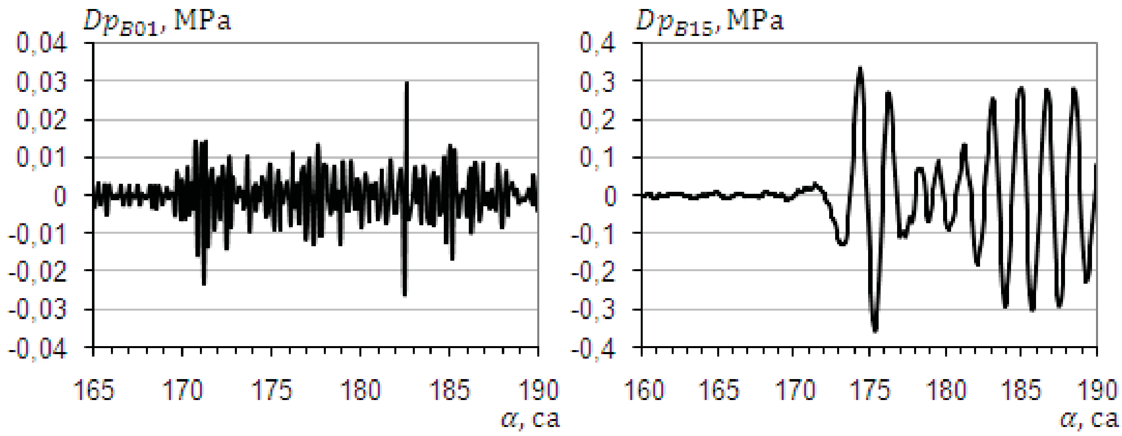

Fig. 3. Example B of separation of measurement disturbance Dp_{B01} and gas channel disturbances Dp_{B15} determined as a consequence of the course smoothing p in five passes

Separation of disturbances shown in Figure 3 was obtained by means of course smoothing p in 5 passes on the same interval of data as in example A. For separation of high-frequency disturbances, at the first pass, one has made a double approximation with $K = 2$. This decomposition of disturbances could also have been implemented approximating the total course of deviations Dp_{A04} or the constituent part of deviations Dp_{A01} (Fig. 2) with an accordingly small K. Using appropriate filters, one can independently pursue objectives of smoothing and the task of separation of disturbances, which is interesting for us.

5. Filtering abnormally big deviations of an impulse nature

Sometimes the measurement results may to include an abbnormally big deviations of an impulse (peak) nature, which can achieve significant values. Using the multiple approximation, one can obtain sufficient level of smoothing, but significant phase and amplitude errors may introduced (Fig. 4).

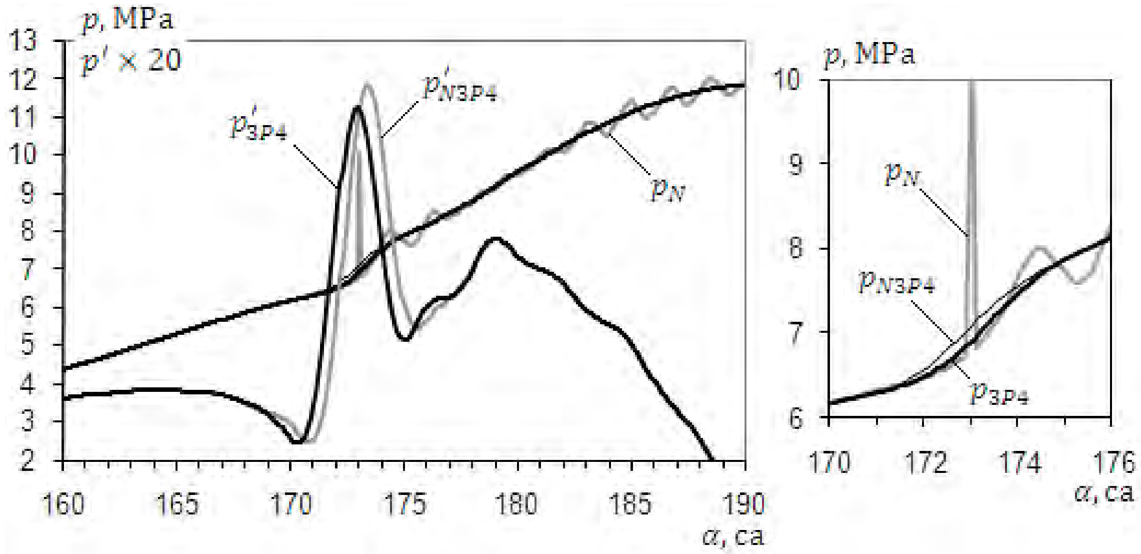

Fig. 4. Example of the impact of an impulse interference on the result of course smoothing: $p_N = p + \Delta p_N$ $-\Delta p_N$ disturbance added to the course p in point α= 173° OWK

This kind of disturbance should not be smoothed with the averaging method. In the case of an occurrence of abnormal impulse deviations, the averaging filter, used at the entry to the measurement device, would introduce identical signal distortion.

To detect and remove the impulse deviations, we have used a simple algorithm to move approximation into the central interval [[-K, -K_w), (K, K_w]], in which the given central sector of measurement data [-K_w, K_w] is excluded from approximation. In particular, the point excluded can be one point – a central point where). For each pass, one determines the standard deviation, there are fixed deviations in the points excluded from approximation and compared with the standard deviation. In the case of excessive differences, the values in the points with excessive deviation are replaced with the values obtained from the approximation of the neighbouring points. The exclusion of the points of high deviation from the approximated set will result in departure from the model with parallel knots.

6. Selection of approximation polynomial and approximation parameters

The basic influence on smoothing results has selection of the approximation polynomial. In the developed programs, one has been provided with a possibility of selection of approximating polynomials from zero to the fifth degree. At this point, one should note that for a central point of the symmetric interval of data, the results of smoothing with a polynomial degree zero and one, then of degree two and three as well as of degree four and five are identical. The results of approximation differ with existence of the derived of the highest degree, e.g. for five degree there is a derivative five of this grade, but the smoothing result is the same as the smoothing result for the polynomial of four degree.

Figure 5 compares the results of smoothing sample course p by means of a moving average as well as the polynomial of the third and fifth degree.

As one can see in Fig. 5, to obtain the desired smoothness, along with the growth of the degree of approximation polynomial, one has to use more and more wide intervals of approximation as well as a growing number of passes. Increasing the approximation interval width will cause an increasing impact of remote points on the results of approximation on the concerned point, which one may note in the concerned example, comparing derivative p'_{3P4} with p'_{5P5} (Fig. 5) to the left from point 170° OWK where on the course of derived p'_{5P5} one can notice a visible distortion (wave – hump).

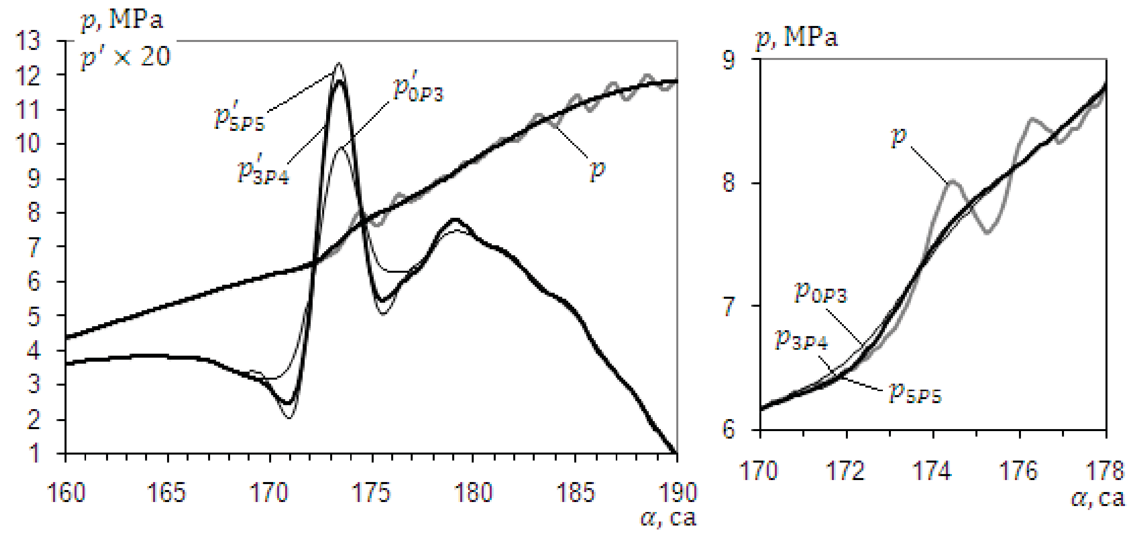

Fig. 5. Comparison of the results of p course smoothing with the mutinomials of different degrees: p_{0P3}, p'_{0P3} – 3pass approximation with a moving average for K = ; p_{3P4}, p'_{3P4} – 4-pass approximation with the polynomial of 3 degree for K =16; p_{5P5}, p'_{5P5} – 5-pass approximation with the polynomial of 5 degree for K = 24

Too small a value for the degree of approximation multinomial introduces excessive phase and amplitude errors, which one may note comparing derivative p'_{0P3} with p'_{3P4} (Fig. 5). In the concerned example, the approximation with a polynomial of three degree is optimal. Selection of the width of approximation intervals and number of passes will depend also on the quality of the expected smoothing. Apart from polynomials, one may use other functions or approximation objects with bonds [1, 2].

The required parameters of the filter may be different if the aim is separation and decomposition of disturbances. The previous experience has shown that the selection of filter parameters is, to a large extent, subjective and depends on the knowledge of the essence of the analyzed curve, but for the particular family of curves, e.g. for indicator charts or for the curves of angular speed of a crankshaft, the optimal parameters of the filters are identical or close to each other, regardless of the type of the examined object. At this point, one should note that the criteria for selection of wavelet filter parameters used in a wavelet analysis also do not lead to the smoothing or decomposition results of disturbances of an objective nature without the operator's intervention, knowing the essence of the analyzed curve.

7. Comparison of smoothing with the method of moving approximation with smoothing with wavelet filters

Currently there are available several packages of programs for mathematical analysis, containing wavelet filters that may be used for smoothing the courses and decomposition of disturbances. To compare the smoothing results with the methods of moving approximation and by means of wavelet filters, course p has been subjected to wave decomposition by means of several types of wavelets [4], using Wavelet Explorer package for this purpose, acting in the environment of Mathematica program.

Figure 6 compares the results of smoothing curve p by means of known wavelets of Daubechies and Meyer with different parameters with the results of smoothing with the method of multiple approximation with a multinomial of three degree. The derivatives of smoothed curves with wave methods were determined with the method of a moving approximation.

On the courses of derivatives and $p'_{D1}, p'_{D2}, p'_{D3}, p'_{D4}, p'_{M}$ (Fig. 6) there are visible substantial waves of identical nature that are not present on the derived p'_{3P4} These are the waves, typical for the whole-interval approximation with catenated functions [1, 2].

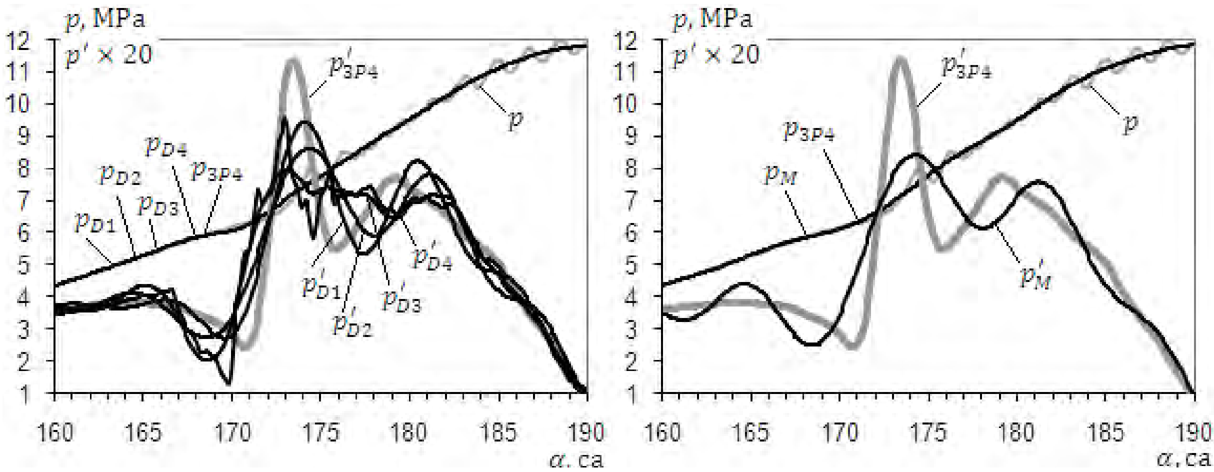

Fig. 6. Comparison of the results of p curve smoothing with a multinomial of three degree and wavelet filters: p_{3P4}, p'_{3P4} – smoothed curve and the derivative received from 4-pass approximation with polynomiall of three degree for $K = 16$; p_{D1}-p_{D4}, p'_{D1}-p'_{D4} – smoothed curves and their derivatives received by means of Daubechies wavelet filters; p_M, p'_M – smoothed curve and its derivative received by means of Meyer wavelet filter

One should pay attention to the fact that these wavelet filters generate non-existing deviations (wave form). Similar symptoms were observed in the application of other types of wavelet filters, apart from Shannon wavelet filter, but in this case the obtained results require additional smoothing with the method of moving approximation [2], which questions the purpose of applying this filter.

8. Conclusions and program implementations of the method

The smoothing filters based on multiple moving least-square approximation with polynomials enables smoothing the measurement series, generation of derivatives, as well as decomposition and separation of disturbances. Characterized by simplicity of algorithms and great speed of calculations, they enable obtaining better or identical results than in the case of wavelet filters. They are particularly useful for measurement series with on-line processing.

One can relatively easily create a program for measurement data processing with the method of moving approximation. The authors prepared the programs in Excel and Delphi programming environments. As demonstrated by our previous experience in the field of measurement data processing, the programs executed in Excel are mostly sufficient for the initial treatment (identification) of measurement data. In these programs, we've used power multinomials to the fifth degree. We should emphasize that, while processing the indicator charts of different types of engines for the curves of angular speeds of shafts and the curves of vibrations, we have, so far, not encountered the need to apply the polynomials of higher degrees than three.

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