CONCEPTION OF VALUATION
OF COMBUSTION ENGINE OPERATION

Jerzy Girtler

Gdansk University of Technology
Faculty of Ocean Engineering & Ship Technology
Department of Ship Power Plants
tel.: +48 58 3472430, fax: +48 58 3471981
e-mail: jgirtl@pg.gda.pl

Abstract

The work provides a proposal of interpretation showing operation in terms of values being compared to a physical quantity with the unit of measure called joule-second, just like operations of Hamilton and Maupertius demonstrated in classical mechanics. Operation in this interpretation is also considered in quantum mechanics, referring to body electromagnetic radiation source of which operation is characterized by Planck constant. An original method of analysis and evaluation of machine operation is herein presented in the energetic aspect, as the result of applying the theory of semi-Markov processes. In order to give grounds for usability of such interpreted operation it has been employed the model describing changes in energy of combustion engines, in the form of state-discrete and time-continuous semi-Markov process. The process is thus a semi-Markov model of getting worse combustion engine operation run with the lapse of time and regaining the initial energetic state of the engine after its refurbishment. The limiting distribution of the process has been determined with probabilities of occurring the distinguished energetic states of mentioned engines. The paper shows that the proposed interpretation for the operation enables making a graph of operation, which demonstrates engine operation in the form of field. The attention is paid that the engine operation in the presented version has got the advantage that can be expressed as a quantity with the joule-second as a unit of measure and as a graph in the form of operation field. The coordinate system for presenting the operation as a field, proposed in this paper, refers to the known in thermodynamics possibility of expressing the energy conversion sorts like work and heat (according to the conception of Clapeyron and Belpaire) in the form of fields.

Key words: heat, operation, energy, work, semi-Markov process, combustion engine

1. Introduction

Operation of combustion engines (self-ignition engines, spark-ignition engines, turbine combustion engines) in accordance with the second law of thermodynamics is characterized by energy dissipation in the form of work or heat. The dissipation increases with the lapse of operation time of the engines in consequence of their wear [7, 8, 13] which causes worse conversion (transformation and transmission) of energy being delivered to them. This is the reason that the engines work at lower and lower efficiency with the lapse of time [3, 4, 16]. Thus, it comes a need of assessment of combustion engines operation. The assessment should take into account not only the value of energy available while using the given combustion engine at determined time, but also the time at which the energy can be conveyed by the engine to a receiver. It is sensible then to consider the combustion engines operation in terms which enable determining it with energy and time at which the energy is available, at the same time. Determination of the engine operation as the result of energy transformations proceeding inside at the defined time is interesting because there exist some analogies for such approach to operation referring to physical systems. The classical physics considers operations of Hamilton and Maupertius [18] and the quantum mechanics - operation of body electromagnetic radiation source characterized by Planck’s constant [15]. The mentioned above interpretation of the engine operation is party presented by the author of papers [5, 6, 8, 9, 10, 11, 12, 13]. The author applies
therein the homogeneous process of Poisson. However, according to the author, the theory of semi-Markov processes can also be applied for such interpretation of engine operation [1, 7, 8, 11]. In order to get clearance of considerations the general interpretation of engine operation is presented as the first in turn.

2. General Interpretation of Engine Operation

Combustion engine operation consists in transforming and transmitting the energy delivered in the form of work or heat while first the chemical energy enclosed in fuel changes into thermal energy and then the thermal energy - into mechanical. This interpretation of energy conversion in the mentioned engines considers the fact that heat is a form of chemical energy conversion into thermal energy and work - a form of thermal energy conversion into mechanical energy [17].

In general conception, operation of each engine, consisting in transforming and transmitting the energy, can be interpreted as follows [6, 8, 9, 12]:

\[
D = \int_{t_0}^{t_1} E(t) \, dt
\]  

(1)

where:

\(D\) - engine operation,

\(E\) - converted (obtained) energy enabling realization of a task,

\(t\) - time of \(E\) energy consumption.

For combustion engines there are considered two ways of energy exchange during the time interval \([t_1, t_2]\), that is work \(L\) and heat \(Q\) [17]. Thus, operation of the engines can be expressed with the following formulas:

\[
D_L = \int_{t_1}^{t_2} L \, dt; \quad D_Q = \int_{t_1}^{t_2} Q \, dt
\]  

(2)

Such understood operation can be presented in accordance with the dependence (1) as the „E-t“ coordinate system, so in the form of graph which I propose to call operation graph. An example of such operation graph for any selected times \(t_1\) and \(t_2\), is shown in Fig. 1 [6, 8]. Following the formula (2) the operation can be obviously expressed also as the coordinate system „L-t“ or „Q-t“.

Operation of any combustion engine can be considered as: demanded operation \((D_W)\) and possible operation \((D_M)\) [6, 8, 12]. Therefore, one can accept that each engine finds itself in the ability state (and can perform a task) when:

\[
D_M \geq D_W.
\]  

(3)

Otherwise, when \(D_M < D_W\) one can assume that the engine finds itself in the disability state or the state of partial ability [7, 8]. Usability of the given engine can be concluded after making comparison between fields (Fig. 1) of the demanded operation \((D_W)\) and the possible operation \((D_M)\). This issue is presented in the papers [8, 9, 12, 13].

Fig. 1. Exemplary graph of combustion engine operation: \(E\) - energy, \(t\) - time
It is obvious that in order to determine the operation field \( D \) the time \( t \) dependence of the functional energy \( E \) is needed to know. Because \( D = f(E, t) \) the operation of machines can also be presented as the system of coordinates \( (D, E, t) \).

In case when energy of a given combustion engine, being transferred to a receiver is constant \( (E = \text{idem}) \) in time interval \([t_1, t_2]\), the operation, considering the formula (1), is:

\[
D = \int_{t_0}^{t_1} E(t)dt = E(t_1) - E(t_0) = E(t_2 - t_1).
\] (4)

Taking into account the ways of \( E \) energy conversion (4) in combustion engines, in the forms of work \( (L) \) and heat \( (Q) \), operation of the engines can be determined by use of the formulas (2) as follows:

\[
D_L = \int_{t_1}^{t_2} Ldt = L(t_2 - t_1); \quad D_Q = \int_{t_1}^{t_2} Qdt = Q(t_2 - t_1).
\] (5)

Thus, operation of combustion engines can be considered as a physical quantity that characterizes energy consumption at defined time.

When the combustion engines undergo wear the conversion of fuel chemical energy into thermal energy \( (Q) \) and then into mechanical energy \( (L) \) gets worse. The consequence is that the energy \( (E) \) being delivered to a receiver during engine operation \( (t) \), decreases. In practice one can record each decrease of energy \( E \) by a defined fundamental particle \( e \) which may be called a quantum (so \( \Delta = e \)). This is because of possibility of applying a diagnosing system fitted with measuring devices having measurement resolution adequate for that purpose.

The decrease of energy \( E \) by the particle \( e \) can be demonstrated in the form of the scheme (6)

\[
E_{\text{max}} = E_1 \xrightarrow{e} E_2 \xrightarrow{e} \ldots \xrightarrow{e} E_{n-2} \xrightarrow{e} E_{n-1} \xrightarrow{e} E_n = E_{\text{min}}
\] (6)

where:

- \( E_i (i = 1, 2, \ldots, n-1) \)- energies defined in result of recording (by a diagnosing system) the successive decreases of engine energy \( (E) \), in the form of the particles (quantum) \( e \);
- \( E_{\text{max}} \)- maximum energy that can be generated by the engine at time of correct operation, being possible to record by a diagnosing system,
- \( E_{\text{min}} \)- minimum energy that can be generated by the engine at time of its failure (recording it by a diagnosing system does not need to be possible).

Combustion engines which gained the energy \( E_{\text{min}} \) as the result of wear, are refurbishable if it is cost-effective. In consequence of refurbishment made with transforming the energy according to the scheme (6), the engine may get back its initial energy \( E_{\text{max}} \). Changes in operation of combustion engines, following from the scheme (6), and the possibility of their refurbishment can be transparently demonstrated in the form of a model of which elaboration needs of employing the theory of semi-Markov processes.


In case of combustion engines it can be assumed [7, 8] that time of duration of any energetic state \( E_i \in E_z (26) \) reached in the moment \( \tau_n \) and the state reached it the moment \( \tau_{n+1} \) do not depend stochastically on states which occurred before and the time intervals of their duration. Thus, it can be accepted that the process \( D(t); t \geq 0 \) of energy transformations of the engines is a semi-Markov
process with the set of states \( E_\varepsilon = \{ E_i; i = 1, 2, \ldots, k \} \) and the graph of state transitions, presented in Fig. 2.

Distinguished energetic states \( E_i \in E_\varepsilon \) of the process \( \{ D(t): t \geq 0 \} \) for an arbitrary combustion engine can be recognized with help of adequate diagnosing systems (SDG) [2, 7, 8].

The process \( \{ D(t): t \geq 0 \} \) presented as a model of changes in the energetic state of combustion engines can be useful in practice only when refurbishment of the engines assures increase of their energetic capabilities by the only one quantum. In practice, however, there is no need to interfere so often in combustion engine structure to perform the refurbishment. The refurbishment is performed when engine wear reaches the conventional limiting state. Also this should ensure the reliable operation of the engine. This means that while determining the limiting values of engine wear one should tend to determine the energy \( E \) which is indispensable to perform the given task. This can be done by determining the indexes of transformation of chemical energy contained in fuel into thermal energy and the indexes of transformation of thermal into mechanical energy. The control over realization of the mentioned types of engine transformation can be provided by presenting the engine operation in terms of values and employing the model of changes in engine energetic states, in the form of semi-Markov process \( \{ D^*(t): t \geq 0 \} \) with a graph of the state transitions showed in Fig. 3.

Exemplary one can assume that in operating practice of a given combustion engine when its operation runs rationally it is sufficient to distinguish the energetic states which form the set of energetic states

\[
E_\varepsilon = \{ E_1, E_2, E_3, E_4 \},
\]

having the following interpretation:

- \( E_1 \) - energetic state referring to capability of engine load in full range for which the engine was destined in the designing phase and manufacture,
- \( E_2 \) - energetic state referring to capability of engine load like in the state \( E_1 \) but at limited time of operation, for instance a few hours,
- \( E_3 \) - energetic state referring to capability of engine load in partial range but sufficient to perform a task which is to be performed,
- \( E_4 \) - energetic state which makes impossible using the engine in accordance with its destination but still enables performing a diagnosis of its state and assessing the scope of works for refurbishment.

The set of energetic states \( E_\varepsilon \) (7) can be accepted as the set of the states of the process \( \{ D^*(t): t \geq 0 \} \). Transition of the states \( E_i \) (\( i = 1, 2, 3, 4 \)) proceed according to the graph of state transitions showed in Fig. 3. Fig. 4. presents an exemplary realization of this process.
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Fig. 3. Graph of state transitions in the process \( \{ D^i(t): t \geq 0 \} \)

Fig. 4. Exemplary course of one-dimension process \( \{D^i(t): t \geq 0 \} \) when refurbishment of a machine takes place only after the state \( E_4 \) occurs what makes the further machine operation impossible.

Determination of the process \( \{D^i(t): t \geq 0 \} \) requires fixing the initial distribution and its functional matrix.

The initial distribution of the process \( \{D^i(t): t \geq 0 \} \) is:

\[
P_i = P\{D(0) = E_i\} = \begin{cases} 1 & i = 1 \\ 0 & i = 2, 3, 4 \end{cases}
\]  

(8)

Following the graph of state transitions presented in Fig. 3 the functional matrix if of the following form:

\[
Q(t) = \begin{bmatrix}
0 & Q_{12}(t) & 0 & 0 \\
0 & 0 & Q_{23}(t) & 0 \\
0 & 0 & 0 & Q_{34}(t) \\
Q_{41}(t) & 0 & 0 & 0 \\
\end{bmatrix}
\]  

(9)

The functional matrix \( Q(t) \) is a model of transition of energetic states of any combustion engine (with self-ignition or spark ignition or turbine combustion engine).

Non-zero elements \( Q_{ij}(t) \) of the matrix \( Q(t) \) depend on distribution of random variables being time intervals of the \( \{D^i(t): t \geq 0 \} \) process’ stay in states \( E_i \in E_4(i = 1, 2, 3, 4) \). The elements are
probabilities of the process transition from the state $E_i$ to the state $E_j$ ($E_i, E_j \in E_2$) at time not longer than $t$, which are determined as follows [14]:

$$Q_{ij}(t) = P\{Y(\tau_{n+1}) = E_j, \tau_{n+1} - \tau_n \leq t | Y(\tau_n) = E_i\} = p_{ij}F_{ij}(t),$$  \hspace{1cm} (10)$$

where:

- $p_{ij}$ - probability of single-step transition in homogenous Markov chain,
- $p_{ij} = P\{Y(\tau_{n+1}) = E_j | Y(\tau_n) = E_i\} = \lim_{t \to +\infty} Q_{ij}(t)$,
- $F_{ij}(t)$ - distribution of random variable $T_{ij}$ indicating duration of the state $E_i$ of the process $Y_1(t)$: $t \in T$ on condition that the state $E_j$ is the successive one for this process.

Because the matrix $Q(t)$ is a stochastic matrix the probability matrix of transition of embedded Markov chain in the process $\{D^\ast(t): t \geq 0\}$ is as follows:

$$P = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
\end{bmatrix}. \hspace{1cm} (11)$$

The process $\{D^\ast(t): t \geq 0\}$ is irreducible [8, 14] and random variables $T_{ij}$ have got finite positive expected values. Hence, its limiting distribution

$$P_j = \lim_{t \to +\infty} P_{ij}(t) = \lim_{t \to +\infty} P\{Y(t) = E_j\}, E_j \in E(j = 1, 2, 3, 4),$$  \hspace{1cm} (12)$$
is of the following form [14]:

$$P_j = \frac{\pi_j E(T_j)}{\sum_{m=1}^4 \pi_m E(T_m)}.$$  \hspace{1cm} (13)$$

Probabilities $\pi_j(j = 1, 2, 3, 4)$ in the formula (13) are limiting probabilities of the embedded Markov chain in the process $\{Y(t): t \in T\}$.

Determination of the limiting distribution (13) requires solving the following system of equations:

$$[\pi_1, \pi_2, \pi_3, \pi_4] \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
\end{bmatrix} = [\pi_1, \pi_2, \pi_3, \pi_4]$$

$$\sum_{k=1}^4 \pi_m = 1 \hspace{1cm} (14)$$

In the consequence of solving the system of equations (14) and using the formula (13) the following dependences can be obtained:
\[ P_1 = \frac{E(T_1)}{M}, P_2 = \frac{E(T_2)}{M}, P_3 = \frac{E(T_3)}{M}, P_4 = \frac{E(T_{4-3})}{M}, \]

at which:
\[ M = E(T_1) + E(T_2) + E(T_3) + E(T_4). \]

\( P_1 \) is a limiting probability that at \( t \to \infty \) the engine finds itself in the energetic state \( E_1 \). Thus, this probability determines possibility of engine operation at maximum load for which the engine was destined in the designing phase and manufactured. \( P(j = 2, 3, 4) \) are limiting probabilities that at \( t \to \infty \) engine finds itself accordingly in the energetic state \( E(j = 2, 3, 4) \). \( P_4 \) is a limiting probability that at \( t \to \infty \) engine finds itself in the energetic state \( E_4 \) that can be identified with the energetic state making the engine operation impossible (\( E_4 = E_{\text{min}} \)) but after diagnostic tests it enables to determine the needs for reconditioning the engine technical state and obtaining the energetic state \( E_1 \). That means that the engine in state \( E_4 \) should undergo refurbishment if it is profitable or liquidation (withdrawal from use) in case the refurbishment is not profitable.

\section*{4. Conclusion}

Operation of any combustion engine, in the presented proposal, is considered as generation of energy \( (E) \) at determined time \( (t) \). That is why it is demonstrated as a physical quantity (like the operations of Hamilton and Maupertius in classical physics) which can be expressed with a numerical value and unit of measure called joule-second. The direct effects of engine operation in such interpretation are the energy \( (E) \) generated by the engine and the time \( (t) \) at which it works. Therefore the energy and the time are the quantities which unequivocally characterize such understood operation of combustion engines.

Additionally, engine operation in the presented version has got this advantage that can be tested by performing precise measurements and then expressed in the form of:
- number with the unit of measure (formula 1 or 2),
- graph as operation field (Fig. 1).

From the presented considerations follows that the transformation process of energetic properties of combustion engines can be tested with help of models built in the form of semi-Markov processes.

Semi-Markov processes as the models of real processes taking place in the phase of operation of combustion engines are more useful models in practice than Markov processes. This is because semi-Markov processes with continuous time parameter and finite set of states have the property that the time intervals of their stay in particular states are random variables with arbitrary distributions concentrated in the set \( R_+ = [0, \infty) \). This makes them different from Markov processes of which intervals are random variables with exponential distributions.

Semi-Markov models of processes proceeding in the phase of operation of combustion engines are time-continuous processes with finite sets of states.

\section*{References}

J. Girtler


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