PREVENTIVE MAINTENANCE WITH IMPERFECT REPAIRS OF VEHICLES

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Abstract

A vehicle ability to realise transport tasks may be restored by repairing only failed elements. This is called imperfect repair as the vehicle is not as good as new after such a repair. Preventive replacement is an example of imperfect repair as well. The advantage of such maintenance is that it enables controlling a reliability level of a fleet of vehicles. Sets of vehicles elements which should be replaced in that aim are derived on a base of statistical diagnosing with use of data about elements failures. The acceptable level of a failure risk while executing transportation tasks has been taken as a criterion. An algorithm for selecting elements for preventive replacement has been developed. It was shown that a level of a fleet reliability can be controlled by changing an order of a quantile function in coordination with a number of redundant objects. A computer simulation model of the vehicle fleet was used as an example to illustrate derived dependencies.

In particular algorithm for selecting elements for preventive replacement, graph of model states, graphical interpretation of calculating new quantile orders, simulation experiments results for system n out of n, simulation experiments results for d = 2,5 are presented in the paper.

Keywords: *transport, maintenance, preventive replacement, imperfect repair, statistical diagnosis*

1. Introduction

Preventive replacements are used to maintain demanded reliability of vehicles in a transport firm. They enable avoiding failures of individual vehicles in a fleet. However a need for high reliability of fleet of vehicles being used can effects in great amount of elements replaced during preventive actions. As it can not be considered full restoration of the vehicle reliability after the service, so only elements of the vehicle should be replaced. This can be called as imperfect repairs of the vehicle.

High reliability is achieved in practice by services when specific elements are replaced by new ones. A criterion of selecting elements depends on level of reliability that is expected. In a case of a homogeneous set of vehicle, a range of prophylactic activities depends on a reliability level of the whole fleet and on its reliability structure.

This takes in account redundancy, that enables first off all to replace failed objects enabling execution of transportation tasks. A number of redundant objects depend on the acceptable probability of failure during the task implementation period. In order to minimize the size of redundancy one should, on the one hand, be using objects of high reliability, and keep their reliability in the operating process at possibly high level, on the other hand.

Instead of known from the literature method of replacing object in a given rate [6], the method of replacing sets of chosen elements that enables achieving demanded level of the fleet reliability is proposed. This method uses statistical characteristics of the vehicles instead of applying measurable parameters of its elements.

The ability of the object of fulfilling given tasks with demanded probability could be statistically measured by quantile of given order. For this measure, a method of statistical diagnosis was developed. It points out at given moment to a set of elements that should be replaced by new ones to achieve demanded reliability of the whole fleet of vehicles.

2. Preventive replacements

A method that is known from literature and used for defining of a scope and deadlines of preventive replacements is to include the costs of attentive replacements and the costs generated by the occurring failures [1, 5]. As a result of application of the method, minimum average costs per unit of time related to maintenance of objects in a proper reliability status are achievable. However, in order to benefit from that effect there is a need to replace individual elements in various time intervals, usually uncoordinated with the performance of tasks, which may wipe out advantages effecting from the implemented optimisation. Therefore, a possibility should be considered to make preventive replacements of selected elements of objects in the assumed time intervals whose scope is defined on the basis of assessment of reliability of the elements and the assumed reliability level of the entire fleet [3]. The fleet maintained in such a way preserves its ability to realize transportation tasks with a given probability.

In case of complex objects, a failure appears whenever an element, which creates a series reliability structure with the others, has failed. A repair usually involves a replacement of the element for a brand new one. However, the replacement of the damaged element for the new one does not effect in recovery of such a reliability level as that before occurrence of the failure. This is because the value of the reliability function of the damaged element before the failure was less than 1, and following the replacement it was equal to 1. In effect, the condition of the object after the repair is – and must be – slightly better than before the failure. So, practically there are no possibilities to recover such a status of the object following the repair, as the one right before the failure.

Both the objects and their components are considered when developing the preventive replacements strategy. Properties of the components are more predicable than those of objects which they are part of.

Dynamic determination of a scope of preventive replacements could be based on a statistical assessment of present status of objects' elements. The statistical diagnosis is a maintenance methodology in the area of maintaining objects with non-exponential distributions. It identifies preventive maintenance tasks to realise the inherent reliability of equipment at a minimum expenditure of resources. In order to do that, data is required about a distribution of time to failure and its parameters as well as about its operational use so far (since being new or from the moment of its replacement). The statistical diagnosis uses data gathered during normal utilization of objects. They concern failures, repairs and replacements of object elements. Next, the probability distribution function of time or mileage to failure for each of these elements is determined. It can be done either with the use of data collected in the past or by relying upon experts' opinions at the start.

On that basis, defined is a set of those elements the replacement of which will effect in a situation that a failure probability will not exceed its assumed value in the duration of the scheduled task. The main target is increasing the availability of the equipment by reducing the amount of technical tasks to its minimum, and it reaches this target by substituting the technical inspection with the statistical diagnosis.

The procedure statistically predicts failures at part level by calculating the mean residual lifetime to failure (MRL).

However, the MRL compared to required work period effects in that about half of objects would be serviced before failure and the rest would fail without any treatment. Thus, it is better to apply a quantile function of residual lifetime instead of the MRL to enlarge the probability of preventive service. This measure directly relates to predicted work period and the reliability of the system. For any moment **t** the following conditions have to be met:

$$q_p(t) \geq d\,, \tag{1}$$

where:

d – tasks implementation period,
$q_p(t)$ – quantile of residual lifetime function, order p.

Function $q_p(t)$ shall be defined as follows [2]:

$$q_p(t) = F_t^{-1}(p) = \inf\{x : F_t(x) \geq p\}\,, \tag{2}$$

where:

$$1 - F_t(x) = R_t(x) = \frac{R(t+x)}{R(t)}, \qquad x, t \geq 0\,, \tag{3}$$

$F_t(x)$ – cumulative distribution function of the residual lifetime,
$R_t(x)$ – conditional reliability function.

Statistical control can be performed at any moment because it retrieves data gathered in the informational area of the means-of-transport maintenance management system. It could be done either in a constant period of time or during planned service or during current repair. The distribution parameters are modified when either repair or replacement of the element has been done. The actual technical condition of the object is not taken into consideration here as that would require for the object to be excluded from its operational use. Having data, reliability characteristics of elements, updated working time of individual elements, a period for execution of the transportation task, it is possible to define elements that require preventive replacement in order for the project implementation probability not to decline below its assumed value.

It can be applied as well to elements as to complex objects (i.e. functional sets or the whole vehicle). In a case of complex object, its reliability structure is to be considered as well as special procedure of choosing elements to replace, which enables achieving demanded probability of proper work of the object [3, 4].

A particular example of complex object is a fleet of vehicles in a transport firm. It could be characterized by reliability and as well as by a reliability structure. Probability of a failure during a task period can be determined in both cases, that is, when the replacements either have or have not been made. Additionally, the assessment may refer to the entire fleet of objects that have been assigned for execution of the transportation tasks.

3. Imperfect repair

Majority of theoretical conclusions are derived with assumption of perfect (ideal) object restoring. However such processes with use of models of full renewal are adequate only when object is replaced by a new one or in a case of a general repair.

In the case of corrective repairs made after failing of any vehicle element, a model of minimal repair is often used [1]. This means that the object is to be restored to the condition just before failure. However it is not possible practically, as object reliability status after repair of its element is better than before failure. Those are reasons that theoretical models of either perfect or minimal repairs have limited applicability. Real repair restores object reliability to an intermediate value. Thus it is called an imperfect (incomplete) repair [4]. However a degree of object restoration by replacing one or more its elements can be estimated only after repair.

Modelling exploitation process with use of the imperfect repairs means defining characteristics of random variable X_k concerning time of proper work after (k-1)th repair. Object's reliability function after repair is given by the following formula:

$$R_2(x) = [R_1(x)]^{\alpha} \left[\frac{R_1(t+x)}{R_1(t)} \right]^{1-\alpha}, \tag{4}$$

where:

$R_1(x)$, $R_2(x)$ – reliability functions of the object before and after the repair respectively,
α – degree of the object restoration.

The formula for the failure rate function relation before and after the repair is as follows:

$$\lambda_2(x) = \alpha\lambda_1(x) + (1-\alpha)\lambda_1(t+x), \tag{5}$$

hence

$$\alpha = \frac{\lambda_1(t+x) - \lambda_2(x)}{\lambda_1(t+x) - \lambda_1(x)}, \tag{6}$$

where:

$\lambda_1(x)$, $\lambda_2(x)$ are the failure rate functions before and after the repair respectively.

4. Algorithm for selecting elements to be replaced

A preventive replacement of elements is made if the value of function (11), which has been calculated for the entire set of objects, is lower than the duration of the scheduled task planned for that set of objects. In order to select a set of elements to be replaced at given moment, an updated value of the reliability function is calculated including operational time of each and every one of them. Then a quantile of a given order is calculated for a distribution of the residual lifetime of each element. The elements are put in order according to the growing quantile value.

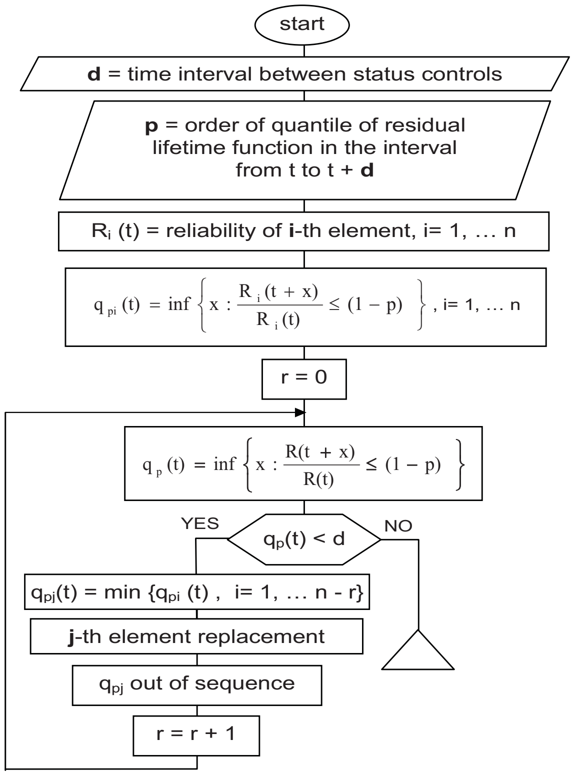

Fig. 1. Algorithm for selecting elements for preventive replacement

Subsequent elements, starting from an element of the lowest quantile value until the quantile of the entire fleet of objects – calculated by having included the replacement of assigned elements for brand new ones – is not lower than the duration of the scheduled task (algorithm in Fig.1), are assigned for replacement. The replacement of elements that have been assigned in that way ensures the assumed probability that the object will not fail during implementation of the transport task.

5. Redundancies in a fleet of vehicles

Sustaining a high reliability level of technical objects in their operational use process – served by preventive replacements of components being threatened by a failure – can be accompanied by adding redundant objects to the fleet.

Let us assume that **n** objects are essentially required for carrying out transportation tasks. If the entire fleet consists of **n** objects, then an assumption can be made that reliability structure of the fleet is in series. This imposes large requirements on reliability of each object, which is often not achievable. Then, in order to keep reliability of the fleet at its required level, redundant objects can be introduced into the fleet. Adding **k** redundant objects to the fleet allows for considering the fleet reliability structure as a threshold structure, in this case „**n out of n+k**".

The fleet reliability model depends on the way the redundant objects are operating in. Redundant objects may play a role of the cold (unloaded) reserve, that is, they passively wait for one of the objects to fail, or the hot (loaded) reserve, thus increasing the entire fleet capacity until one of the objects has failed.

In case of structure „**n out of n+1**" with the cold reserve, the fleet reliability function $R_{n+1}(t)$ will be a sum of probabilities for occurrence of the following situations:

1) until moment t no object will fail out of **n** objects establishing a series reliability structure,
2) at any moment $\tau < t$ one out of **n** objects shall fail and will be replaced with a reserve object that will not fail along with the remaining objects in a range of (τ, t).

Probabilities for occurrence of the above situations are respectively:

$$P_1 = R^n(t), \tag{7}$$

$$P_2 = \int_0^t f_n(\tau) R_n(\tau, t) d\tau , \tag{8}$$

$$R_n(\tau, t) = \left[\frac{R(t)}{R(\tau)}\right]^{n-1} R(t-\tau), \tag{9}$$

$$f_n(\tau) = \frac{d}{d\tau}\left[1 - R^n(\tau)\right] = n R^{n-1}(\tau) f(\tau), \tag{10}$$

$$P_2 = nR^{n-1}(t)\int_0^t f(\tau) R(t-\tau) d\tau , \tag{11}$$

$$R_{n+1}(t) = P_1 + P_2 = R^{n-1}(t)\left[R(t) + n\int_0^t f(\tau) R(t-\tau) d\tau\right], \tag{12}$$

where:

R(t) – object's reliability function,

$f_n(\tau)$ – probability density function of a failure of one out of **n** identical objects establishing a series reliability structure,

$R_n(\tau, t)$ – probability of a non-failure in the range of (τ, t) of the fleet consisting of (**n-1**) objects aged τ and one new object.

Probability density function of a failure of the fleet with a structure of „**n out of n+1**" with the cold reserve is expressed by relation:

$$f_{n+1}(t) = n\,R^{n-1}(t)\int_0^t \left[f(t-\tau) + (n-1)\frac{f(t)}{R(t)} R(t-\tau) \right] dF(\tau), \tag{13}$$

and no recurrence formulas are known.

In case of the structure „**n out of n+2**", the analytical description becomes more complex, as there is the second reserve object. This means that in the fleet, established at the moment τ and consisting of (**n-1**) objects aged τ and one new object, one of the objects may fail and be replaced by the second reserve object before the moment t.

In case of the fleet with structure of „**n out of n+k**" with the hot reserve, we may use the following relation:

$$R_{(n,n+k)} = \sum_{i=n}^{n+k} \binom{n+k}{i} R^i (1-R)^{n+k-i} \tag{14}$$

and the recurrence formula:

$$R_{(n,n+k)} = R \cdot R_{(n-1,n+k-1)} + (1-R) \cdot R_{(n,n+k-1)}. \tag{15}$$

Complexity of the analytical description, regardless of simplifying assumptions that have been made (i.e. identical objects, omission of the reliability structure of objects alone), indicates that there is a need for using a computer simulation for issues being considered here.

If **k** vehicles works as the hot reserve, the system can be treated as "**n out of n+k**" structure and the order p represents demanded level of reliability. However, in the case of **k** redundant vehicles working as the cold reserve with n vehicles presenting a series reliability structure there is a need for calculations new value for the level of reliability.

$$1-\alpha_k = \frac{1-p}{(1-R)^k \left[\binom{n+k}{n} + \sum_{i=1}^{k} \binom{n+k}{n+i} \left(\frac{R}{1-R} \right)^i \right]}, \tag{16}$$

where:

α_k – probability of failure one of n vehicles ($\alpha_0 = p$),
p – acceptable probability of system failure,
R – reliability of a single vehicle,
n – number of vehicles needed for transportation tasks execution,
k – number of redundant vehicles.

This order is less then that for the whole fleet according to this formula, derived from the formula (14). The relation between orders (**p, α_k**) is shown in Fig.2.

For the order **p** it can be calculated quantile for the structure "**n out of n+k**". Then the reliability **R** of a single vehicle for the same quantile is calculated for the structure "**1 out of 1**".

With use of these values the order α_k can be calculated, which is the order for the structure "**n out of n**".

6. Simulation experiments

The above consideration was confirmed with use of a computer simulation. In the model, objects were applied, that were fully replaced at steady intervals of mileage, according to results of statistical diagnosis. The planned process of replacements was combined with random process of failures and repairs. A graph of model states is presented in Fig. 3.

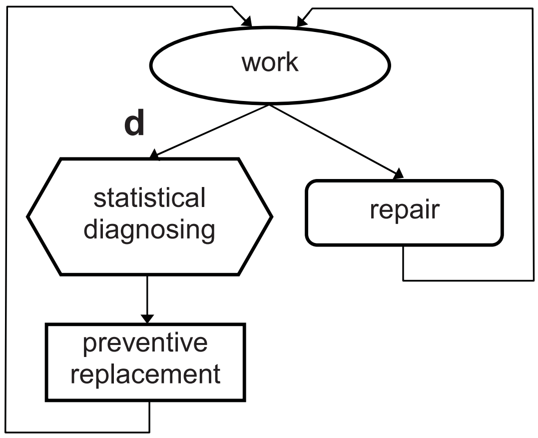

Fig. 2. Graph of model states (work – object is working, statistical diagnosing – set of objects is selected, preventive replacement – selected objects are replaced by new ones, repair – failed object is replaced by a new one)

The fleet of **n** objects was used for execution of tasks in the model. Each object is composed of three groups of different elements. The mileage to failure of a single element was Weibull distribution with a reliability function:

$$R(x) = \exp\left[-\left(\frac{x}{b}\right)^{a}\right]. \tag{17}$$

The acceptable probability of the fleet unavailability was **p**. The required reliability was maintained by preventive replacements of objects. Statistical diagnosing was done in intervals of length **d**.

The following operational strategies were applied:

- n objects used – fleet without redundancy,
- n+1 objects used – one redundant object, hot reserve,
- n+2 objects used – two redundant objects, hot reserve.

Parameters of the model were as follows: **n** = 50, **p** = 0.1, **d** = 3, $\mathbf{a_1}$ = 2.5, $\mathbf{b_1}$ = 65, $\mathbf{a_2}$ = 2.5, $\mathbf{b_2}$ = 80, $\mathbf{a_3}$ = 2.5, $\mathbf{b_3}$ = 100. The range of simulation is **T** = 1000, and experiments were repeated 10 times. As a result of simulation, numbers of replacements and failures of objects and unavailability of the whole fleet were estimated.

First, the number of failures in the system "n out of n" without any prophylactic and then with statistical diagnosing with p = 0.1 and d = 2,5 was estimated. The results show that it is possible to achieve demanded reliability with significant decreasing the number of random brakes in work but with a very big number of preventive replacements (Tab.1).

Tab. 1. Simulation experiments results for system "n out of n"

Number of:	Without preventive replacements with perfect object repair	Without preventive replacements with imperfect object repair	With preventive replacements (d = 2,5) p = 0,10
preventive replacements	-	-	33568
fleet unavailability	1102	2065	42
object failures	1102	2065	42

Imperfect repairs effect in greater number of fleet unavailability with compare to perfect object repairs. This is obvious because when repairing only elements the object resources are not restored, and the object is not good as new. Numbers of fleet unavailability with perfect and imperfect repairs are equal only for exponential distribution.

The aim of preventive replacements is to decrease number of random object failures by avoiding them with assumed probability, as they break fleet utilization and bring many unpredictable consequences. However to achieve this significant decreasing the number of unavailability, up to 60 % of all opportunities to replace elements were realized (maximum possible replacements of element was 60 000 in the example). Such a great number of replacements in this case were a result of rather low reliability of object components.

The reliability of the fleet can be also enlarged by adding redundancy to the system. This decreases the number of preventive replacements in system "n out of n+k" as - according to formula (15) - the result is analogical to appropriate increasing the quantile order of the system "n out of n". Such modification of the quantile order can be designated graphically as it is shown in Fig. 3. The modified orders of quantile calculated with use of formula (15) for systems with n+1 and n+2 objects are as follows: $\alpha_1 = 0.410$, $\alpha_2 = 0.660$.

But the result is only valid for a perfect repair after every statistical diagnosing, i.e. each object is replaced by the new one. For complex objects, i.e. composed of some elements this condition could be fulfilled when the interval of statistical testing where long enough. However, such a long interval is useless and practically should be lower than quantile of given order at x = 0.

The imperfect repair – done by replacing selected elements of serviced objects – are useful only when the interval between statistical diagnosing is shorter than the initial quantile at t = 0. After a number of such replacements of objects' elements the fleet do not consists of new objects. So the probability of tasks fulfilling by a single vehicle should be calculated for systems „n out of n+1" and „n out of n+2" separately on the base of appropriate experiments (Tab. 2), instead of basing on reliability function of a new object. Then the modified orders for system „n out of n" should be calculated with use of formula (15).

Simulation results in Tab.2 show that the numbers of replaced elements in systems "n out of n" with $p = \alpha_1$ and "n out of n+1" with p = 0.1 are similar, as well as in systems "n out of n" with $p = \alpha_2$ and "n out of n+2" with p = 0.1.

The perfect repair means restoring the object after each repair. Considering complex objects this also means replacing all of their elements to achieve object status as good as new. This can not be accepted in practice. Therefore imperfect repairs are more appropriate for practical applications.

The results of simulations confirmed a natural supposition, that elements of the lowest reliability constituted a dominating group of replaced elements. The share of such elements in the whole number is greater than without statistical diagnosing. These element where recognized and such a shifting was done by the algorithm guarantying the demanded level of the fleet reliability.

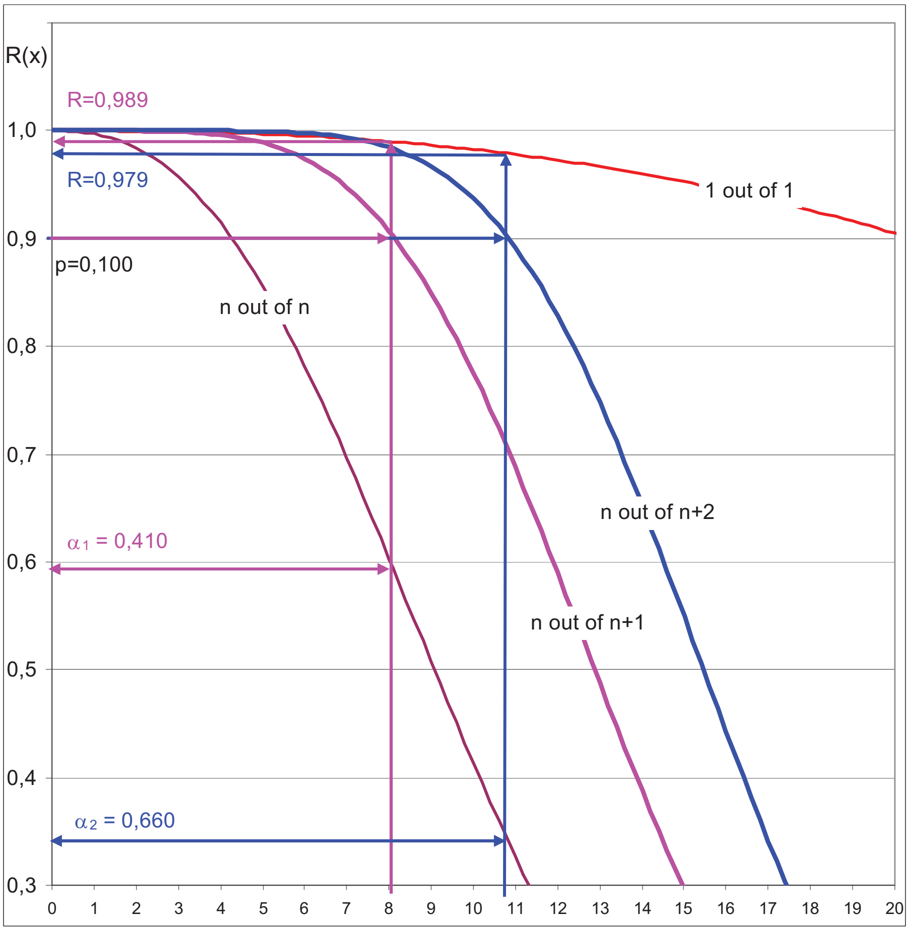

Fig. 3. Graphical interpretation of calculating new quantile orders

Tab. 2. Simulation experiments results (d = 2,5)

Number of:	Model			
	n out of n+1 p=0.10	n out of n α_1=0,346	n out of n+2 p=0.10	n out of n α_2=0,482
preventive elements replacements	14405	14638	10886	9807
group 1	7434	10213	5340	6582
group 2	4516	2982	3435	2175
group 3	2455	1443	2112	1050
fleet unavailability	23	174	16	256
object failures	154	174	232	256
Reliability of a single vehicle	0.992	0.991	0.989	0.987

7. Conclusions

The imperfect repairs are natural way of maintaining vehicles ability to performing transport tasks. They better fit to real situations, since the perfect repair policy is quite unrealistic in case of vehicles. A kind of imperfect repair is a preventive replacement of vehicle elements as it restores fleet capacity partially. This way a considerable reduction in a number of incidental failures of vehicles, compared to a use without any prophylaxis, is achievable through application of the statistical control. However, maintaining a high reliability of a fleet of vehicles is accompanied by a great amount of preventive replacements of vehicles' elements. This means that there are many more preventive replacements than random failures of objects because of relatively low reliability of a single object.

Thus, it would be easier to achieve the required fleet availability by adding redundant vehicles that replace damaged ones than to maintain a high reliability of the fleet of vehicles without redundancy. So, by adding a redundant object, more failures of vehicles can be accepted as well as a number of preventive replacements is reduced. It would be useful to combine redundancy and preventive replacement based on statistical diagnosing.

Required level of fleet reliability could be achieved by adding surplus vehicles and properly matching them with the quantile order applied to the main part of the fleet. By those two measures, random failures of the vehicles fleet are significantly reduced in number of replaced elements being much lower than those without redundancy.

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