HARMONICAL AND MODAL ANALYSIS TO DETERMINE THE ANISOTROPIC PROPERTIES OF AVIATION COMPOSITE MATERIALS

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Abstract

The value of all composite elastic modules has the crucial influence on dynamic behaviour of aviation load-bearing structures. Numerous experimental techniques and standards are used to characterize the elastic properties of polymeric composite materials. Among this is the large group of static tests, acoustic methods based on longitudinal, lateral or shear surface sound wave speed measurements and also on vibrating surfaces amplitude measurements. However, preparation specimens for static test with shape required by standards not always possible from already manufactured piece. Again the designated dynamic experiment introduces difficulties of investigated specimen acoustic isolating, impossibility of some required wave type excitation and performing of displacements field measurement with guaranteed precision. In this study an acoustic method is developed using the measure of all specimen's eigenfrequencies in a certain frequency range. Small rectangular composite specimen is excited by glued piezoelectric actuator and response is measured by piezoelectric sensor. Preliminary performed finite-element (FE) analyses serve to think the vibration natural modes of linked mechanical system - specimen and piezoelectric element. In this FE analyse the rough estimations of all elastic constants obtained from other independents (as a rule static) experiments were used. Further the amelioration of initial elastic modules was performed. Thereby the target nonlinear functional dependent on all quest modules was minimized by genetic and (or) Levenberg-Marquardt algorithm.

Keywords: *polymeric composite, elastic module, acoustic measurements, eigenmodes, finite element method*

1. Introduction

The dynamic behaviour of carrier composite structures in particular passed through significant elastic strains essentially depends on all material elastic constants [7]. Therefore the problem of a ready components material elastic constant monitoring has the large importance, especially, in aircraft manufacture. Despite of a series of the standards, designed and used for trial of composite materials, the problem of polymeric composite elastic constants identification has not lost urgency. It is stipulated both development of new materials, and use of composite workpieces, which shape does not allow to make samples of a standard configuration. Besides the actually used standards provide an excise of samples from the specially prepared plates, the technique of which manufacture (pull-up of winding, conditions of a curing) can not match to the work piece.

Among sources of rough errors and bad reproducibility of composite elastic modules determination results in static tests [9, 10, 13, 15 - 17] is the nonuniformity of stress distribution in a working area of specimen, stipulated by the testing scheme [9, 13], or numerical instability of a calculation method at experimental data handling, and also the geometrical errors of samples preparation and grabbing of a testing machine [15].

One of utilized techniques of elastic properties investigation is the comparative analysis of a surface displacement field [8, 13, 17], eigenfrequencies and natural modes derived from experimental data and numerical simulations [6, 12, 17], and employment of calculated results elaboration by iterative methods [2, 11, 12], genetic algorithms (GA) [6], etc.

In the present article experimental dynamic techniques for specimens cut out from a ready workpiece (polymeric composite spar of the helicopter main rotor blade) is considered. These techniques complement the early designed by authors' FE-based means [15] for orthotropic composite static tests. The offered dynamic tests include an evaluation of specimen's amplitude-frequency response, determination of fundamental frequencies and vibration modes of specimens both in natural experiments and numerical FE - simulations.

The identification process consists of several stages. In series of static tests are determined all allowable modules. Further a complete matrix of elastic constant is constructed, but some modules specified by approximated values (in particular, interlaminar shear modules). A series of dynamic tests executed in which the periodical excitation in samples and the frequency response is recorded by means of piezoelectric actuators and sensors. Then on basis of early defined modules of composite and experimentally founded eigenfrequencies by means of FE modelling the vibration natural modes, and also the boundaries of the damping factors variation are identified. By combination of FE modelling, GA, and Levenberg-Marquardt method the specification of composite mechanical properties is evaluated.

2. Identification of Piezoelectric Elements Properties

The identification of piezoelectric elements properties is indispensable for an adequate numerical modelling of a compound structure including a composite specimen with sensor and actuator. With this purpose for polarized on width piezoelectric rectangular plate ($14.6 \times 6.8 \times 0.28$ mm) the frequency response of impedance is measured (see Fig. 1).

The mathematical description of dynamic problem is carried out within the framework of the linear theory of electroelasticity. This boundary value problem for electro elastic body $\underline{x} \in V_P$ consists of the differential equations [14]:

$$\sigma_{ij,j} = \rho \ddot{u}_i \; (-\rho \omega^2 u_i) \,, \; i = 1,2,3 \,, \tag{1}$$

$$D_{i,i} = 0 \,, \tag{2}$$

constitutive equations:

$$\sigma_{ij} = c_{ijkl} \, u_{k,l} + e_{ijk} \varphi_{,k} \,, \; D_i = e_{ijk} \, u_{j,k} - \varepsilon_{ij} \, \varphi_{,j} \,, \tag{3}$$

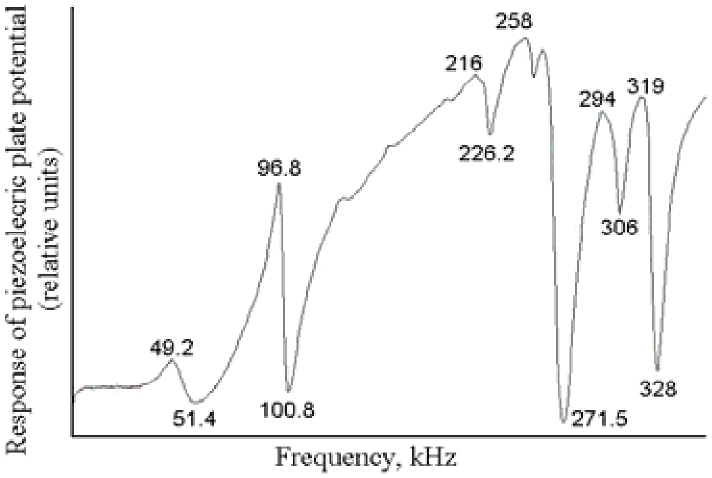


Fig. 1. The frequency response of piezoelectric plate potential (experimental data)

mechanical boundary conditions:

$$t_i = \sigma_{ij} n_j |_S = 0 \qquad (4)$$

and electrical boundary conditions on $S = \bigcup_{m=1}^{M} S_{E_m} \cup S_D$:

$$\varphi |_{S_{E_m}} = \varphi_m^0 = const\,,\ D_n |_{S_D} = D_i n_i |_{S_D} = D_n^e \quad (=0)\,, \qquad (5)$$

where: ρ=7910 kg/m^3 – material density, σ_{ij} – elements of stress tensor, u_i, $u_{i,k}$ – components of displacements and deformations respectively, D – electrostatic induction, c_{ijkl} – stiffness matrix, e_{ijk} – piezoelectric constants tensor, ε_{ij} – electric permeability tensor, t_i – normal stress vector on i – boundary, n_i – boundary unit normal vector, φ – electric potential, S_{Em}, S_D –coated by electrodes and free boundaries respectively, and m – boundary number.

The unknown potential on m – electrode was determined from equation:

$$\int_{E_m} \dot{D}_n dS = I_m\,, \qquad (6)$$

where: I_m – electric current in external circuit.

The FE - simulation of piezoelectric elements was performed in software ACELAN [4]. The FE - meshing is shown in a Fig. 2. The properties of piezoelectric plate can be determined on basis of frequency response (see Fig. 1) and method combining FE and genetic algorithms [1, 3]. However applying this procedure needs identification of oscillations modes, which one will match to splash on the frequency response curve (Fig. 1). The used piezoelectric ceramics composition has matched to properties represented in Tab. 1.

Tab. 1. Properties of sensor and actuator piezoelectric material

Stiffness matrix, GPa					
c_{11}	c_{12}	c_{13}	c_{33}	c_{44}	c_{66}
109	61	54	93	24	$(c_{11}- c_{12})/2$

Piezoelectric and dielectric constants				
e_{31}, $N/(V \cdot m)$	e_{33}, $N/(V \cdot m)$	e_{15}, $N/(V \cdot m)$	$\varepsilon_{11}/\varepsilon_0$	$\varepsilon_{33}/\varepsilon_0$
-4.9	14.9	10.6	820	840

The results of a harmonic analysis in ACELAN are shown in a Fig. 3, on which the dependence of admittance amplitude on excitation frequency is figured. Some unconformity between the graphs of a Fig. 1 and Fig. 3 in the frequencies range 40 - 50 kHz, in which one there are some flexural modes (second flexural mode of a plate along a short edge and first flexural mode in a plane along the longest edge), is stipulated by their excitation at the irregularity of the geometrical shape, polarization and specimen's electrode coating. In numerical experiment at the perfect prism geometry these modes are not piezoelectric.

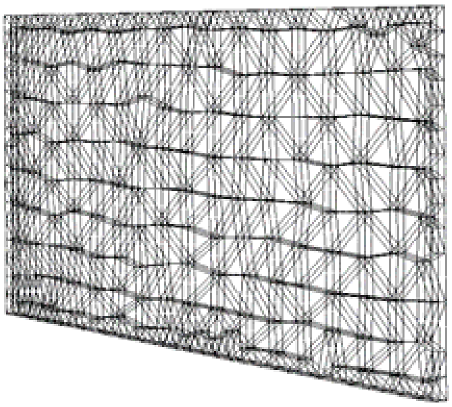


Fig. 2. The finite element meshing of piezoelectric plate

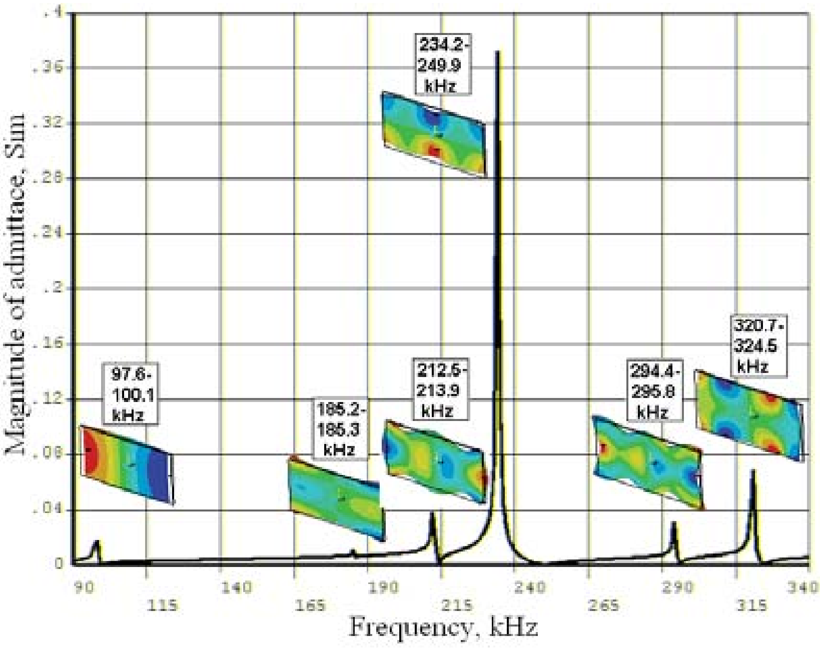


Fig. 3. The frequency response of piezoelectric plate's admittance (FEM - simulation results)

The graphic portrait of displacements amplitude on six foremost oscillations modes are linked with correspondent splashes on the frequency response curve (see Fig. 3). The results of the modal analysis have allowed to identify piezoelectric oscillations modes and to receive improved values of elastic and piezoelectric modules, which one will be utilized further in a problem of composite elastic modules identification.

3. Experimental Setup, Specimens and Results of Measurements

The installation for dynamic tests of mechanically coupled piezoceramic plates and polymeric composite specimen include the ultrasound generator, digital frequency meter, oscilloscope, digital voltmeter, optical meter of the surface point displacement amplitude and specimen header. In experiment the different schemes of piezoelectric elements connection utilized (see. Fig. 4). The inset (a) corresponds to the connection «actuator - sensor», (b) - «actuator - passive element», (c) - «two parallel actuators», (d) - «bimorph actuator».

The polymeric composite material utilized for a specimen's excision is the multilayered laminate obtained by spool of unidirectional tape (prepreg) with a further curing of epoxy resin matrix at heating up in a mold tool. This material used at production of spar for the helicopter main rotor blade. The shape of the sample - parallelepiped ($25.0 \times 14.5 \times 5.5$ - sizes in mm). Unidirectional fiberglass tapes are parallel to major plane of specimen and are located under angles $\pm 30^0$ to a direction of the greater edge (Fig. 5a). At centre of the major plane, are symmetrically attached the piezoelectric plates. The used connection circuits allow exciting symmetric, flexural and other oscillations modes with different efficiency.

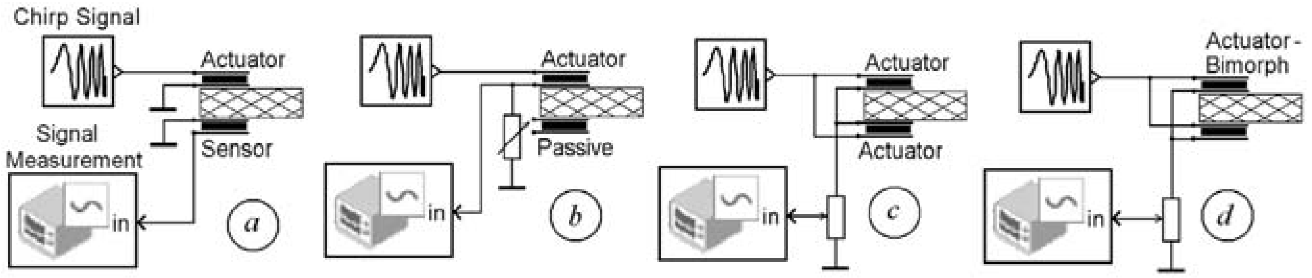
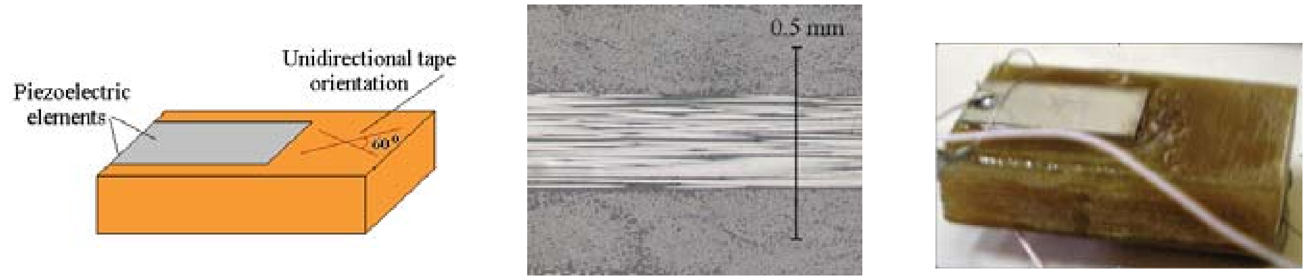


Fig. 4. Electrical circuits for specimen's excitation on different oscillation natural modes



a) b) c)

Fig. 5. Composite specimen for determination of elastic constant by dynamic test:
a) schematic view, b) magnified transversal cross-section normal to largest plane,
c) snapshot of specimen with installed piezoelectric elements

The Fig. 6 depicts a frequency response of a sample oscillations obtained by optical meter of displacements, and Fig. 7 - frequency response of potential on sensor's contact, when the oscillations are excited by actuator. The obtained information on eigenfrequencies will be utilized by identification of oscillations modes corresponding to peaks of frequency response.

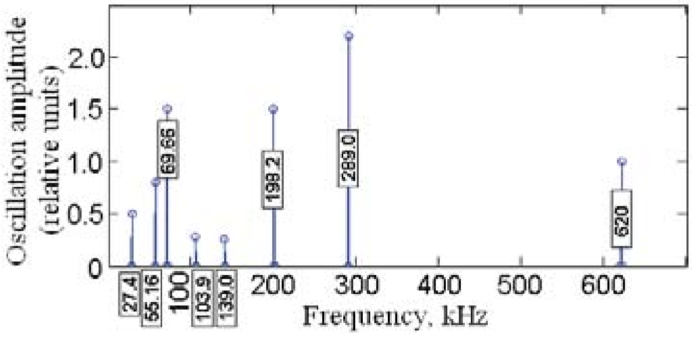


Fig. 6. Frequency response of polymeric composite specimen
(experimental data obtained by displacements optical meter)

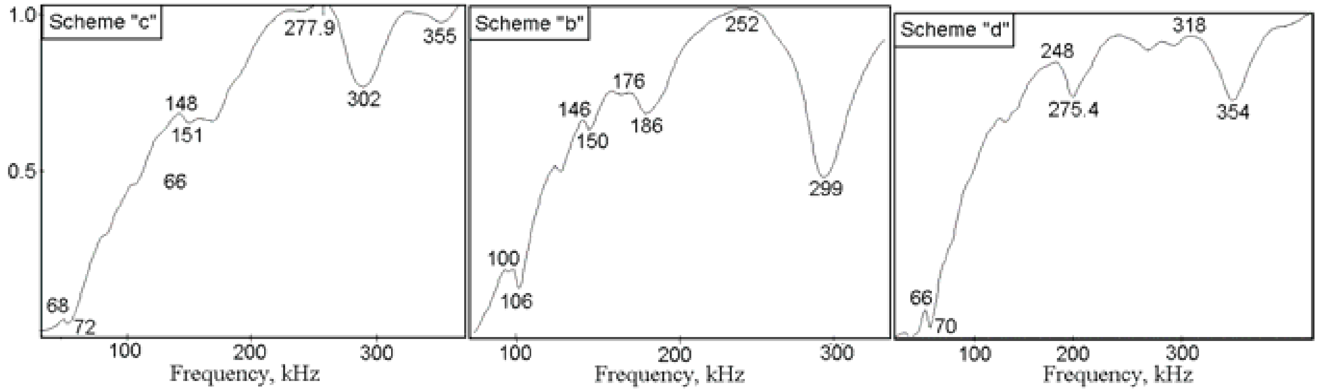


Fig. 7. Frequency response of polymeric composite specimen (experimental data – explanation in text)

4. Finite element models and specimen's natural modes identification

The problem description includes relations (1) - (5) for an electroelastic body, and for $\underline{x} \in V_E$ - elastic body: the equations of motion (1), constitutive relations:

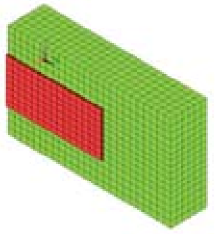
$$\sigma_{ij} = c^{(E)}{}_{ijkl}\, u_{k,l} \quad i = 1,2,3 \tag{7}$$

and mechanical boundary conditions (acting forces):

$$t_i = \sigma_{ij} n_j \,|_S = p_i \ . \tag{8}$$

The components of an elastic tensor in two index matrix notations $C = ((c^{(E)}{}_{rt})),\ \ r,t = 1,2,\dots,6$ have values determined in static tests [15] (see Fig. 8,b).

The simulation of specimen (Fig. 8,a) oscillations was carried out in ACELAN taking into account a damping, both in elastic, and in electroelastic bodies. The purpose of this simulation was the identification of specimen's eigenfrequencies obtained in experiment, and oscillations modes, correspondent to them.



a)

$$C = \begin{pmatrix} 8.378\cdot10^{10} & 3.939\cdot10^{10} & 3.357\cdot10^{10} & & & \\ 3.939\cdot10^{10} & 3.348\cdot10^{10} & 1.899\cdot10^{10} & & 0 & \\ 3.357\cdot10^{10} & 1.899\cdot10^{10} & 2.414\cdot10^{10} & & & \\ & & & 5\cdot10^{9} & 0 & 0 \\ & 0 & & 0 & 1.3\cdot10^{10} & 0 \\ & & & 0 & 0 & 1.5\cdot10^{10} \end{pmatrix}$$

b)

Fig. 8. FE – model of specimen with attached piezoelectric elements (a) and stiffness matrix for used orthotropic composite material (b)

Vibration modes forced by piezoelectric actuator were selected in numerical experiment (harmonic analysis). The numerical calculation results of admittance frequency response (Fig. 9a) and of displacement module of a point on the sample upper surface (Fig. 9b) are submitted.

In the Tab. 2 the results of the modal analysis are presented. The cells of this table contain frequencies of a resonance and anti-resonance, image of the oscillations modes, and numbers of the circuits (Fig. 4), which excite the given oscillations mode. The data of the table specify, that use of the different commutation allow exciting founded by numerical simulation the majority of oscillations modes in a wide frequency range. Note, that at a spectrum there are frequencies with small factor of electromechanical linking (No. 3 and 4 in Tab. 2), which experimental detection needs more sensitive instrumentation, than that was used in the conducted experiments.

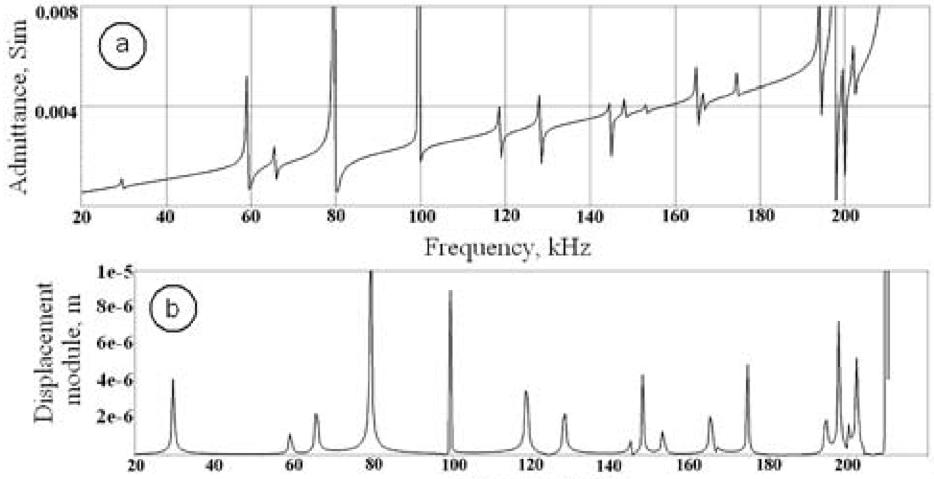
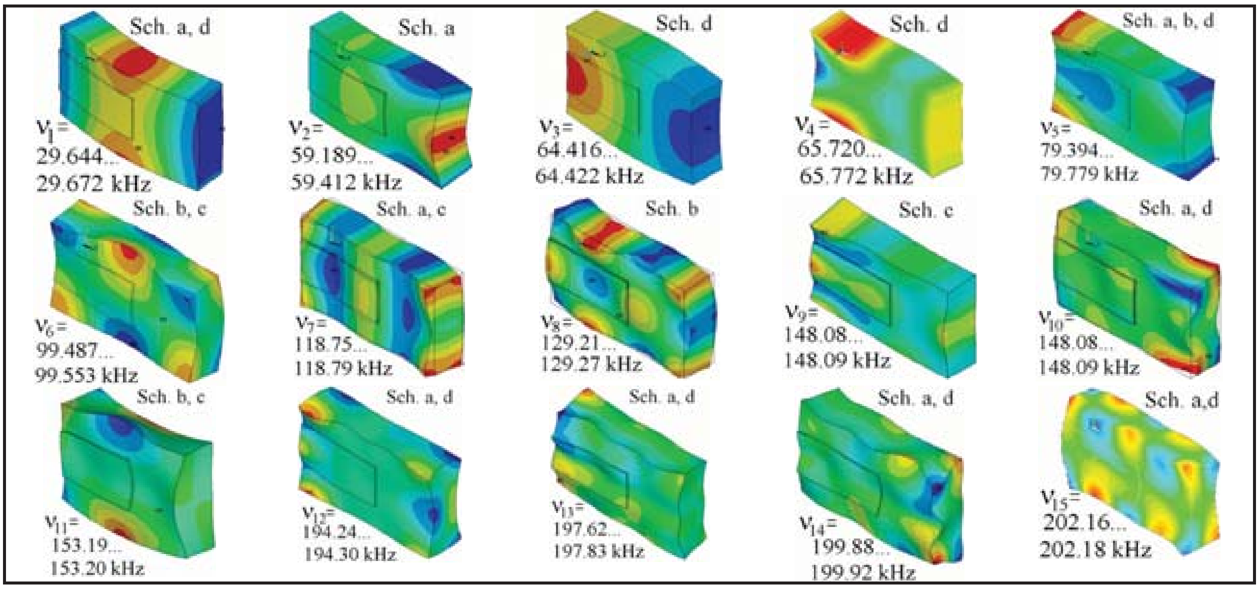


Fig. 9. Frequency response of admittance (a) and the surface point's displacement modulus for studied polymeric composite specimen (simulation results)

Tab. 2. The natural modes of composite specimen's oscillation (simulation results)



5. Procedure of Effective Elastic Modules Improvement

Elastic constants used in FE analysis are obtained in static tests. Their precision is not identical, in particular it concerns to interlaminar shear modules [5, 10, 13, 16]. The improvement of these modules can be carried out by minimization of some nonquadratic functional of a discrepancy between measured and calculated magnitudes. As basic experimental information we used a spectrum of eigenfrequencies and frequency response of potential on piezoelectric plate (Fig. 7). The values of surface points displacement was used in [3, 8, 13] for determination of elastic modules. Unfortunately, for a sample with used shape the exactitude of displacement measurements inaccessible to the available instrumentation was required. Therefore problem of nine modules $C_{ij}\ i,j=1,2,...6$ improvement is formulated as minimization of an objective function

$$G(c_{11},c_{12},c_{13},c_{22},c_{23},c_{33},c_{44},c_{55},c_{66})=\sum_{i=1}^{n}k_i\left(\frac{\omega_i-\psi_i}{\psi_i}\right)^2+\sum_{i=1}^{n}\sum_{k=1}^{m}h_{ik}(a_{ik}-u_{ik})^2\ , \qquad (10)$$

where $\psi_1,...,\psi_n$ - set of measured, and $\omega_1,...,\omega_n$ - set of the FE - simulated eigenfrequencies; k_i and h_{ik} - weighting coefficients; a_{ik} - displacement of k - point, obtained from numerical experiment on the i – natural form; u_{ik} - displacement in the same point obtained as a result of measure. Usage of second summand in (9) eliminates an "intermixing" of eigenfrequencies at consecution of minimization epochs. For this purpose on a specimen's surface the set of reference points was selected (see Tab. 2). These points should be placed near minimum and maximum of oscillations on all observable modes. Thus to maximum amplitude corresponded values $u_{ik}=\pm 1$ (sign there corresponds to a phase shift on π), and to minimum amplitude – the values $u_{ik}=0$. All these points should be accessible to optical amplitude measurement. The minimization of a functional (9) was performed by GA using the "strong selection", the "elite" strategy, Gray's codes [3] and procedure of improvement (searching in narrow area by Levenberg-Marquardt method). The obtained experimental results and numerical calculations for fibreglass reinforced orthotropic composite have shown a good efficiency and reliability of proposed identification method.

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