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MATHEMATICAL AND GRAPHICAL MODELLING OF AEROBATIC FLIGHT TRAJECTORIES

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Abstract

The article presents the current state of knowledge related to mathematical and graphical modelling of aircraft trajectories, particularly those of aerobatic flights. The first part of the article is an introduction to the subject of training and aerobatic flight. In the second chapter of this work, the authors describe the basic definitions related to mathematical modelling, rules for glider aerobatics (rules for the pilots and for the judges) and specification of navigation software, which is using during competitions. In the further part of the article the authors present example possibilities of application of the mathematical modelling tools in General Aviation and describe the possibilities of implementing the obtained models in selected navigation and tracking systems. In the same part of the article there is a description of the AeroSafetyShow Demonstrator PL – an intelligent system of flights supervision and safety (ASSD+PL), which will be using to compare the real flights trajectory and the models build trough the mathematical software. ASSD+PL in also only tracking system using by the judges at glider aerobatic World Championships. The aim of this article is to validate the applicability of the system comparing to the actual aircraft trajectory with the models referred to as 'ideal' for aerobatic purposes.

Keywords: *mathematical models, flight trajectory, aerobatics, flight safety, glider aerobatics*

1. Introduction

Both, during pilot training and in aerobatics, it is extremely important accurately to determine the position of an aircraft in space along with the basic flight parameters. An early detection of even small deviations from the performed task will reduce the level of risk of unwanted incidents. Additionally, having the airspace supervision system, the ground operator will be able to react immediately, which will positively contribute to the quality of pilot training. The need to introduce an intelligent and autonomous system that will control the course of the flight in real time has become a priority. The software allowing tracking of the flights and comparing them with predefined reference models (aerobatic figures) would define a new level of not only the flight operations but also the safety of their performance.

The possibility of the application of a real time supervision and safety system of flight operations that at the same time would be based on model flight trajectories enables an increase in the quality of pilot training and safety of aerobatic operations.

2. Mathematical and graphical modelling of trajectories of an aircraft

The problem of trajectory modelling or the modelling of selected aerobatic figures has been described in literature [10-12, 14-19] from purely mathematical considerations through 2 and 3D models. These descriptions, however, do not treat aerobatic figures as part of the flight in its entirety, nor do they relate to the judge evaluations or perfecting of the art of piloting. In this article, the authors will primarily discuss the competition-related aspect, which will result in an increase in the safety of such competition operations. The aim of this article is to define the need to apply mathematical modelling in general aviation aerobatics.

A mathematical model is a set of rules and relations, based on which, through calculations, one may predict the course of a modelled process. A mathematical model may be mathematical equations describing the process and all relations describing limitations and simplifications [1]. Modelling is all actions aiming at creating a mathematical or physical model [1].

According to the above-mentioned details, one may model all stages and components of the flight. The problem of particular importance is defining the stages and key components of the flight. In this article, the authors will discuss the examples of glider aerobatics.

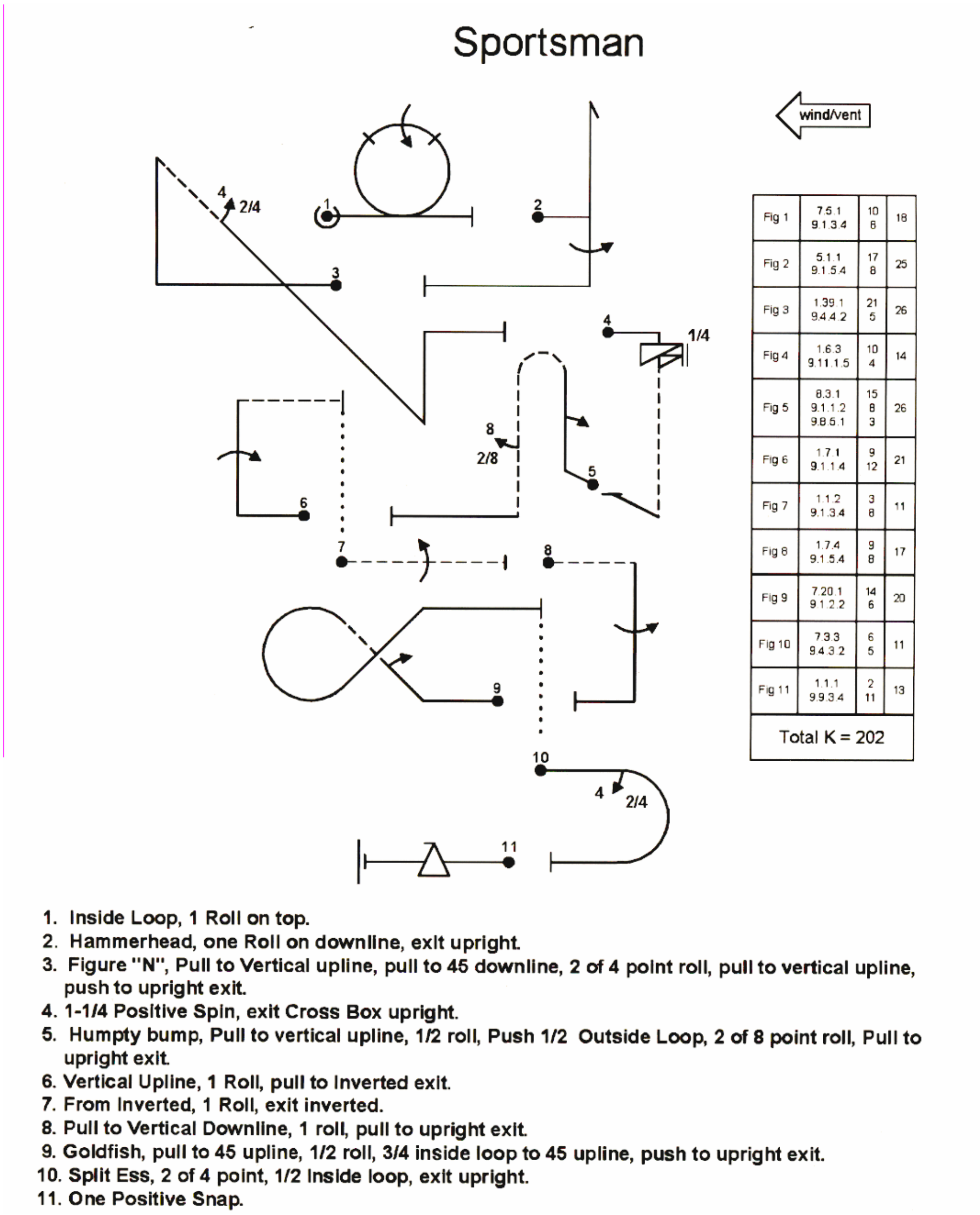

Fig. 1. A sequence of glider aerobatic figures [2]

Figure 1 presents a sequence of aerobatic figures and their names. The entire flight has to be performed by the pilot in a strictly determined aerobatic zone (Fig. 2). This zone is specified by a three-dimensional cube whose length and width are 1000 m each. The bottom boundary of the competition flights is determined on the level of 100 m and the top boundary is 1000 m. For each zone transgression, the pilot receives penalty points. The judges are familiarized with the zone boundaries prior to the start of the competition with the assistance of a test pilot who is not a competitor. Additionally, the zone is defined by horizontal signs on the airstrip.

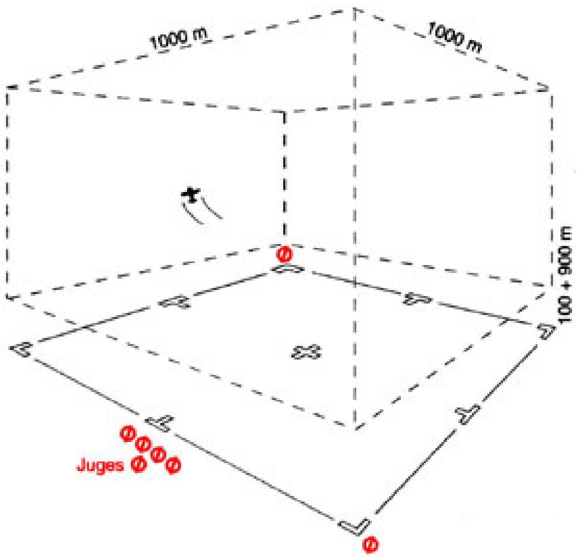

Fig. 2. Aerobatic zone (Box) [3]

The judges evaluate both the quality of the figures and the maintenance of the position in the box [4]. The mathematical and graphical models must not only allow for the parameters of the flight but also refer to a given three-dimensional space, because the judges are not capable of observing every zone transgression that may affect the final score of the contestant.

A tool in general aviation that enables tracking of an aircraft and identifying basic flight parameters is the AeroSafetyShow system (ASSD) developed by a scientific consortium composed of Żelazny, Wojciech Krupa and Poznan University of Technology financed by the EU. The ASSD judge-assisting software system is currently the only one acknowledged and used in European level competitions [5].

The functionalities of the ASSD software and the developed and implemented mathematical models for individual stages of the flight and figures allow an automatic evaluation of the competition flights, thus providing objectivism and reliability. For the creation of three-dimensional models of individual figures, the MATLAB mathematical programming tool can be applied. Fig. 3 presents an example simulated flight trajectory. Such a trajectory not only includes individual figures performed by the pilot but also the maintenance of position within the box.

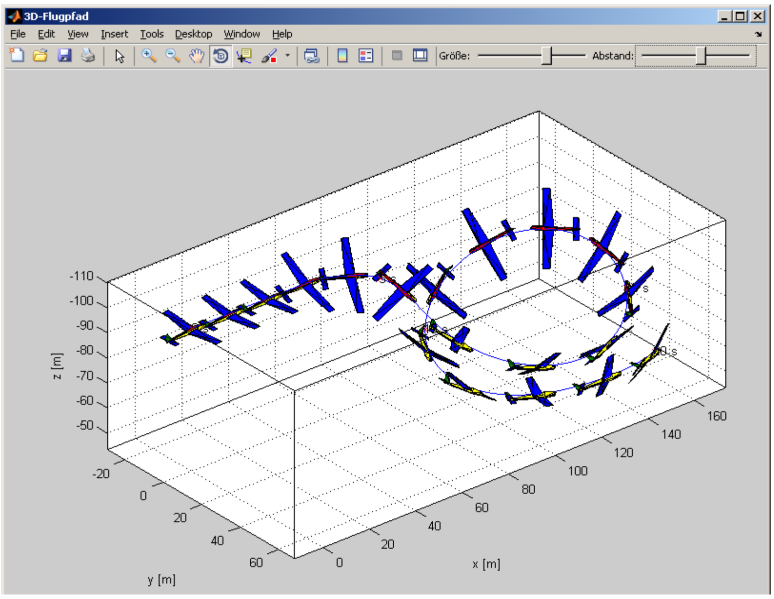

Fig. 3. A three-dimensional model developed in the MATLAB environment [6]

The model may be implemented in the ASSD software and used for comparison with the report obtained from the application. At the first stage of the development of the flight evaluation software, research is planned based on the reports printed after the sequence of figures during the competition. Fig. 4-6 present the flight trajectory. The reports in this form are printed automatically after the flight has been completed. Additionally, the images in Fig. 4, 5 and 7 are variants available for the ground operator/judge that can be switched arbitrarily at any moment.

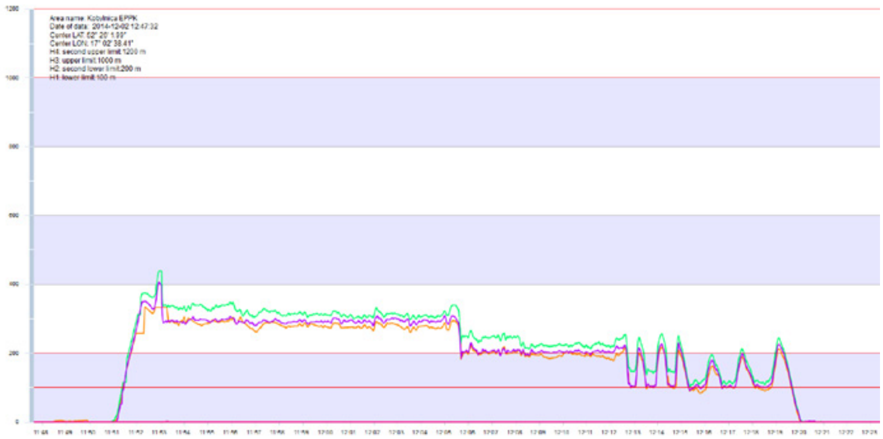

Fig. 4. Altitude curve of the pilot-competitor

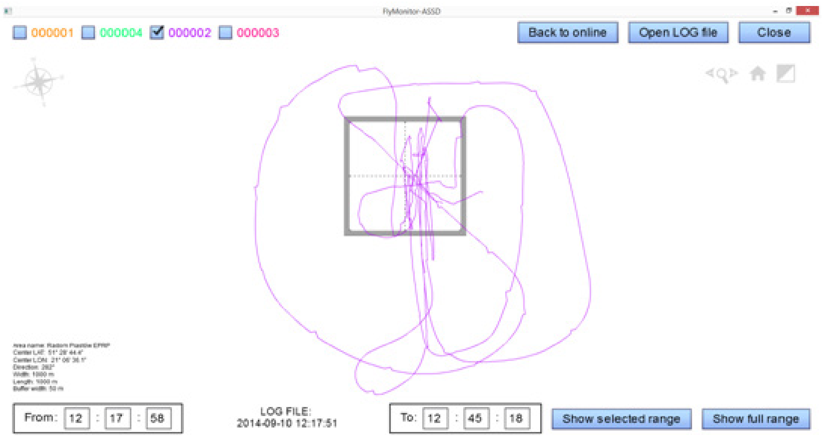

Fig. 5. Aircraft trajectory on the horizontal plane

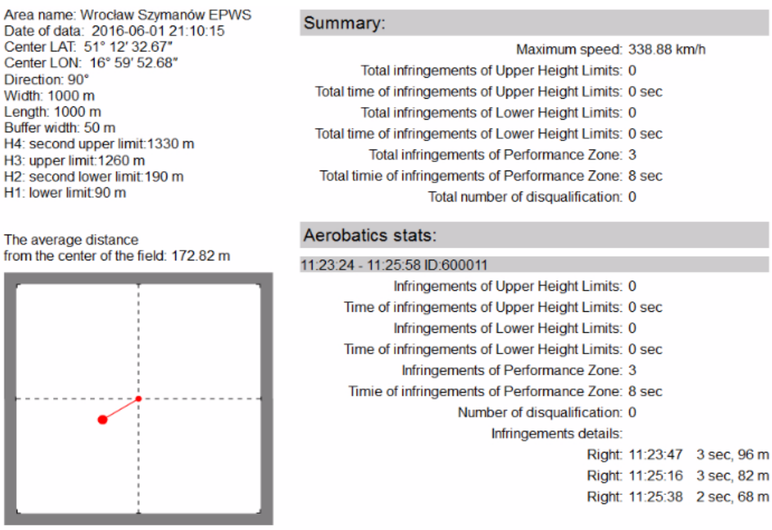

Fig. 6. Collective results of the pilot-competitor related to his position in the box

The views in Fig. 5 and 4 have an additional function. The operator may choose any number of aircraft for comparison at the same time. The flight trajectory of each of the aircraft will then be marked with a different colour and the judge will be able directly to compare two or more flights. This function may turn out useful, particularly if any doubts arise or in case of complaints of the competitors performing the same figures.

Figure 7 presents the main view of the ASSD interface. On the left of the screen, there is a list of all aircraft fitted with the mobile transmitters at a given time, sending a signal to the base station, between which the operator may freely switch for tracking (a visible trail made by the aircraft). On the right of the screen, the altitude curve of a selected competitor is shown. Any transgressions of the box top and bottom limits are marked red. An additional signal for the operator/judge related to the unallowed position (including the transgression of the left right front and rear sides of the box) is the red illumination of a given wall of the box. The trail marked orange created in real time by the aircraft defines the contours of the performed aerobatic figure.

Fig. 7. Real time recording of the pilot-competitor position in the box

At this time, the ASSD system independently cannot evaluate the correctness of the performed aerobatic figure. The evaluations for individual stages of the flight are made following the judges' subjective decisions that cannot be validated in an appropriate way. Such a situation has led to an attempt to develop an independent system based on data obtained through the ASSD software and simulations created in MATLAB.

3. The application of the mathematical and graphical models in General Aviation

The works in the discussed field were taken up because of the lack of systems allowing on-going flight supervision in the sector of general aviation that at the same time would allow evaluating the correctness of the performed figures [7]. Additionally, collaboration with experts within the AeroSafetyShow project has proven that a need for the development of such software appeared recently. Consultations with aerobatic pilots from the National Representation also confirm this thesis. It is noteworthy that, in military aviation, a similar method of aerobatic flight evaluation was developed, based on creating graphical models of flight trajectories with the help of flight recorders, which enables investigating the deviations from correct aircraft manoeuvres [8, 9]. Mathematical modelling of figures and flight trajectories has been discussed in world literature [10-12, 14-19], which indicates substantial interest in this scientific aspect. However, there are no works strictly related to modelling of aerobatic figures with a view to its implementation in the supervision system in order to form autonomous and complete flight evaluation software.

The application of mathematical and graphical modelling in the airspace supervision system will enable not only objective judge evaluation. On-going monitoring of the flight trajectory and flight parameters will allow ground operators directly to react to the course of the performed task. Such standards of supervision over the performed flights (aerobatic in particular) will improve the quality of the pilot training and, consequently, increase the safety of the flight operations.

When analysing the applicability of modelling of aerobatic figures for evaluation and pilot training purposes, it should be noted that a compilation of all available aerobatic figures should be created in the first place. Such a compilation should be a part of the Aresti catalogue, which defines aerobatic figures in a two-dimensional plane [13]. The content of the catalogue presents, *inter alia*, sets of figures grouped into families:
Family 1 Lines and Angles,
Family 2 Turns and Rolling Turns,
Family 3 Combinations of Lines,
Family 4 Spins (Not in Use),
Family 5 Stall Turns,
Family 6 Tail Slides,
Family 7 Loops and Eights,
Family 8 Combinations of Lines, Angles and Loops,
Family 9 Rolls and Spins [13].

At the initial stage, for each family, a reference figure will be selected. The modelling of the figure will include the critical moments (angles, tilts, velocity) and spatial frames. For this reason, the most suitable tool for the model design appears to be the MATLAB Simulink environment, as has been described in the previous chapter. Upon development of the set of models, a new catalogue of aerobatic figures will be created reflecting the figures in a spatial Cartesian system.

The developed models will then be compared with reports and trajectories obtained through the ASSD application. This will allow determining the deviations from the correct trajectory. The quality of the performed aerobatic figures and the position in the box was maintained. The final effect of the works will be the implementation of the models in the ASSD software, which will consequently result in a complete and autonomous system for the betterment of the aerobatic pilot training.

Additionally, the authors consider an extension of the software by a system summing the judge scores that currently being developed at Poznan University of Technology based on the FAI and CIVA regulations [4].

4. Conclusions

Objective evaluation of aerobatic flights is an important issue not only for the competitors but also for the instructors and judges. Owing to the application of methods of mathematical programming, it will be possible to create a catalogue of spatial aerobatic figures constituting the components of the competition flights. The implementation of such computer simulations in an appropriate navigation system will be an achievement supporting pilots at all stages of their professional advancement. The system will operate in real time, which will not only enable an evaluation of the flights but also ensure the upkeep of high safety standards, which was, thus far uncommon in general aviation.

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