

DETERMINATION OF THE MATERIAL CONSTANTS OF COMPOSITE MATERIALS ON THE EXAMPLE OF MODIFIED WOOD

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Abstract

Homogenization is the transition from the level of microscopic heterogeneity to a homogeneous macroscopic level. The analysis of the value of equivalent parameters and their variability requires prior determination of the influence of microstructure on the values of these parameters [15]. Wood is a composite material and has a layered structure in the form of alternate layers of soft and hard wood. One of the main issues was the determination of constants materials of individual layers of natural wood and modified and then equivalent material constants of natural and modified wood. The material constants of single layers and the material constants of composite were determined on the basis of experimental studies. For this purpose, a homogenization method has been used to determine equivalent material constants on the basis of material constants of single layers of soft and hard wood [1, 2, 14]. A representative cell consisting of a softwood and hardwood layer has been isolated from the sample-measuring portion. On the basis of this cell have been developed mathematical model of equivalent material constants. A sample consisting of two layers was subjected to an even stretch in the direction of the axes x_1 , x_2 , x_3 . The equivalent material constants have been defined by using equilibrium conditions, geometric conditions, and Hooke's generalized law. Each wood component on the micro-level is homogeneous, continuous with its constitutive equation, conservation laws, and boundary conditions at the boundary of the separation of components. The equivalent material constants of natural and modified wood have been determined using the homogenization method [12, 13]. The results obtained from the research and the results obtained from the calculations are very similar.

Keywords: modified wood, polymethylmethacrylate, homogenization, equivalent material constants

1. Description of studies

The research was conducted on natural and modified pinewood. The modification of wood consists of 2 stages: the impregnation of the wood with methyl methacrylate and the polymerization process. The basic purpose of wood modification is to increase the mechanical properties. The detailed course of the modification process is presented in the works [3 ÷ 9]. Wood is an orthotropic material composed of alternating soft and hard layers. One of the main issues was the determination of materials constants of natural and modified wood and materials constants of individual layers of natural and modified wood.

The materials constants of examined materials and of single layers were determined on the basis of the experimental studies. The use of a homogenization method to determine the equivalent material constants on the basis of material constants of single layers of soft and hard wood was the fundamental issue. [1, 2, 14].

The method and course of the studies concerning the determination of materials constants of wood were conducted according to current standards and presented in the works [11-13]. The results of material constants of natural wood K0.0 and modified wood K0.56 are presented in Tab. 1 item a). Numbers 0 or 0.56 denote the amount of kg of polymethylmethacrylate / 1 kg of dry wood [11].

The homogenization method is used to describe the properties of rocks, reinforced ground, reinforced concrete, bone tissue and human muscles [15]. The homogenization method has also been used to describe the properties of a composite such as wood. The determination of individual wood layers properties is a complex issue due to the nature of the fibres.

A detailed description of the research is presented in the works [1, 2, 11-14]. The test results of material constants of single layers are presented in the descriptions [11-13].

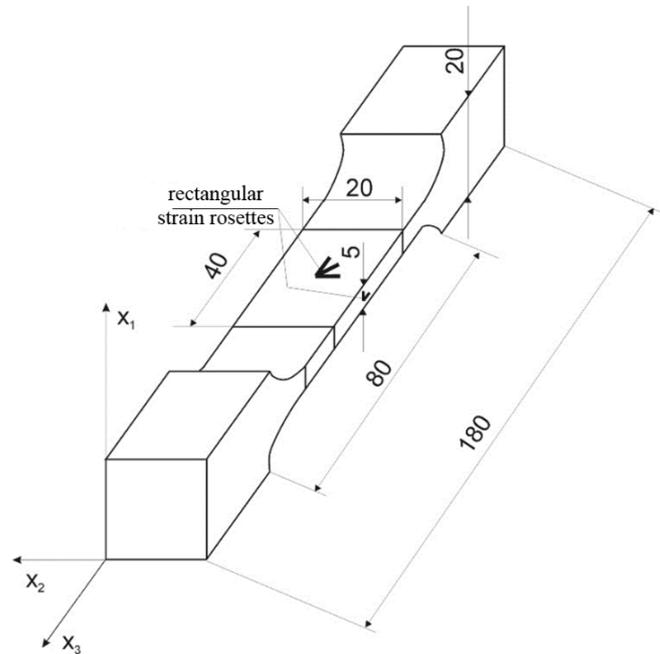


Fig. 1. Geometry of the sample, dimensions and anatomical directions x_1 – radial direction, x_2 – tangent direction, x_3 – longitudinal direction

Figure 1 shows the shape and dimensions of the samples for determining material constants of single layers of soft and hard wood. The sample axes were aligned with the direction of the fibres and were perpendicular to the fibres. Due to the limited transverse dimensions of the wood trunk, the measuring parts of samples were made separately and then glued to the gripping parts [10, 12, 13]. The method of determining equivalent material constants (so called material constants determined by the homogenization method) is presented on the samples which geometry is shown in Fig. 2.

In this sample, the measuring part consists of alternating layers of soft and hard wood. The method of equivalent material constants wood is based on the theory of elasticity [14].

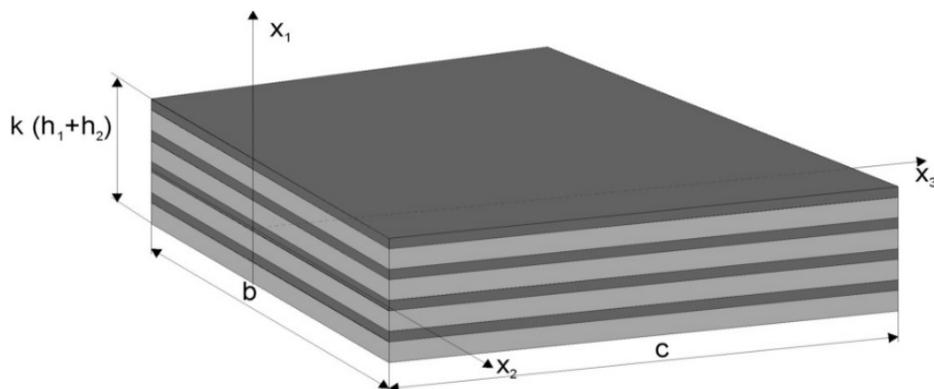


Fig. 2. The geometry of the measuring part of the sample (see Fig. 1): c – length of the measuring part, b – measurement part width, k – measurement part thickness

A representative cell consisting of a softwood and hardwood layer (Fig. 3) was isolated from the sample-measuring portion of k thickness (Fig. 2). On the basis of this cell, a mathematical model of equivalent material constants was determined. A sample consisting of two layers was subjected to an even stretch in the direction of the axes x_1, x_2, x_3 . The equivalent material constants have been defined by using force equilibrium conditions, geometric conditions and Hooke's generalized law. The equilibrium conditions, geometric conditions as well as physical conditions were used to determine material constants.

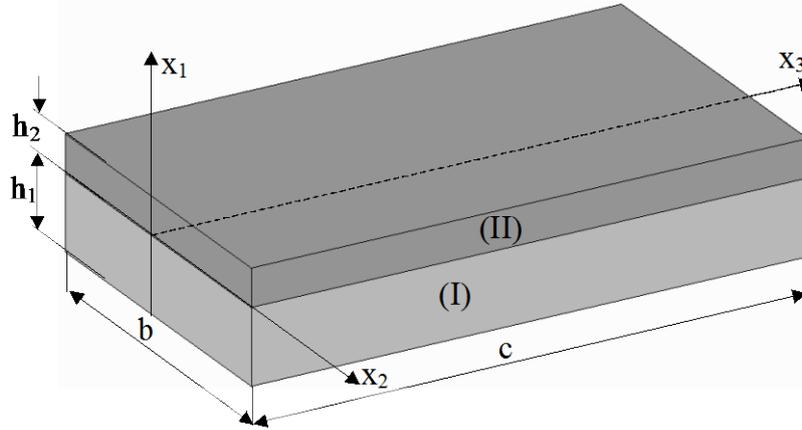


Fig. 3. Sample measuring part oriented in a rectangular coordinate system: I – soft layer, II – hard layer

Taking into account the sample dimensions from Fig. 3, the cross-section F , being transverse to x_3 axis can be expressed by the equation: $F = F_1 + F_2$ or $b(h_1 + h_2) = b h_1 + b h_2$. Dimensional ratio was adopted

$$n = \frac{h_2}{h_1} = 0.5, \quad (1)$$

hence $F = F_1(1 + n)$.

2. Calculation model

The results show that there is a large scattering of values obtained from the study of single layers of wood. Therefore, the average stresses have been accepted in the areas under consideration. $\tilde{\sigma}_{22}^I, \tilde{\sigma}_{22}^{II}$ and $\tilde{\sigma}_{33}^I, \tilde{\sigma}_{33}^{II}$.

Average values of tangential stresses are zero. In the equations, the upper indices refer to softwood and hardwood areas, the absence of an upper index means that the value determines the properties of the homogeneous sample. The components stress in the axle direction x_1 in both areas is equal and equivalent to the component occurs in the homogeneous sample with uniform load.

$$\sigma_{11}^I = \sigma_{11}^{II} = \sigma_{11}, \quad (2)$$

Therefore, the resulting forces in the direction of the axis $x_2 x_3$ must be equivalent:

$$h_1 b \tilde{\sigma}_{22}^I = -h_2 b \tilde{\sigma}_{22}^{II}, \quad \tilde{\sigma}_{22}^I = -n \tilde{\sigma}_{22}^{II}, \quad (3)$$

$$h_1 c \tilde{\sigma}_{33}^I = -h_2 c \tilde{\sigma}_{33}^{II}, \quad \tilde{\sigma}_{33}^I = -n \tilde{\sigma}_{33}^{II}. \quad (4)$$

If a geometric condition is used in the direction of the x_1 axis of the deformation surface:

$$h_1 \varepsilon_{11}^I + h_2 \varepsilon_{11}^{II} = (h_1 + h_2) \varepsilon_{11} \quad \text{or} \quad \varepsilon_{11}^I + n \varepsilon_{11}^{II} = (1 + n) \varepsilon_{11}. \quad (5)$$

For a heterogeneous and a homogeneous sample, the increase in the cross-sectional area to the axis x_2 and x_3 must be the same. Based on this condition, the following equations were obtained:

$$h_1 b (\varepsilon_{11}^I + \varepsilon_{22}^I) + h_2 b (\varepsilon_{11}^{II} + \varepsilon_{22}^{II}) = (h_1 + h_2) b (\varepsilon_{11} + \varepsilon_{22}),$$

$$h_1 c (\varepsilon_{11}^I + \varepsilon_{33}^I) + h_2 c (\varepsilon_{11}^{II} + \varepsilon_{33}^{II}) = (h_1 + h_2) c (\varepsilon_{11} + \varepsilon_{33}).$$

Introduction of equations (1) and (5) into these equations results in:

$$\begin{aligned} \varepsilon_{22}^I + n \varepsilon_{22}^{II} &= (1+n) \varepsilon_{22}, \\ \varepsilon_{33}^I + n \varepsilon_{33}^{II} &= (1+n) \varepsilon_{33}. \end{aligned} \quad (6)$$

Hooke's generalized law has been used to express the relationships between the components of strain and stress:

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{21} \\ \varepsilon_{32} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{21} & s_{22} & s_{23} & 0 & 0 & 0 \\ s_{31} & s_{32} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{55} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{21} \\ \sigma_{32} \end{bmatrix}. \quad (7)$$

After taking into account the physical relationship (7) and equations (3), (4) and (5) the following equations were obtained:

$$s_{11}^I \sigma_{11} + s_{12}^I \tilde{\sigma}_{22}^I + s_{13}^I \tilde{\sigma}_{33}^I + n (s_{11}^{II} \sigma_{11} + s_{12}^{II} \tilde{\sigma}_{22}^{II} + s_{13}^{II} \tilde{\sigma}_{33}^{II}) = (1+n) s_{11} \sigma_{11}, \quad (8)$$

$$(s_{21}^I \sigma_{11} + s_{22}^I \tilde{\sigma}_{22}^I + s_{23}^I \tilde{\sigma}_{33}^I) + n (s_{21}^{II} \sigma_{11} + s_{22}^{II} \tilde{\sigma}_{22}^{II} + s_{23}^{II} \tilde{\sigma}_{33}^{II}) = (1+n) s_{21} \sigma_{11}, \quad (9)$$

$$(s_{31}^I \sigma_{11} + s_{32}^I \tilde{\sigma}_{22}^I + s_{33}^I \tilde{\sigma}_{33}^I) + n (s_{31}^{II} \sigma_{11} + s_{32}^{II} \tilde{\sigma}_{22}^{II} + s_{33}^{II} \tilde{\sigma}_{33}^{II}) = (1+n) s_{31} \sigma_{11}. \quad (10)$$

Stress components $\tilde{\sigma}_{22}^I, \tilde{\sigma}_{33}^I$ and $\tilde{\sigma}_{22}^{II}, \tilde{\sigma}_{33}^{II}$ will be determined depending on the σ_{11} after the system of equations (8), (9) and (10) will be resolved.

If equation (7) will be substituted designated values, then it will get an equation independent of σ_{11} . In this equation, however, there are three unknown material constants s_{11}, s_{21}, s_{31} of homogeneous material. In the both layers was uniformly strain $\varepsilon_{33}^1 = \varepsilon_{33}^2$ if the sample was axially loaded x_3 . In these layers will be stresses $\sigma_{33}^1, \sigma_{33}^2$ and stress components in the direction of the x_2 axis, the mean values of which are determined accordingly σ_{22}^1 and σ_{22}^2 . The resulting forces in the axle direction have been determined based on the geometrical dimensions of the sample (Fig. 1). These forces have to balance, that is to say, must be fulfilled dependence:

$$h_1 b \sigma_{22}^I + h_2 b \sigma_{22}^{II} = 0, \quad \sigma_{22}^I = -n \sigma_{22}^{II}. \quad (11)$$

From the sum of the force projections on the axis x_3 the equation was obtained:

$$\sigma_{33}^I A_1 + \sigma_{33}^{II} A_2 = \sigma_{33} A \quad (12a)$$

or

$$\sigma_{33}^I + n \sigma_{33}^{II} = \sigma_{33} (1+n). \quad (12b)$$

On the basis of geometric conditions in the direction of the axis x_3 the following equation was obtained:

$$\varepsilon_{33}^I = \varepsilon_{33}^{II} = \varepsilon_{33}$$

and taking into account physical dependencies:

$$s_{32}^I \tilde{\sigma}_{22}^I + s_{33}^I \sigma_{33}^I = s_{33} \sigma_{33}, \quad s_{32}^{II} \tilde{\sigma}_{22}^{II} + s_{33}^{II} \sigma_{33}^{II} = s_{33} \sigma_{33},$$

however, after using the dependence (10), the equations for the stress components in the axial direction x_3 were obtained:

$$\sigma_{33}^I = (s_{33} \sigma_{33} + n s_{32}^I \tilde{\sigma}_{22}^{II}) \frac{1}{s_{33}^I}, \quad \sigma_{33}^{II} = (s_{33} \sigma_{33} - s_{32}^{II} \tilde{\sigma}_{22}^{II}) \frac{1}{s_{33}^{II}}. \quad (13)$$

By substituting the expression (13) into equation (12), the $\tilde{\sigma}_{22}^{II}$ was determined, depending on the σ_{33} :

$$\tilde{\sigma}_{22}^{II} = \sigma_{33} \left[(n+1) - s_{33} \left(\frac{1}{s_{33}^I} + \frac{n}{s_{33}^{II}} \right) \right] \left[n \left(\frac{s_{32}^I}{s_{33}^I} - \frac{s_{32}^{II}}{s_{33}^{II}} \right) \right]^{-1}, \quad (14)$$

which can be written in the form:

$$\tilde{\sigma}_{22}^{II} = \sigma_{22a}^{II} + \sigma_{22b}^{II}, \quad (15)$$

where:

$$\sigma_{22a}^{II} = \sigma_{33} (n+1) \left[n \left(\frac{s_{32}^I}{s_{33}^I} - \frac{s_{32}^{II}}{s_{33}^{II}} \right) \right]^{-1},$$

$$\sigma_{22b}^{II} = -\sigma_{33} s_{33} \left(\frac{1}{s_{33}^I} + \frac{n}{s_{33}^{II}} \right) \left[n \left(\frac{s_{32}^I}{s_{33}^I} - \frac{s_{32}^{II}}{s_{33}^{II}} \right) \right]^{-1}.$$

Analogously presented σ_{33}^I and σ_{33}^{II}

$$\sigma_{33a}^I = n \frac{s_{32}^I}{s_{33}^I} \sigma_{22a}^{II}, \quad \sigma_{33b}^I = \left(\sigma_{33} + n \frac{s_{32}^I}{s_{33}^I} \sigma_{22b}^{II} \right) \frac{s_{33}}{s_{33}^I},$$

$$\sigma_{33a}^{II} = -\frac{s_{32}^{II}}{s_{33}^{II}} \sigma_{22a}^{II}, \quad \sigma_{33b}^{II} = \left(\sigma_{33} - \frac{s_{32}^{II}}{s_{33}^{II}} \sigma_{22b}^{II} \right) \frac{s_{33}}{s_{33}^{II}}.$$

In the individual areas, there are the following components of stress $[\sigma_i^I]$, $[\sigma_i^{II}]$, $[\sigma_i]$, where:

$$[\sigma_i^I] = \begin{bmatrix} 0 \\ -n \sigma_{22a}^{II} \\ \sigma_{33a}^I \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -n \sigma_{22b}^{II} \\ \sigma_{33b}^I \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad [\sigma_i^{II}] = \begin{bmatrix} 0 \\ \sigma_{22a}^{II} \\ \sigma_{33a}^{II} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \sigma_{22b}^{II} \\ \sigma_{33b}^{II} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad [\sigma_i] = \begin{bmatrix} 0 \\ 0 \\ \sigma_{33} \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

on the basis of which the elastic energy can be calculated. The sum of the elastic energy from the areas of the heterogeneous sample must be equal to the energy in the homogeneous sample

$$\frac{1}{2}V^I \sum_{i=2}^{i=3} \sum_{k=2}^{k=3} s_{ik}^I \sigma_i^I \sigma_k^I + \frac{1}{2}V^{II} \sum_{i=2}^{i=3} \sum_{k=2}^{k=3} s_{ik}^{II} \sigma_i^{II} \sigma_k^{II} = \frac{1}{2}(V^I + V^{II}) s_{33} \sigma_{33} \sigma_{33}, \quad (16a)$$

or

$$\sum_{i=2}^{i=3} \sum_{k=2}^{k=3} s_{ik}^I \sigma_i^I \sigma_k^I + n \sum_{i=2}^{i=3} \sum_{k=2}^{k=3} s_{ik}^{II} \sigma_i^{II} \sigma_k^{II} = (1+n) s_{33} \sigma_{33} \sigma_{33}. \quad (16b)$$

In each part of the expression above, there is σ_{33} to square. It can be assumed that this equation does not depend on this component of stress, but there is an unknown coefficient s_{33} . On the left side of the equation, it occurs in the stress components having the index “b”. By minimizing the equation with respect to this factor, it can be determined. By comparing the increase in length in the direction of the axis x_1 for a heterogeneous and homogeneous sample, it was obtained:

$$h_1 \varepsilon_{11}^I + h_2 \varepsilon_{11}^{II} = (h_1 + h_2) \varepsilon_{11} \quad \text{or} \quad \varepsilon_{11}^I + n \varepsilon_{11}^{II} = (1+n) \varepsilon_{11}. \quad (17)$$

After the use of physical dependencies, the above relationship can be formed as:

$$s_{12}^I \tilde{\sigma}_{22}^I + s_{13}^I \sigma_{33}^I + n \left(s_{12}^{II} \tilde{\sigma}_{22}^{II} + s_{13}^{II} \sigma_{33}^{II} \right) = (1+n) s_{13} \sigma_{33}. \quad (18)$$

Since the coefficient s_{33} has been calculated, all the components in the left-hand side of the equation can be calculated depending on the σ_{33} . This equation will be independent of σ_{33} and the only unknown s_{13} on its right will be determined. For a heterogeneous and a homogeneous sample, the increase in the cross-sectional area to the axis x_3 must be the same. Based on this condition, the equation was obtained

$$h_1 b \left(\varepsilon_{11}^I + \varepsilon_{22}^I \right) + h_2 b \left(\varepsilon_{11}^{II} + \varepsilon_{22}^{II} \right) = (h_1 + h_2) b (\varepsilon_{11} + \varepsilon_{22}),$$

however, after taking into account (1) and (17), equations are obtained:

$$\varepsilon_{22}^I + n \varepsilon_{22}^{II} = (1+n) \varepsilon_{22} \quad (19)$$

and after taking into account physical dependencies, the following equation was obtained:

$$s_{22}^I \tilde{\sigma}_{22}^I + s_{23}^I \sigma_{33}^I + n \left[s_{22}^{II} \tilde{\sigma}_{22}^{II} + s_{23}^{II} \sigma_{33}^{II} \right] = (1+n) s_{23} \sigma_{33},$$

on the basis of which, an unknown coefficient that was set on the right will be determined.

If the sample is loaded in the direction of the axis x_2 , then values s_{22} , s_{12} and s_{31} can be determined. These values are derived from the equations that were determined for the axial in the direction load x_3 by the corresponding exchange of indices. The condition of equality $s_{23} = s_{32}$ will allow to assess the correctness of the solution. From the designated s_{22} , s_{33} , s_{12} , s_{13} , s_{23} I the coefficient s_{11} can be calculated from the equation. The shear modulus G were determined by considering the load of the layers with force pairs in a plane perpendicular to the direction x_3 x_2 and for the parallel connection and the force pair in the plane perpendicular to the direction for the serial connection (Fig. 1).

Serial system

$$G_{23} = \frac{G_{23}^I \cdot G_{23}^{II} (1+n)}{G_{23}^{II} + n \cdot G_{23}^I} \quad (20)$$

and parallel system:

$$G_{12} = \frac{G_{12}^I + G_{12}^{II} \cdot n}{1 + n}, \quad (21)$$

$$G_{13} = \frac{G_{13}^I + G_{13}^{II} \cdot n}{1 + n}. \quad (22)$$

3. Research results

The equivalent material constants of natural and modified wood were calculated using the homogenization method. Calculations were made on the basis of the samples test results.

The values of the equivalent material constants that have been calculated were verified using the material constants obtained in the experimental studies.

Subsequently, a quantitative analysis of the influence of the values of the soft and hard wood components on the value of substitute parameters has been made using equations (8-22).

The value of equivalent material constants for natural and modified wood is presented in Tab. 1.

Tab. 1. The material constants natural wood K0.0 and modified wood K0.56 a) based on the test, b) specified by homogenization method

a) The material constants natural wood K0.0 and modified wood K0.56 based on the test			b) The material constants natural wood K0.0 and modified wood K0.56 specified by homogenization method		
Material Constant	K0.0	K0.56	Material Constant	K0.0	K0.56
$E_R = E_1$, GPa	2.15	5.09	$E_R = E_1$, GPa	1.98	4.99
$E_T = E_2$, GPa	2.35	5.12	$E_T = E_2$, GPa	2.25	5.13
$E_L = E_3$, GPa	11.85	16.45	$E_L = E_3$, GPa	11.06	16.86
$G_{TL} = G_{23}$, GPa	0.72	2.57	$G_{TL} = G_{23}$, GPa	0.72	2.29
$G_{LR} = G_{31}$, GPa	1.04	2.34	$G_{LR} = G_{31}$, GPa	1.00	2.29
$G_{RT} = G_{12}$, GPa	0.91	1.94	$G_{RT} = G_{12}$, GPa	0.86	1.71
$\nu_{RL} = \nu_{13}$	0.06	0.08	$\nu_{RL} = \nu_{13}$	0.05	0.06
$\nu_{TL} = \nu_{23}$	0.07	0.05	$\nu_{TL} = \nu_{23}$	0.06	0.07
$\nu_{LT} = \nu_{32}$	0.35	0.18	$\nu_{LT} = \nu_{32}$	0.30	0.23
$\nu_{RT} = \nu_{12}$	0.58	0.34	$\nu_{RT} = \nu_{12}$	0.66	0.29
$\nu_{TR} = \nu_{21}$	0.66	0.35	$\nu_{TR} = \nu_{21}$	0.75	0.30
$\nu_{LR} = \nu_{31}$	0.32	0.28	$\nu_{LR} = \nu_{31}$	0.30	0.23

4. Analysis and conclusions

The comparison of the equivalent material constants values for natural and modified wood obtained based on experimental studies and those calculated by the homogenization method (Tab. 1) show that the homogenization method is suitable for wood composites. The difference between the values determined experimentally and the calculated ones does not exceed 15%.

The slight difference in results obtained from the tests and based on the homogenization method results primarily from the discrepancy of values for single layers of fibres. Test specimens were taken from various sites of tree trunk in both length and cross section. The wood on the cross-section has jars characterizing the wood in terms of age, which reflects the strength properties. Therefore, there are discrepancies in the results obtained experimentally and as a result of calculations.

The homogenization method can be used to describe the properties of other layered composite materials.

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