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ANALYTICAL AND NUMERICAL IDENTIFICATION OF LAMB WAVES MODES FOR HYBRID COMPOSITES

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Abstract

The current work is devoted to the problem of analytical and numerical identification of fundamental elastic waves' modes, namely symmetric mode S_0 and antisymmetric mode A_0, in the case of hybrid composite. The investigated material consists of one layer made of aluminum alloy Pa38 and six layers made of glass fabric/epoxy resin. At the very beginning, the dispersion curves are determined with the use of stiffness matrix method. The calculated values of phase velocities are verified by numerical simulation. The semi – analytical finite element method is applied. Next, the numerical simulations of elastic waves propagation are performed. In the studied model, the plane state of strain is assumed. These simulations are carried out with the use of finite element method. The excitation signal is a sine wave modulated by Hanning window. The simulation is repeated for different excitation frequency. The group velocities of wave modes S_0 and A_0 are estimated and compared with the analytical results. The evaluation of the group velocities is based on the analysis of the appropriate components of displacement. The two different method are employed, namely: cross – correlation method and envelope extraction by Hilbert transform. Generally, the obtained results are in a good agreement. However, the method based on envelope extraction by Hilbert transform provides better correlation between analytical and numerical results. The significant discrepancy is observed in the case of symmetric mode S_0 for relatively high values of frequency. It is caused by the dispersion phenomena. The analytical calculations are performed with the use of SCILAB 5.5.2 free software and the numerical simulations are carried out with the use of finite element system ANSYS 13.0.

Keywords: *structural health monitoring, elastic waves, dispersion curves, stiffness matrix method, numerical simulations, finite element method, group velocities.*

1. Introduction

Together with still increasing number of applications of composite materials in engineering structures, the sophisticated methods of damage detection in these structures are also developed. Majority of these methods are based on an analysis of elastic waves propagation in thin-walled composite structures [1]. It seems that the usage of elastic waves in on-line mode is especially promising. These kind of systems are known as Structural Health Monitoring [2]. However, the physical phenomenon of elastic waves propagation in multi-layered structure has a very complex nature. Depending on the direction of polarization of vibrating particles, three main waves' modes can be distinguished, namely symmetric, anti-symmetric and shear horizontal. Symmetric and anti-symmetric modes are known in literature as Lamb waves. Each of mentioned wave's modes is strongly dispersive. Moreover, for sufficiently high frequency, they are also multimodal. It causes that the appropriate interpretation of picked up dynamic response of interrogated structure can be very difficult. First of all the appropriate dispersion curves have to be determined. The shape of these curves depends on mechanical properties and fibre orientation angles in composite material. There are several theoretical methods of dispersion curves determination. The oldest one is the transfer matrix method, which was proposed by Thompson [3] in 1950 and corrected by Haskell [4]. Originally, this approach was dedicated to materials, which consist of isotropic layers. Next, this method was extended for arbitrary composite materials by Nayfeh [5]. Unfortunately, for relatively high product of frequency and composite wall thickness, this method is numerically unstable – *fd problem*, Lowe [6]. In order to cope with this problem, Konopoff [7] in 1964 developed the global matrix method. However, in the case of composite materials with large number of layers, this method is not effective. It seems that the most efficient method of dispersion curve determination is stiffness matrix method proposed by Kausel [8] in 1986. Next, this approach was further developed by Wang and Rokhlin [9, 10]. To the contrary, of the previously discussed methods, the stiffness matrix method is numerically unconditionally stable and only slight less efficient in comparison with transfer matrix method. In the current work, the stiffness matrix method is used in order to estimate the appropriate dispersion curves for investigated composite materials. Besides, in the recent years some modified approaches to this problem have been developed. Karmazin et al. [11] applied the Green's matrix in order to determine dispersion curves for multi-layered composite materials. He Cunfu et al. [12] used the Legendre's orthogonal polynomials approach. Zhaoyang Ma et al. [13] used the so-called reverberation-ray matrix method. Having prepared the dispersion relations for studied composite material, the appropriate experiment should be carried out. It should be stressed here that the identification of the elastic wave modes in reality is challenging task. First of all, due to dispersive nature of the elastic waves, the direct identification with the use of phase velocity is rather difficult. Here can be quoted the paper by Castaings and Hosten [14] or Harb and Yuan [15], where only fundamental anti-symmetric A_0 mode is theoretically and experimentally evaluated for multi-layered material. More often, the elastic wave modes are identified with the use of group velocity. Wang and Yuan [16] compare the theoretically and experimentally obtained group velocities for different composite materials. Similar work is presented by Pant et al. [17], where the hybrid composite materials (aluminium alloy and glass fibres) are also studied. Here it is worth nothing that according to Sang-Ho Rhee et al. [18] there exists discrepancy between experimentally measured Lamb wave group velocity and theoretical group velocity, which is calculated with the Lamb wave dispersion equation. Thus, the authors proposed the advanced algorithm for theoretical determination of group velocity. The calculations are performed for fundamental symmetric mode S_0. The other method of elastic waves mode identification are discussed in the paper by Buli Xiu et al. [19]. It should be noted that the presented theoretical results show a good agreement with experiment.

In the current work, we are going to perform a several numerical simulations of elastic wave propagation in hybrid composite plate. First of all, the theoretical dispersion curves are extracted with the use of relatively new approach, namely stiffness matrix method. The obtained results are verified by the semi-analytical finite element method. Next, a coupled of numerical simulations of elastic waves propagation for different value of excitation frequency are carried out. On the basis of the obtained results, the identification of the fundamental symmetric S_0 and anti-symmetric A_0 modes are performed. The results of numerical simulations and theoretical prediction of the group velocities generally are in good agreement. However, in the case of symmetric mode S_0 for the frequency greater than 200[kHz], the noticeable discrepancy is observed. This problem is caused by the significant dispersion effect.

2. Dispersion curves for hybrid composite plate

The investigated hybrid composite consists of 7 layers. One of them is made of aluminium alloy Pa38 with the mechanical properties as follows: E=69.5 [GPa], υ=0.33, ρ=2.7 [g/cm^3] and thickness d_{layer}=0.5 [mm]. The rest of layers are made of glass fabric/epoxy resin. The "quasi –

isotropic" mechanical properties of this part of composite were evaluated experimentally. Its values are: $E_1=E_2=20.85$ [GPa], $G_{12}=4.15$ [GPa] $\upsilon_{12}=0.127$, $\rho=1.65$ [g/cm³]. The total thickness of the studied composite is equal to d=2.3[mm]. In order to estimate the relationship between the phase velocity and the wave frequency the stiffness matrix [9, 10] method is used. The appropriate numerical procedure is developed in SCILAB 5.5.2 free software. Additionally, the obtained results are verified with the use of semi – analytical finite element method. All computations are carried out with the use of a standard finite element software [20]. In this case, the ANSYS 13.0 is employed. The extracted dispersion curves are shown in the Fig. 1. As it can be noticed, the obtained results are almost identical. Slight discrepancy is observed only in the case of symmetric mode S_0, for the frequency value greater than 700 [kHz]. This discrepancy is caused by the numerical error, which generally increases with frequency value. It is worth noting here that for the frequency value greater than 300 [kHz], the first higher wave mode is present. Thus, in practice, in order to detect the damage in the investigated composite, the useful range of frequency is limited to 300[kHz] frequency value.

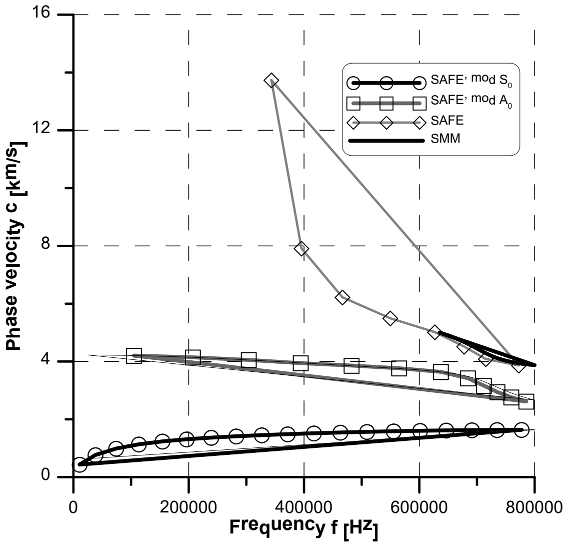

Fig. 1. Dispersion curves (phase velocities) evaluated for studied hybrid composite with the use of stiffness matrix method (SMM) and semi – analytical finite element method (SAFE)

Next, the group velocities c_g is evaluated with the aid of the following formula [2], namely:

$$c_g = c^2\left(c - fd\frac{\partial c}{\partial(fd)}\right), \tag{1}$$

where c, f is the phase velocity and frequency, respectively. The partial derivative in the above equation can be computed with the use of finite difference formula.

3. Numerical simulation of elastic waves propagation

Two-dimensional model of the studied composite plate is shown in Fig. 2. It is assumed plain state of strain ($\varepsilon_{i3}\equiv 0$, i=1, 2, 3). No damping is taken into account. The elastic waves are excited by the single piezoelectric element, which is polarized in the vertical direction. The piezoelectric actuator is made of PZT-5A material. The mechanical and physical properties of this material can be found in ANSYS13.0 help system (vm237) [21]. It is mounted to layer, which is made of glass fabric. It is also assumed that the piezoelectric element is perfectly bonded to the surface of the plate. All simulations are carried out with the use of ANSYS 13.0 software. The structural PLANE183 and coupled field PLANE223 type of the finite element are applied. The size of the finite elements is equal to l_e=0.035 [mm]. This size ensures high accuracy of the obtained results of the dynamic response of the composite plate.

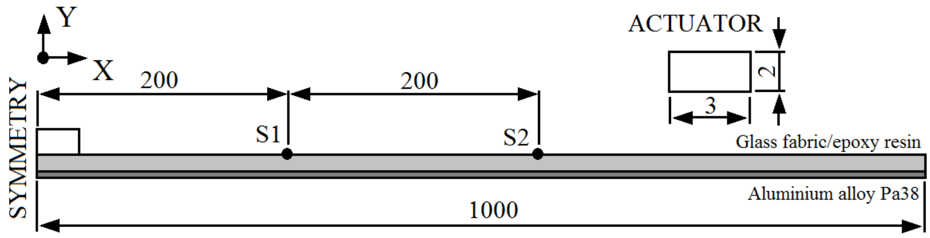

Fig. 2. Finite element model of composite plate

The total time of simulation is equal to t_c=5.52e-4 [s] and the length of time step is equal to dt=2.208e-7 [s]. The piezoelectric actuator is excited by the applied voltage signal with amplitude equal to A_0=100[V]. The signal is a 5 cycles of sine wave modulated by Hanning window. The simulations are carried out for the following excitation frequency, namely: 25, 50, … 300[kHz]. Fig. 3a shows the excitation signal with a frequency of 75[kHz]. Additionally, in the Fig. 3b there are depicted the normalized input signals of different main frequency (50, 150, 250[kHz]) in the frequency domain (Fourier transform of the excitation signals). As it can be observed, when the main frequency of input signal increases and the number of cycles is constant, the frequency bandwidth also increases. It causes that the wave dispersion effect is much stronger. Thus, the evaluation of the group velocities could provide the results, which are slightly different in comparison with those presented in the Fig. 1.

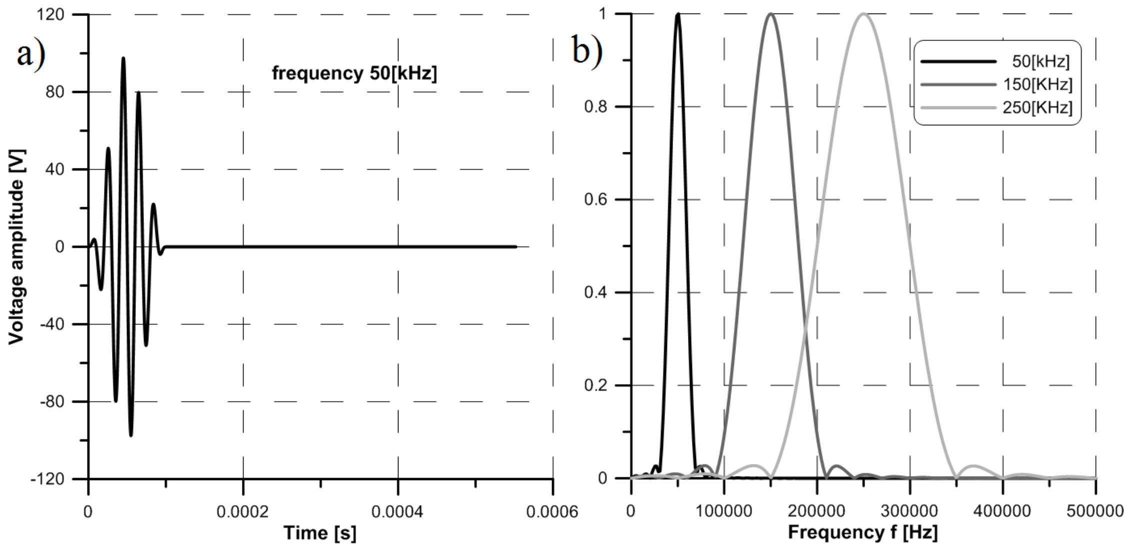

Fig. 3. Input signals: a) excitation signal (voltage amplitude versus time) 5 cycles, frequency 50[kHz], b) frequency bandwidth of input signals with the main frequencies 50, 150 and 250[KHz]

4. Results of numerical simulations of elastic waves propagation

First of all, the group velocities of the fundamental symmetric S_0 and anti-symmetric A_0 wave modes are estimated. In order to deal with this task, the components of displacement are analysed. The estimation of group velocity of fundamental symmetric wave mode is based on UX component of displacement while the group velocity of anti-symmetric wave mode is based on UY, where UX is the direction parallel to the surface of the plate. Among different possible approaches [19], the two methods are applied, namely cross – correlation method and envelope extraction by Hilbert transform. The measurement is performed between the points S1 and S2. In the Fig. 4 there are presented the exemplary results of finite element simulation, namely the component of displacement UX and UY versus time. The central frequency of excitation signal is equal to f=175[kHz]. As it can be observed, the fundamental symmetric S_0 and anti-symmetric A_0 waves' modes are clearly visible. The amplitude of the S_0 mode is significantly less in comparison with anti-symmetric mode A_0. Additionally, it is worth stressing here that the component of displacement UY of the symmetric mode S_0 is almost invisible. However, in the case of A_0 mode the magnitude of both components of displacement are comparable

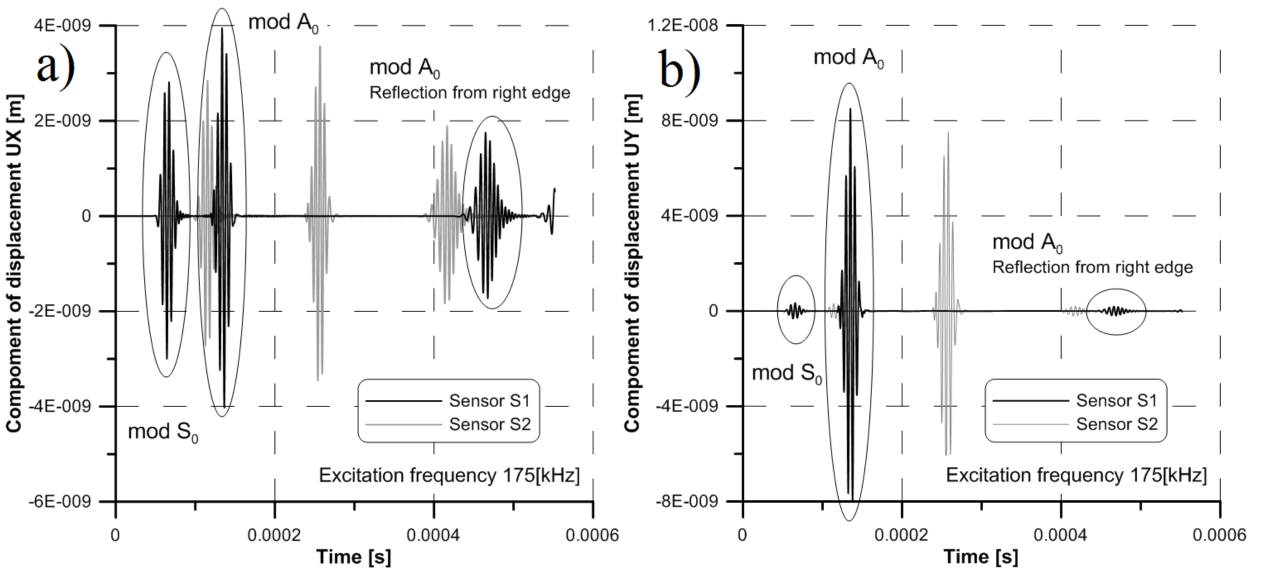

Fig. 4. Received dynamic response of plate in S1 and S2 sensors (main frequency of input signal f=175[kHz]): a) UX component of displacement, b) UY component of displacement

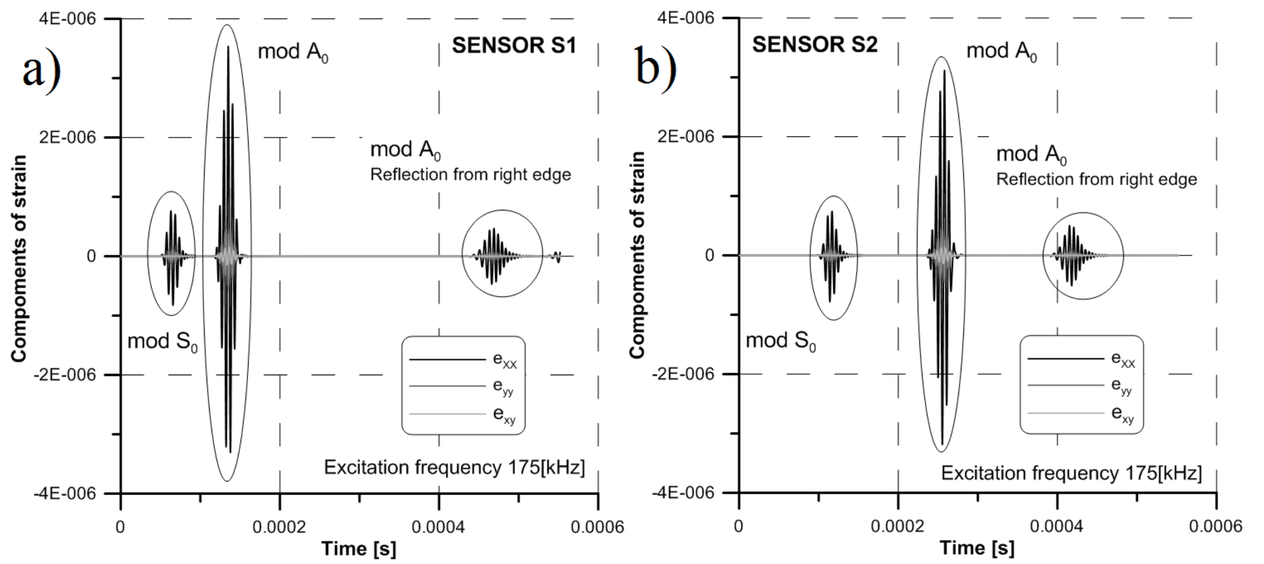

Fig. 5 Components of strain versus time (main frequency of input signal f=175[kHz]: a) received response in sensor S1, b) received response in sensor S2

Moreover, the reflection of the symmetric mode S_0 from the right edge of the studied plate reveals clear dispersion effect. Finally, in the Fig. 5 are shown the components of strain in points S1 and S2. It is worth noting that the component ε_{XX} is several order of magnitude larger in comparison with the rest of components, namely ε_{YY} and ε_{XY}. In the next part of this work, the relationship between excitation frequency and magnitude of amplitude of the ε_{XX} will be also discussed. Moreover, taking under consideration the piezoelectric equations in sensing mode [2] and due to the fact that the component of strain ε_{XX} mainly stimulates the piezoelectric elements, the normalized relationship between ε_{XX} and time t can be considered also as the voltage response of the sensor.

In the Fig. 6 there are shown the results of group velocity evaluation based on the numerical simulations. In the case of anti-symmetric mode A_0, the good correlation between analytical and numerical values of group velocity is observed. Moreover, both applied methods of group velocity provide almost identical results. However, in the case of symmetric mode S_0 and for the frequency greater than 200[kHz] the analytical and numerical group velocities are significantly different. This effect is caused by the dispersion phenomenon. The theoretical values of group velocities of symmetric and anti-symmetric modes are c_S=3.887[km/s] and c_A=1.728[km/s], respectively. In the Tab. 1 there are collected the values of the group velocities (finite element simulations) estimated for the frequency f=250[kHz] and for several different numbers of cycles in the excitation signal, namely 5, 7, 11, 15. Theoretically, if the number of cycles increases, the bandwidth of frequency decreases and it causes that the dispersion effect should be less severe.

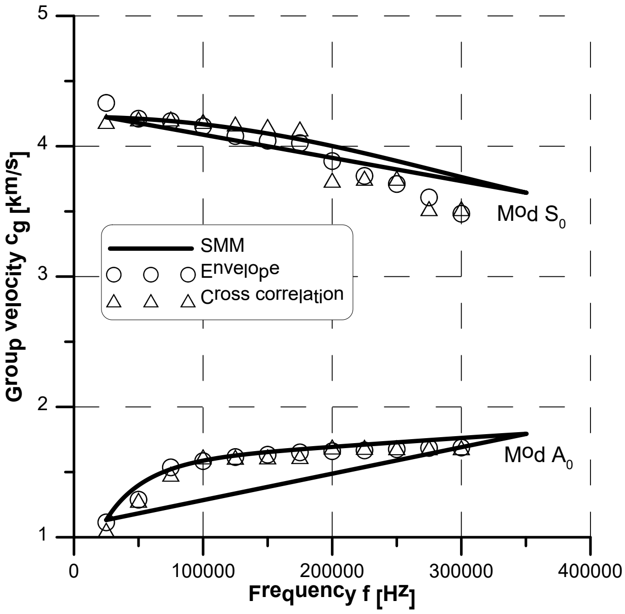

Fig. 6. Comparison of group velocities obtained from SMM and finite element simulations

However now, the slight change of the group velocity is observed only in the case of symmetric mod S_0 and the method based on Hilbert transform, when the number of cycles is equal to n=11 and n=15. The group velocities evaluated with the use of cross correlation method are identical for all numbers of cycles. It worth nothing that the envelope by Hilbert-transform method is slightly more sensitive.

Tab. 1. Estimated from numerical simulations values of group velocities [km/s], f=250[kHz]

Number of cycles	Envelope		Cross – correlation	
	S_0	A_0	S_0	A_0
5	3.712	1.677	3.758	1.690
7	3.712	1.677	3.758	1.690
11	3.743	1.677	3.758	1.690
15	3.790	1.677	3.758	1.690

Finally, in the Fig. 7 there is depicted the comparison between the maximal values of amplitude of strain ε_{XX}. The evaluation of maximum is based on the envelope by Hilbert transform. As it can be observed, both values increases with the excitation frequency in studied range. However, the amplitude of the anti-symmetric A_0 mode increases much stronger in comparison with the fundamental symmetric mode S_0. It means, in practice, that for the relatively low frequencies, the amplitude of S_0 mode is of the magnitude of the measurement noise. Thus, it cannot be properly picked up and distinguished from the rest of data. In the other hand, for the frequencies, where the amplitude of S_0 mode is high enough the dispersion effect could be significant, what makes the process of damage detection very difficult.

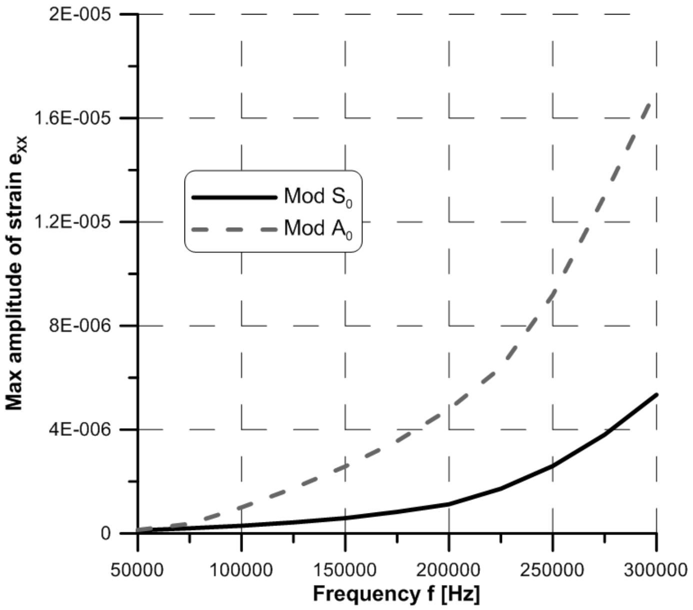

Fig. 7. Comparison of maximal value of strain ε_{XX} for symmetric and anti-symmetric modes measured at sensor S1

5. Conclusions

The current study should be treated as introduction to the problem of damage detection in the case of hybrid composite materials with the use of elastic waves. As it can be observed in the Fig. 1, the upper value of the excitation frequency is limited by the value 300[kHz]. In this frequency the first higher elastic wave mode is present. Further, in the case of fundamental symmetric mode S_0 the dispersion effect is clearly visible for the frequency value greater than 200[kHz]. It causes that the evaluation of the velocity group or time-of-fly of symmetric mode S_0 is associated with significant error. In the other hand, the anti-symmetric mode A_0 for the relatively low frequencies, less than 50[kHz], has also dispersive character. For the frequency greater than 50[kHz] the dispersion is almost not observed. Additionally, for the moderately values of the excitation frequency, for example 100 – 150[kHz] the amplitude of strain ε_{XX} in the case of symmetric mode S_0 is much smaller in comparison with the amplitude of anti-symmetric mode A_0. Thus, it is possible that the amplitude of the S0 mode is of order of measurement noise and its extraction and further analysis could be impossible. This fact in practice could significantly simplify the process of damage detection.

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