

## QUARTER CAR MODEL TO EVALUATE BEHAVIOUR UNDER ROAD AND BODY EXCITATION

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### **Abstract**

*The paper presents a quarter car model two degrees of freedom (wheel and body), with vertical guiding system, the model being equipped with suspension stroke limiters and with excitation by wheel and/ or by body. The model reproduces elastic and damping characteristics of wheel and of rebound and compression stopper bumpers, the spring elastic characteristic and the shock absorber damping characteristic on rebound and compression, function piston speed. The road profile is generated with simple or summation of harmonic functions, or by reproducing real roads. The forces acting on full vehicle body e.g. aerodynamic and inertial forces are reproduced in the proposed quarter model by vertical forces reduced to the analysed quarter part. Thus, the model can be used for evaluation the vertical and horizontal stability at acceleration, deceleration, pitch and roll, at aerial forces, the body ground clearance and the comfort. The model can evaluate the influence of the damping and elastic characteristics of suspension and wheel, of the static position, of the vehicle load state, of the road profile and of the external forces, to the vehicle behaviour.*

**Keywords:** *quarter car model, shock absorber, simulation, comfort, stability, clearance, road profile, external forces*

### **1. Introduction**

The vehicle moving realises under excitation forces generated by the road profile and by the aerodynamic and inertial forces and torques acting directly against the vehicle body.

The aerodynamic forces and torques acting in vehicle pressure centre and inertial forces and torques acting in vehicle gravity centre give forces and moments, which are stabilized by suspension reaction forces, so the vehicle behaviour is influenced by both the road profile and vehicle trajectory and driving speed. This reason we propose a quarter car model able to reproduce both these excitation made by road against the wheel and by aerodynamic and inertial forces against the vehicle body.

Figure 1 shows a vehicle with independent suspensions, all the four suspensions having two degrees of freedom.

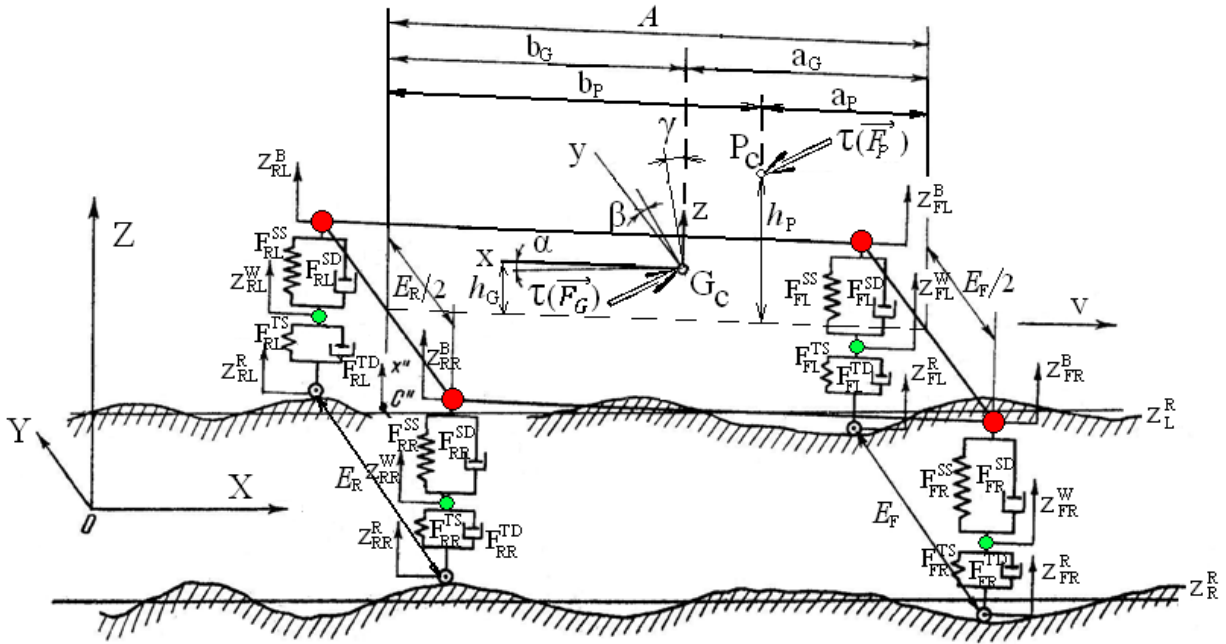


Fig. 1. Vehicle model considering road and body excitations

The vehicle excitations are realised:

- in the vehicle gravity centre  $G_C$  by the inertial twister of forces and moments  $\tau(\overline{F_G}, \overline{M_G})$  generated at acceleration, deceleration or other external actions and trajectory changing,
- in the pressure centre  $P_C$  by the aerodynamic twister of forces and moments  $\tau(\overline{F_P}, \overline{M_P})$  generated by the relative movement air-vehicle,
- in the wheel-road contact points by the road profile unevenness,  $z_{FL}^R, z_{FR}^R, z_{RL}^R, z_{RR}^R$

The equations (1) and (2) define the twisters of force and moment and equations (3)-(6):

$$\tau(\overline{F_G}, \overline{M_G}) = \begin{Bmatrix} \overline{F_G} \\ \overline{M_G} \end{Bmatrix}, \quad (1)$$

$$\tau(\overline{F_P}, \overline{M_P}) = \begin{Bmatrix} \overline{F_P} \\ \overline{M_P} \end{Bmatrix}. \quad (2)$$

The Tab. 1 show the significations of the symbols used in Fig. 1 and in equations (1)-(6):

$$\overline{F_G} = \begin{Bmatrix} F_{Gx} \cdot \vec{i} \\ F_{Gy} \cdot \vec{j} \\ F_{Gz} \cdot \vec{k} \end{Bmatrix}, \quad (3)$$

$$\overline{M_G} = \begin{Bmatrix} M_{Gx} \cdot \vec{i} \\ M_{Gy} \cdot \vec{j} \\ M_{Gz} \cdot \vec{k} \end{Bmatrix}. \quad (4)$$

Tab. 1. Signification of the elements from Fig. 1 and equations (1)-(6)

Symbol	Element	Symbol	Element
A	Wheelbase	$M_{Pz}$	Z component of movement acting in air pressure centre
a	position of $G_C$ relative to the front axle	$z_{RL}^B$	Displacement of the rear left sprung mass
b	position of $G_C$ relative to the front axle	$z_{RL}^W$	Displacement of the rear left wheel
$E_F$	front track	$z_{RL}^R$	Rear left longitudinal profile of the road
$E_R$	rear track	$z_{RR}^B$	Displacement of the rear right sprung mass
$v$ [m/s]	Longitudinal speed	$z_{RR}^W$	Displacement of the rear right wheel
$\tau(\vec{F}_G, \vec{M}_G)$	twister of force and moment acting in gravity centre	$z_{RR}^R$	Rear right longitudinal profile of the road
$\vec{F}_G$	force acting in gravity centre	$F_{FL}^{SS}$	Front left suspension spring force
$F_{Gx}$	X component of force acting in gravity centre	$F_{FL}^{SD}$	Front left suspension damping force
$F_{Gy}$	Y component of force acting in gravity centre	$F_{FL}^{TS}$	Front left tire elastic force
$F_{Gz}$	Z component of force acting in gravity centre	$F_{FL}^{TD}$	Front left tire damping force
$\vec{M}_G$	Movement acting in gravity centre	$F_{FR}^{SS}$	Front right suspension spring force
$M_{Gx}$	X component of movement acting in gravity centre	$F_{FR}^{SD}$	Front right suspension damping force
$M_{Gy}$	Y component of movement acting in gravity centre	$F_{FR}^{TS}$	Front right tire elastic force
$M_{Gz}$	Z component of movement acting in gravity centre	$F_{FR}^{TD}$	Front right tire damping force
$\vec{i}$	unit vector of X axle	$F_{RL}^{SS}$	Rear left suspension spring force
$\vec{j}$	unit vector of Y axle	$F_{RL}^{SD}$	Rear left suspension damping force
$\vec{k}$	unit vector of Z axle	$F_{RL}^{TS}$	Rear left tire elastic force
$G_C$	gravity centre	$F_{RL}^{TD}$	Rear left tire damping force
$h_G$	Gravity centre position	$F_{RR}^{SS}$	Rear right suspension spring force
$\alpha$	Angle of twister of force and moment relative to X axle	$F_{RR}^{SD}$	Rear right suspension damping force
$\beta$	Angle of twister of force and moment relative to Y axle	$F_{RR}^{TS}$	Rear right tire elastic force
$\gamma$	Angle of twister of force and moment relative to Z axle	$F_{RR}^{TD}$	Rear right tire damping force
$\tau(\vec{F}_P, \vec{M}_P)$	aerodynamic twister of force and moment	$P_C$	centre of pressure
$\vec{F}_P$	aerodynamic force acting in air pressure centre	$h_P$	Pressure centre position relative to
$F_{Px}$	X component of force acting in air pressure centre	$z_{FL}^B$	Displacement of the front left sprung mass
$F_{Py}$	Y component of force acting in air pressure centre	$z_{FL}^W$	Displacement of the front left wheel
$F_{Pz}$	Z component of force acting in air pressure centre	$z_{FL}^R$	Front left longitudinal profile of the road
$\vec{M}_P$	aerodynamic movement acting in air pressure centre	$z_{FR}^B$	Displacement of the front right sprung mass
$M_{Px}$	X component of movement acting in air pressure centre	$z_{FR}^W$	Displacement of the front left wheel
$M_{Py}$	Y component of movement acting in air pressure centre	$z_{FR}^R$	Front left longitudinal profile of the road

$$\vec{F}_P = \begin{cases} F_{Px} \cdot \vec{i} \\ F_{Py} \cdot \vec{j}, \\ F_{Pz} \cdot \vec{k} \end{cases} \quad (5)$$

$$\vec{M}_P = \begin{cases} M_{Px} \cdot \vec{i} \\ M_{Py} \cdot \vec{j}, \\ M_{Pz} \cdot \vec{k} \end{cases} \quad (6)$$

### 2. Triaxial quarter car model

For studying longitudinal and transversal stability uses a triaxial quarter car model. Fig. 2 shows a triaxial quarter car model for rear left sidecar.

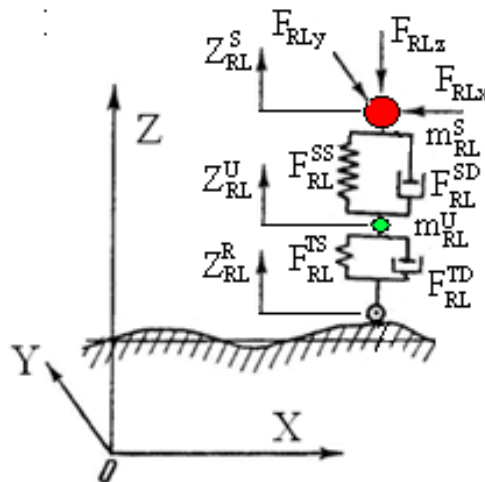


Fig. 2. Triaxial quarter car model

The elements from Fig. 2 are presented in Tab. 2.

Tab. 2. The elements from Fig. 2

Symbol	Element	Symbol	Element
$m_{RL}^S$	sprung mass of rear left car part	$F_{RLx}$	X component of forces acting against the sprung mass
$m_{RL}^U$	unsprung mass of rear left car part	$F_{RLy}$	Y component of forces acting against the sprung mass
$F_{RL}^{SS}$	suspension spring force of rear left car part	$F_{RLz}$	Z component of forces acting against the sprung mass
$F_{RL}^{SD}$	suspension damping force of rear left car part	$z_{RL}^S$	vertical displacement of sprung mass
$F_{RL}^{TS}$	tire elastic force of rear left car part	$z_{RL}^U$	vertical displacement of unsprung mass
$F_{RL}^{TD}$	tire damping force of rear left car part	$z_{RL}^R$	vertical road profile

Usually, for suspension studies, it uses biaxial quarter car model similar those presented in the next chapter.

### 3. Biaxial quarter car model

Figure 3 presents the biaxial quarter car model, the elements being defined in the Tab. 2.

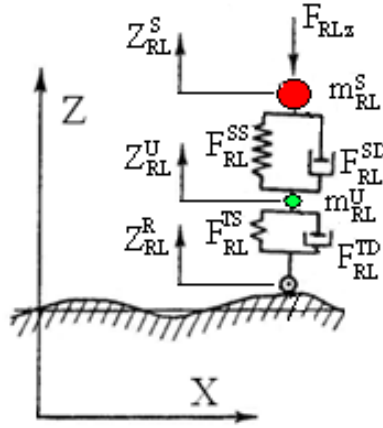


Fig. 3. Biaxial quarter car model

#### 4. Vertical force acting against the sprung mass of quarter car model

$F_{RLz}$  the vertical force acting against the sprung mass of quarter car model is generated by the forces and moments  $F_{Gx}$ ,  $F_{Gz}$ ,  $F_{Px}$ ,  $F_{Pz}$ ,  $M_{Gx}$ ,  $M_{Gy}$ ,  $M_{Px}$ ,  $M_{Py}$ .

Considering the system symmetric to the plane XOZ and the rigidities of each suspension identically and the same front and rear track, the contributions of previous forces and moments to the vertical force acting against the sprung mass of quarter car model  $F_{RLz}$  are according next equations:

$F_{Gx}$  gives:

$$F_{RLz} = \frac{h_G}{2l} F_{Gx}, \quad (7)$$

$F_{Gy}$  gives:

$$F_{RLz} = \frac{h_G}{2 \cdot E_R} F_{Gy}, \quad (8)$$

$F_{Gz}$  gives:

$$F_{RLz} = \frac{a_G}{2l} F_{Gz}, \quad (9)$$

$F_{Px}$  gives:

$$F_{RLz} = \frac{h_P}{2l} F_{Px}, \quad (10)$$

$F_{Py}$  gives:

$$F_{RLz} = \frac{h_P}{2E_R} F_{Py}, \quad (11)$$

$F_{Pz}$  gives:

$$F_{RLz} = \frac{a_P}{2l} F_{Pz}, \quad (12)$$

$M_{Gx}$  gives:

$$F_{RLz} = \frac{M_{Gx}}{2E_R}, \quad (13)$$

$M_{Gy}$  gives:

$$F_{RLz} = \frac{M_{Gy}}{2l}, \quad (14)$$

$M_{Px}$  gives:

$$F_{RLz} = \frac{M_{Px}}{2E_R}, \quad (15)$$

$M_{Py}$  gives:

$$F_{RLz} = \frac{M_{Py}}{2l}. \quad (16)$$

The vertical force acting against the sprung mass of quarter car model  $F_{RLz}$  obtains by summation all previous forces, according the equation (17).

$$F_{RLz} = \frac{h_G}{2l} F_{Gx} + \frac{h_G}{2E_R} F_{Gy} + \frac{a_G}{2l} F_{Gz} + \frac{h_P}{2l} F_{Px} + \frac{h_P}{2E_R} F_{Py} + \frac{a_P}{2l} F_{Pz} + \frac{M_{Gx}}{2E_R} + \frac{M_{Gy}}{2l} + \frac{M_{Px}}{2E_R} + \frac{M_{Py}}{2l} . \quad (17)$$

## 5. Proposed quarter car model

The virtual model was realized with ADAMS View software and it is presented in Fig. 4.

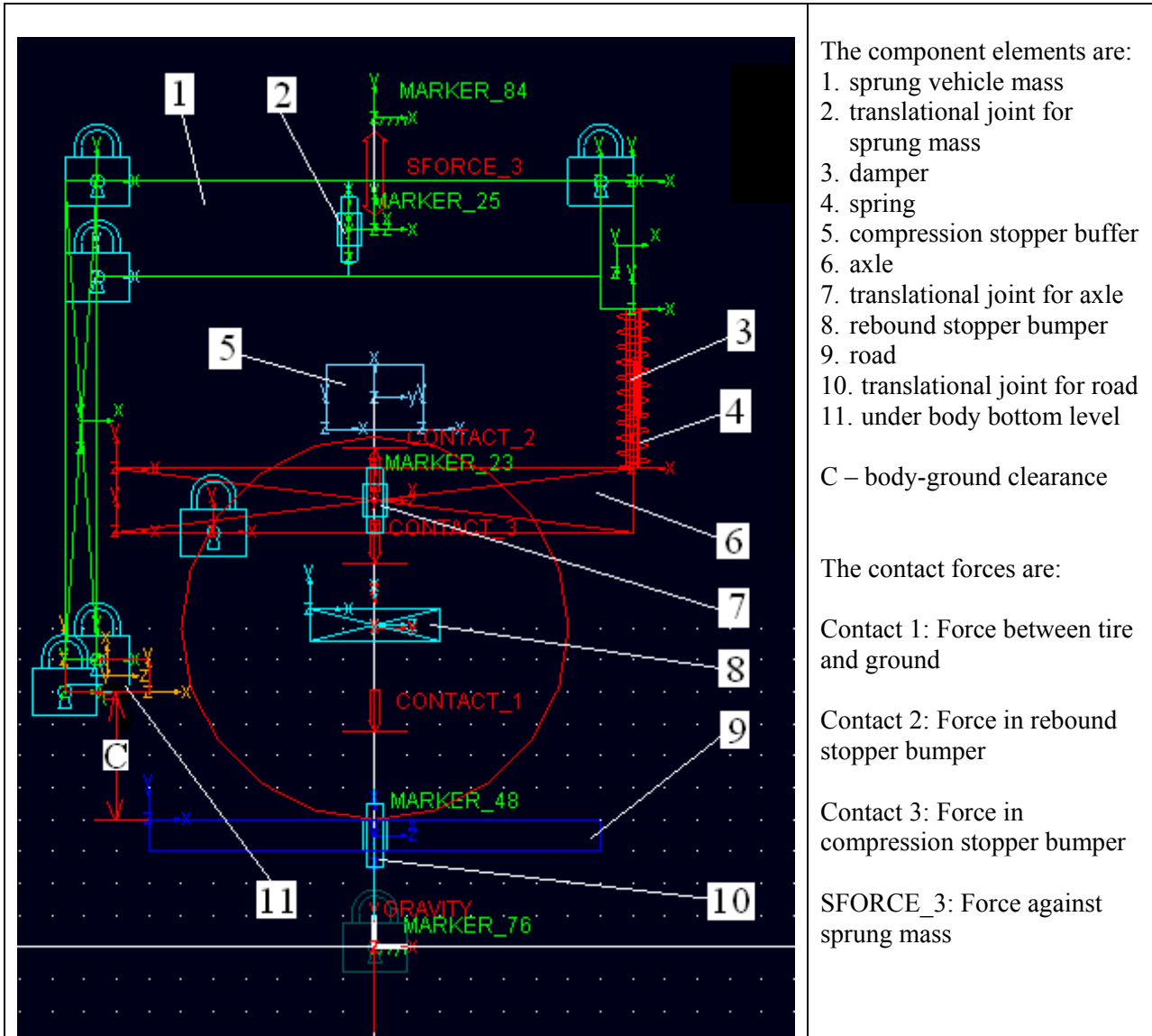


Fig. 4. The quarter car model realized with Adams software, View module

## 6. Simulation to evaluate the influence of force acting directly against the sprung mass

Simulations on two identically quarter car model, the both being excited by the same road, one of them being excited supplementary against the sprung mass, reveal the influence of the body direct excitation.

This case the excitation against the vehicle body is oscillatory.

### 6.1. Test conditions

Both models are excited by road with a harmonic function according equation (18).

$$z = 0.04 \cdot \sin(2\pi \cdot 3 \cdot \text{time}), \quad (18)$$

where:

oscillation amplitude 0.04 [m],

frequency 3 [Hz].

The model 1 is supplementary excited against body by a force according (19).

$$F = 800 \cdot \sin(2\pi \cdot 0.2 \cdot \text{time}), \quad (19)$$

where:

force amplitude 800 [N],

frequency 0.2 [Hz].

## 6.2. Numerical application

The vertical interaction has been simulated using ADAMS software View module. The characteristics of the considered model of suspension are presented in Tab. 3.

Tab. 3. Suspension characteristics

Symbol	Value	Units	Parameter
$m_{U}^{\dot{S}}$	240	[kg]	sprung mass at Minimal loaded (noted Unload)
$m_U$	35	[kg]	unsprung mass
$m_{T0}$	275	[kg]	total mass at minimal loaded
$l$	0.236	[m]	overall suspension stroke
$k_P$	14085	[N/m]	pneumatic suspension rigidity
$k_T$	200000	[N/m]	tire rigidity
$k_{CB}$	250000	[N/m]	compression stopper buffer rigidity
$k_{RB}$	500000	[N/m]	rebound stopper buffer rigidity

The used damping characteristic is presented in Tab. 4.

Tab. 4. Shock absorber damping characteristic

Speed [m/s]		0.05	0.1	0.2	0.3	0.4	0.55	0.75	0.95	1.5	3
Force [N]	Rebound	70	170	410	650	800	1030	1320	1600	2450	4600
	Compression	170	210	320	440	530	650	830	1000	1500	2740

## 6.3. Results

The suspension quality will be evaluated by the body stability, comfort and body ground clearance, by adherence and road protection and by body-axle protection.

Figure 5 shows the diagrams for model M1 excited by road and body and Fig. 6 for model M2 excited only by road.

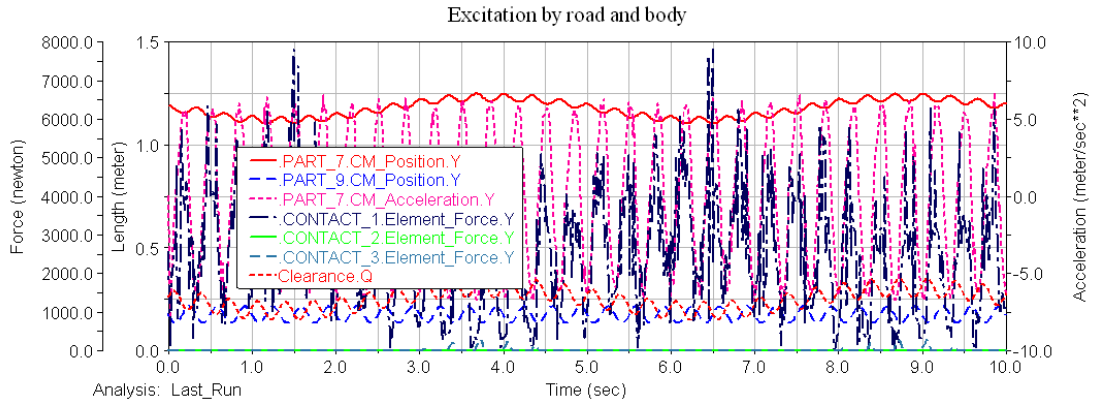


Fig. 5. Diagrams for model M1 excited by road and body

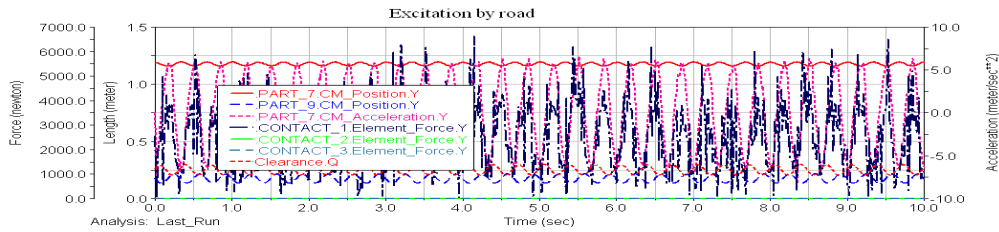


Fig. 6. Diagrams for model M2 excited only by road

In the Tab. 5 are presented the significant parameters for evaluate the influence of the supplementary body excitation.

Tab. 5. Comparative results

Model	Excitation	STABILITY Body vertical position [m]			Comfort Body Acceleration [m/s <sup>2</sup> ]	CLEARANCE [m]	ROAD PROTECTION/ ADHERENCE Tire Contact Force [N]		VEHICLE PROTECTION Forces in Bumpers [N]	
		Max	Min	Δ			Max	Avg	Rebo- und Max	Compres sion Max
M1	Road + Body	1.249	6.72	0.147	6.72	0.148	7805	2608	0	296
M2	Road	1.195	6.57	0.039	6.57	0.206	6614	2541	0	0
M2 better [%]		+ 4.3	+ 31.4	+ 2.2	+ 2.2	+39.2	+39.2	-2.6	0	+100

## 7. Conclusions

The paper gives solution to evaluate on a quarter-car model the influence of the road and body combined excitation acting on a vehicle.

The example shows the supplementary excitation worsen the vehicle performances.

## References

- [1] Niculescu, A. I., Alexandru, C., Jankowski, A., *A Solution for Crash and High Ground Unevenness Effect Reducing*, Journal of KONES, Vol. 15, No. 3, ISSN 1231-4005, pp. 389-396, 2008.



- [2] Niculescu, A. I., Jankowski, A., Kowalski, M., Sireteanu, T., *Advantages Conferred by Shock Absorbers with Cylindrical Actuator Application to Landing Gear*, Journal of KONES, Vol. 23, No. 1, ISSN 1231-4005, pp. 255-262, 2016.
- [3] Niculescu, A. I., Jankowski, A., Kowalski, M., Sireteanu, T., *On the new concept and advantages of the integrated shock absorber – air spring – ISAS*, Proceedings of the European Automotive Congress EAEC-ESFA, Springer International Publishing, ISBN: 978-3-319-27276-4, pp. 93-103, 2015.
- [4] Niculescu, A. I., Jankowski, A., Sireteanu, T., *On „VZN“ damper behaviour at crash*, Journal of KONES Powertrain and Transport, Vol. 16, No. 2, pp. 369-374, 2009.
- [5] Sireteanu, T., Gundisch, O., Paraiian S., *Vibratiile aleatoare ale automobilelor*, Editura Tehnica, Bucuresti 1981.

