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APPLICATION OF MODEL PREDICTIVE CONTROL FOR AUTONOMOUS MAINTENANCE OF SATELLITES CONSTELLATION

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Abstract

This investigation deals with the problem of satellite constellation maintenance. An autonomous control strategy is presented in this paper. Main task of the proposed control algorithm is relative station keeping where the relative positions of the satellites are maintained. The approach utilizes model predictive control with successive linearization of a nonlinear model of relative motion between the satellites. The predictive algorithm takes into account the variability of the future model parameters within assumed prediction horizon. Since the future model parameters are dependent on future control actions, the presented strategy employs a heuristic method for preliminary control trajectory estimation. The proposed method also enables the constellation deployment, reorganization, relative station keeping for large separations between the satellites, and control for the case where the reference point is moving in a highly elliptical orbit. Performance of the algorithm was verified using numerical simulations.

The presented algorithm is able to cope with the tasks of deployment, reorganization and maintenance of satellite constellation.

Keywords: *satellites constellation, spacecraft control, orbital mechanics, model predictive control*

1. Introduction

The concept of satellites constellation gives a series of advantages over a single and bigger spacecraft. The advantages include synchronous measurements over a dispersed area, improvements in performance and survivability, ability to be launched in multiplies, requiring possibly cheaper dedicated launch vehicles. They also allow for cheaper designs.

The need for constellation maintenance is an outcome of two reasons: first, each satellite is moving in an orbit slightly different from intended. If left uncorrected, these small differences will accumulate with time to disorganize the overall structure of the constellation. The second reason arises from orbit perturbations of a various manner.

Deployment and maintaining of constellation structure for a long term can be a major element of economic cost and risk. Both can be significantly reduced by low-cost, autonomous orbit maintenance methods and algorithms. Autonomous constellation maintenance system enables to maintain the constellation structure to very high precision for much lower cost and risk than is currently available with ground-based orbit maintenance.

An autonomous maintenance strategy proposed in this paper is based on model predictive control (MPC) for relative motion between each of controlled spacecraft and a reference satellite (a mothership) or a chosen reference point. Since fuel consumption is a critical parameter in the space industry, the proposed controller enables calculation of quasi-optimal control trajectories, wherein the cost function includes metrics responsible for minimization of the fuel use. As shown in [1], the proposed control algorithm allows for asymptotically stable control process, even with assumption of separations up to tens of thousands kilometres, what far exceeds the requirements for the orbital station keeping problem. Additionally, the assumed control algorithm allows for control in the case where the constellation is moving in a highly elliptical orbit. The algorithm enables to apply constraints on relative motion state, what allows for collision-free and safe reorganization of the satellite constellation.

The proposed controller employs a full, time-variant, nonlinear model of relative motion (relative position and relative velocity), while the vast majority of MPC approaches to relative motion control utilizes linear models, such as Hill-Clohessy-Wiltshire (HCW) equations [2, 3] or Tschauner-Hempel model [4]. Applicability of the linear models is limited to cases where the relative distance is small, not exceeding 1000 meters.

The proposed algorithm considers a variability of model parameters over a prediction horizon. Such feature is obtained by generation of local linear models for every controller step within a prediction horizon. The local linear models are derived by a linearization of the full nonlinear model.

Parameters of the nonlinear model are dependent on state, control and time. In order to estimate future model parameters, it is necessary to estimate future control actions. In this investigation, this problem is solved by an application of a finite impulse response (FIR) filter as a heuristic method for preliminary estimation of the future control trajectory.

2. Mathematical Model of Relative Motion

The control strategy proposed in this paper requires an internal model of relative motion between each of controlled spacecraft and mothership (reference satellite or a reference point). Let us call further the controlled satellite as a deputy spacecraft. A set of nonlinear equations of a single deputy spacecraft motion relative to the reference satellite will be used to formulate a model of relative motion for all the deputy satellites within the constellation.

2.1. Reference Frame

The relative motion is described in Cartesian Local Vertical – Local Horizontal (LVLH) frame depicted in Fig. 1.

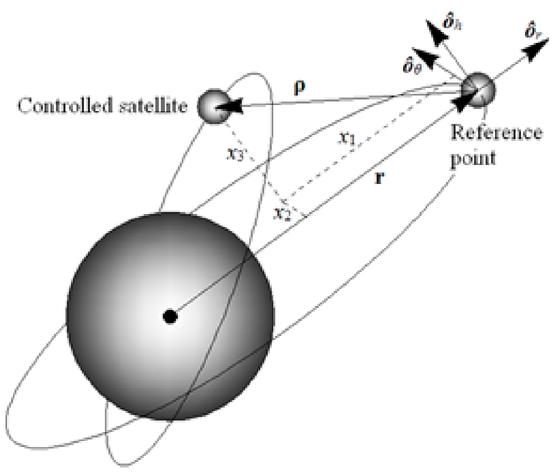

Fig. 1. LVLH reference frame

The LVLH frame is attached to the reference satellite (or a chosen reference point) and rotates with the radius vector $\mathbf{r}$ of this basis. Orientation of the frame is determined by the unit vector triad $\{\hat{\mathbf{o}}_r, \hat{\mathbf{o}}_\theta, \hat{\mathbf{o}}_h\}$ where vector $\hat{\mathbf{o}}_r$ lies in the radial direction of the reference satellite, $\hat{\mathbf{o}}_h$ is parallel to the orbit angular momentum vector, and $\hat{\mathbf{o}}_\theta$ completes the right-handed orthogonal triad.

Position of *j*-th deputy satellite relative to the reference satellite can be expressed by Cartesian coordinate vector $\boldsymbol{\rho}_j$:

$$\boldsymbol{\rho}_j = x_{j,1}\hat{\mathbf{o}}_r + x_{j,2}\hat{\mathbf{o}}_\theta + x_{j,3}\hat{\mathbf{o}}_h. \tag{1}$$

2.2. Model of True Anomaly

Most of orbital relative motion models utilize the notion of *true anomaly* [5], [6]. Here we will develop a strategy to express the true anomaly as a function of time. First, let us recall the notion of *mean anomaly*:

$$M = M_0 + n(t - t_0), \tag{2}$$

where *t* denotes the time, M_0 is an initial value for the mean anomaly at an initial time moment t_0, while *n* denotes *mean angular motion*:

$$n = \sqrt{\frac{\mu}{a^3}}, \tag{3}$$

wherein *a* is semi-major axis of the orbit and μ is the standard gravitational parameter.

Given mean anomaly *M*, we can solve the Kepler's Equation for *eccentric anomaly E*:

$$M = E - e\sin E, \tag{4}$$

where *e* is the orbit eccentricity. Equation 4 can be solved using simple numerical method, such us the Newton's method.

Finally, using eccentric anomaly *E* we can find the true anomaly *f*:

$$\tan\frac{f}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}. \tag{5}$$

First derivative of the true anomaly is an important model parameter:

$$\dot{f} = \sqrt{\frac{\mu p}{r^4}}, \tag{6}$$

wherein r is a distance from the Earth's center to satellite's centre of mass and *p* is a parameter called *semilatus rectum*:

$$p = a(1 - e^2). \tag{7}$$

2.3. Model of Relative Motion for a Single Deputy Satellite

Derivation of the exact, time-variant, nonlinear equations of relative motion in LVLH frame, further called NERM, can be found in [1] and [5]. The equations are presented below:

$$\ddot{x}_{j,1} - 2\dot{f}\left(\dot{x}_{j,2} - x_{j,2}\frac{\dot{r}}{r}\right) - x_{j,1}\dot{f}^2 - \frac{\mu}{r^2} = -\frac{\mu}{r_j^3}\left(r + x_{j,1}\right) + \frac{u_{j,1}}{m_j}, \tag{8}$$

$$\ddot{x}_{j,2} + 2\dot{f}\left(\dot{x}_{j,1} - x_{j,1}\frac{\dot{r}}{r}\right) - x_{j,2}\dot{f}^2 = -\frac{\mu}{r_j^3}x_{j,2} + \frac{u_{j,2}}{m_j}, \tag{9}$$

$$\ddot{x}_{j,3} = -\frac{\mu}{r_j^3}x_{j,3} + \frac{u_{j,3}}{m_j}, \tag{10}$$

wherein $\dot{x}_{j,1}$ and $\ddot{x}_{j,1}$ are relative velocity and acceleration of j-th satellite in the direction pointed by the unit vector $\hat{\mathbf{o}}_r$, respectively, $\dot{x}_{j,2}$ and $\ddot{x}_{j,2}$ are velocity and acceleration of j-th satellite in the direction pointed by the unit vector $\hat{\mathbf{o}}_\theta$, respectively, whereas $\ddot{x}_{j,3}$ is acceleration in the direction pointed by the $\hat{\mathbf{o}}_h$ vector.

Further, f is a true anomaly of the reference satellite, r and r_j is the current distances from the Earth's center to the reference and j-th deputy satellite respectively, wherein $u_{j,1}$, $u_{j,2}$ and $uj,_3$ are components of a j-th control vector representing control forces acting on j-th deputy satellite and m_j is the current mass of j-th deputy satellite.

Since it is assumed that the deputy satellites manoeuvre using expulsion of significant amount of mass, an impact of j-th deputy spacecraft mass m_j variability has been considered.

Let us assume that $\dot{\boldsymbol{x}}_{j,1} = \boldsymbol{x}_{j,4}$, $\dot{\boldsymbol{x}}_{j,2} = \boldsymbol{x}_{j,5}$, $\dot{\boldsymbol{x}}_{j,3} = \boldsymbol{x}_{j,6}$ and that the positions and velocities of j-th satellite relative to the reference satellite or point can be expressed using the following state vector:

$$\mathbf{x}_j = [\boldsymbol{x}_{j,1} \quad \boldsymbol{x}_{j,2} \quad \boldsymbol{x}_{j,3} \quad \boldsymbol{x}_{j,4} \quad \boldsymbol{x}_{j,5} \quad \boldsymbol{x}_{j,6}]^{\boldsymbol{T}}. \tag{11}$$

Then, the relative motion of j-th deputy satellite can be represented in a state-space:

$$\dot{\mathbf{x}}_j = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \dot{f}^2 - \frac{\mu}{r_j^3} & 2\dot{f}\frac{\dot{r}}{r} & 0 & 0 & 2\dot{f} & 0 \\ 2\dot{f}\frac{\dot{r}}{r} & \dot{f}^2 - \frac{\mu}{r_j^3} & 0 & -2\dot{f} & 0 & 0 \\ 0 & 0 & -\frac{\mu}{r_j^3} & 0 & 0 & 0 \end{bmatrix} \mathbf{x}_j + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m_j} & 0 & 0 \\ 0 & \frac{1}{m_j} & 0 \\ 0 & 0 & \frac{1}{m_j} \end{bmatrix} \mathbf{u}_j + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \mu\left(\frac{1}{r^2} - \frac{r}{r_j^3}\right) \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

or:

$$\dot{\mathbf{x}}_j = \mathbf{A}_j\mathbf{x}_j + \mathbf{B}_j\mathbf{u}_j + \mathbf{V}_j, \tag{13}$$

where $\mathbf{A}_j$ is a state matrix, $\mathbf{B}_j$ is an input matrix and $\mathbf{V}_j$ is a matrix of nonlinear term. The control vector is expressed as:

$$\mathbf{u}_{\boldsymbol{j}} = [\mathbf{u}_{j,1} \quad \mathbf{u}_{j,2} \quad \mathbf{u}_{j,3}]^{\boldsymbol{T}}. \tag{14}$$

2.4. Model of Relative Motion for All Deputies

Let us assume that the constellation consists of N controlled satellites (deputies). In case where the reference point is associated with a physical satellite, the constellation also includes this uncontrolled mothership (a reference satellite). Then, the motion of deputy satellites relative to the reference point can be expressed using the following state vector:

$$\mathbf{x}_c = [\mathbf{x}_1 \quad \mathbf{x}_2 \quad \cdots \quad \mathbf{x}_N]^{\boldsymbol{T}}, \tag{15}$$

wherein $\mathbf{x}_1$, $\mathbf{x}_2$, ..., $\mathbf{x}_N$ are the $\mathbf{x}_j$ vectors defined by Equation 11, where $j \in \{1, 2, \dots, N\}$.

Using this state vector, we can express the motion of all the deputy satellites using the following state-space representation:

$$\dot{\mathbf{x}}_c = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0}_{6,6} & \mathbf{0}_{6,6} & \mathbf{0}_{6,6} \\ \mathbf{0}_{6,6} & \mathbf{A}_2 & \mathbf{0}_{6,6} & \mathbf{0}_{6,6} \\ \mathbf{0}_{6,6} & \mathbf{0}_{6,6} & \ddots & \mathbf{0}_{6,6} \\ \mathbf{0}_{6,6} & \mathbf{0}_{6,6} & \mathbf{0}_{6,6} & \mathbf{A}_N \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 & \mathbf{0}_{6,3} & \mathbf{0}_{6,3} & \mathbf{0}_{6,3} \\ \mathbf{0}_{6,3} & \mathbf{B}_2 & \mathbf{0}_{6,3} & \mathbf{0}_{6,3} \\ \mathbf{0}_{6,3} & \mathbf{0}_{6,3} & \ddots & \mathbf{0}_{6,3} \\ \mathbf{0}_{6,3} & \mathbf{0}_{6,3} & \mathbf{0}_{6,3} & \mathbf{B}_N \end{bmatrix} \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \vdots \\ \mathbf{u}_N \end{bmatrix} + \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \vdots \\ \mathbf{V}_N \end{bmatrix}, \tag{16}$$

wherein $\mathbf{A}_1, \mathbf{A}_2, ..., \mathbf{A}_N = \mathbf{A}_j$ and $\mathbf{B}_1, \mathbf{B}_2, ..., \mathbf{B}_N = \mathbf{B}_j$ while $\mathbf{V}_1, \mathbf{V}_2, ..., \mathbf{V}_N = \mathbf{V}_j$, for j=1,2, …, N.
In order to provide an internal model for the discrete predictive controller, Equation 16 is transformed into discrete state-space representation.

3. Controller Design

Model predictive control (MPC), also referred to as moving horizon control or receding horizon control, is an advanced method of dynamic systems control. This investigation considers discrete time predictive control only.

Model predictive controllers computes control action by periodical solving an optimal control problem over finite future horizon, possibly subject to constraints on the inputs and outputs. The optimal control problem is solved using the current state estimate of the controlled process as the initial state. The optimization yields an optimal control sequence, however only the first control action in this sequence is applied to the plant. The whole procedure is repeated at the next sampling instant. An excellent overview of MPC algorithms and their history can be found in [7]. An introduction to theoretical and practical aspects of the most commonly used MPC strategies is presented in [8]. A survey on the contemporary MPC approaches is provided by [9].

In this investigation, the control of spacecraft relative motion is calculated using a novel formulation of discrete model predictive control algorithm. The basic scheme of the proposed algorithm is presented in Fig. 2. The ultimate goal was to design a relatively simple and reliable controller for the deployment, reorganization and maintenance problems.

The controller is equipped with a discrete, time-variant, nonlinear model of relative motion, obtained by discretization of Equation 16. In Fig. 2, this model is designated as nonlinear model I and takes the following form:

$$\mathbf{x}_c(k+1) = \mathbf{A}_c(k)\mathbf{x}_c(k) + \mathbf{B}_c(k)\mathbf{u}_c(k) + \mathbf{V}_c(k). \tag{17}$$

where k is a discrete time sampling instant. A simplified version of the above model is nonlinear model II, where the matrix of nonlinear term $\mathbf{V}_c(k)$ is omitted and further treated as disturbance to the process dynamics:

$$\mathbf{x}_c(k+1) = \mathbf{A}_c(k)\mathbf{x}_c(k) + \mathbf{B}_c(k)\mathbf{u}_c(k). \tag{18}$$

The algorithm obtains a set of local, time-invariant, linear models by calculation of parameters of nonlinear model II for the current and predicted operation points within prediction horizon. Each local model corresponds to each prediction horizon step. The set of local linear models distributed over the prediction horizon enables for consideration of the model parameters time-variance.

However, the model parameters over the prediction horizon are dependent on future states and future control input values. Since the model parameters estimation is performed before calculation of the current control action, a preliminary control trajectory estimate is found in a heuristic manner using a filter with finite impulse response (FIR).

Each local model is augmented using an embedded integrator, forming a set of Increment-Input-Output (IIO) models. This allows the controller to reject constant disturbances without steady-state errors.

Further, the set of augmented IIO models is used to formulate an output prediction system, in order to predict the output trajectory (of relative motion) within the prediction horizon. The output trajectory prediction system is then provided to the cost function. In its most general form, the cost function can be expressed by:

$$J = \left(\mathbf{R}_s - \widehat{\mathbf{Y}}\right)^T \left(\mathbf{R}_s - \widehat{\mathbf{Y}}\right) + \Delta\mathbf{U}^T \overline{\mathbf{R}} \Delta\mathbf{U}, \tag{19}$$

where $\mathbf{R}_s$ denotes a set-point signal, $\widehat{\mathbf{Y}}$ is the predicted output trajectory (predicted relative motion), $\Delta\mathbf{U}$ is trajectory of control increments to be calculated and $\overline{\mathbf{R}}$ is a weight matrix for the control increments.

$$\mathbf{x}_1^{init} = [-18264 \quad -24438 \quad -326 \quad -14 \quad 26 \quad 3]^T, \text{ meters, meters per second,} \tag{20}$$

$$\mathbf{x}_2^{init} = [6528 \quad 48102 \quad 301 \quad 18 \quad -17 \quad -3]^T, \text{ meters, meters per second,} \tag{21}$$

$$\mathbf{x}_3^{init} = [-6633 \quad -48191 \quad 167 \quad -18 \quad 17 \quad -1]^T, \text{ meters, meters per second.} \tag{22}$$

It is assumed that an initial mass of each of the deputies is equal to 1500 kg, including 500 kg of the propellant. Specific impulse of the deputies' thrusters was assumed as 400 s.

An example of control history (for the deputy 0 case) is depicted in Fig. 3. Position trajectory for all the deputies is presented in Fig. 4-7 presents an example of relative velocity trajectory – case of deputy 0. Finally, Fig. 8 depicts a history of amount of fuel expelled by the deputy 0.

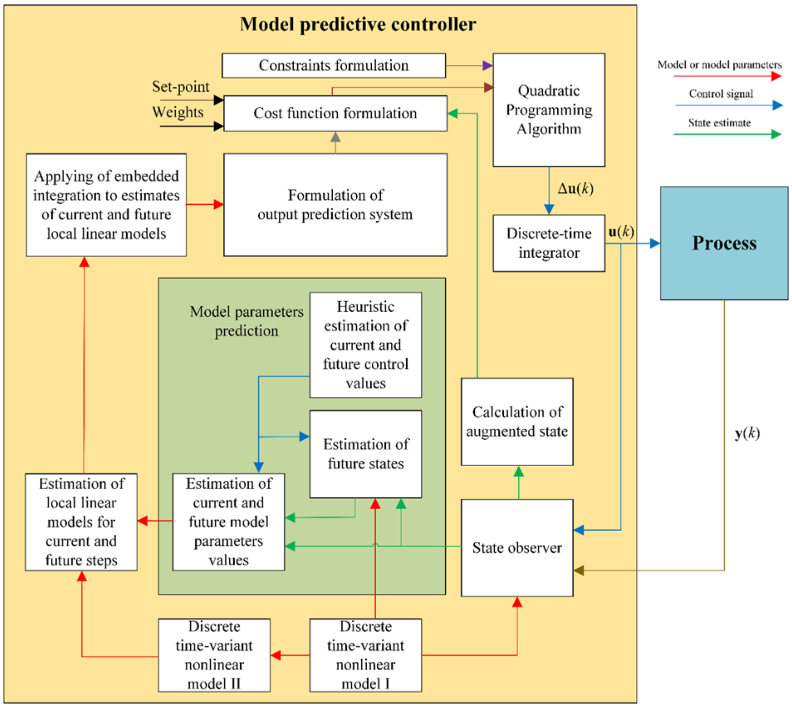

Fig. 2. Operation diagram of the proposed algorithm

The cost function, equipped with the output prediction system, given augmented estimate of the current state, formulated by taking into account of the set point and weights, is fed to the optimization algorithm together with constraints matrices.

Although utilization of the nonlinear model leads to nonlinear, nonconvex optimization, treatments such as generation of the local linear models allowed for reducing of the optimization problem to a quadratic optimization procedure. A detailed description of the algorithm architecture and operation principles is presented in [1].

4. Simulation and Results

The proposed control algorithm was applied to simulation of satellite constellation deployment and maintenance problem, which seems to be much more challenging than only maintenance problem. The constellation consisted of a mothership and three controlled spacecraft (deputies 0, 1 and 2). Initial conditions for the constellation deployment and maintenance process are presented in Tab. 1, in a form of classical orbital elements in order to facilitate their interpretation. As can be seen, the constellation moves in a highly elliptical orbit. The initial conditions for deputy 0 corresponds to state vector given by Equation 4-1, initial conditions of deputy 1 are given by Equation 4-2, while Equation 4-3 provides initial conditions of relative motion for deputy 2. The simulated process would last 30000 s (over 8 hour) in the real world.

Tab. 1. Initial conditions for the simulated process

Orbit parameter	Symbol	Value for the mothership	Value for deputy 0	Value for deputy 1	Value for deputy 2	Unit
Semi-major axis	a	25000	25000	25000	25000	km
Eccentricity	e	0.700	0.701	0.699	0.701	—
Inclination	i	30.00	30.02	29.98	29.99	°
Longitude of the ascending node	Ω	45.0	45.0	45.0	45.0	°
Argument of periapsis	ω	60.0	60.0	60.0	60.0	°
Mean anomaly at epoch 0	M_0	350.00	350.02	350.01	349.99	°

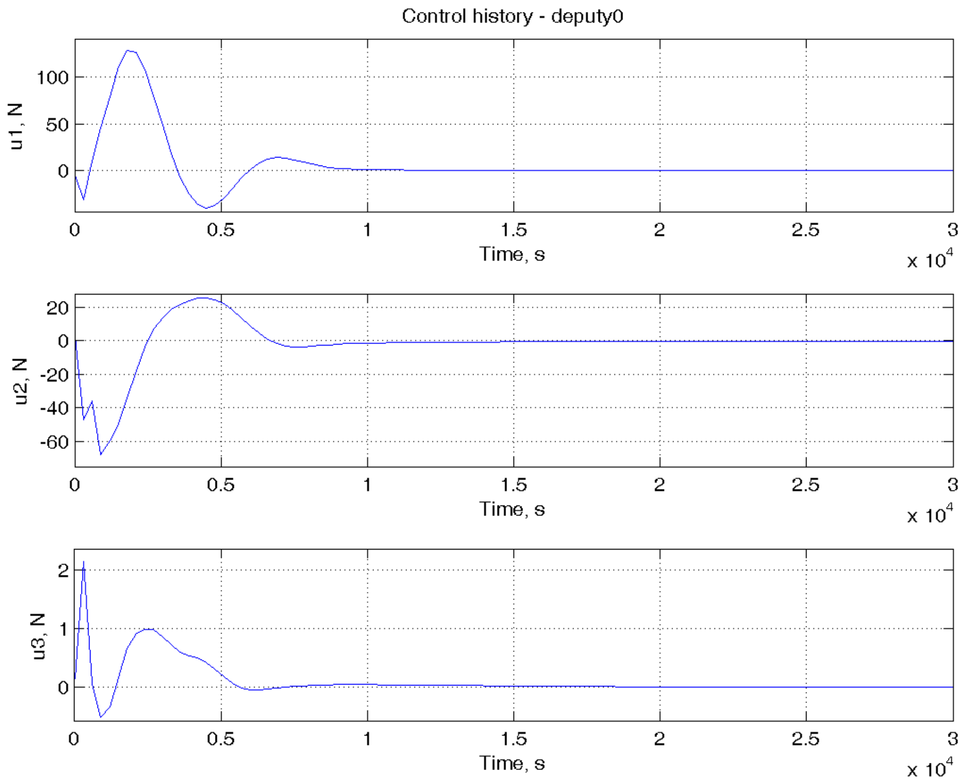

Fig. 3. History of control for deputy 0

5. Conclusion and future work

The simulation results show that the presented algorithm is able to cope with the tasks of deployment, reorganization and maintenance of satellite constellation. Although a nonlinear model of relative, motion was employed as an internal model for the model predictive control algorithm, what leads to a nonconvex optimization, several treatments allowed for reducing of the optimization problem to a standard quadrating optimization problem. The algorithm enables to solve for quasi-optimal control trajectories, where the fuel consumption is minimized, what plays a crucial role in the space industry. As the simulation indicates, the proposed control algorithm allows for asymptotically stable control process even in the case where the constellation is moving in a highly elliptical orbit (eccentricity approximately 0.7).

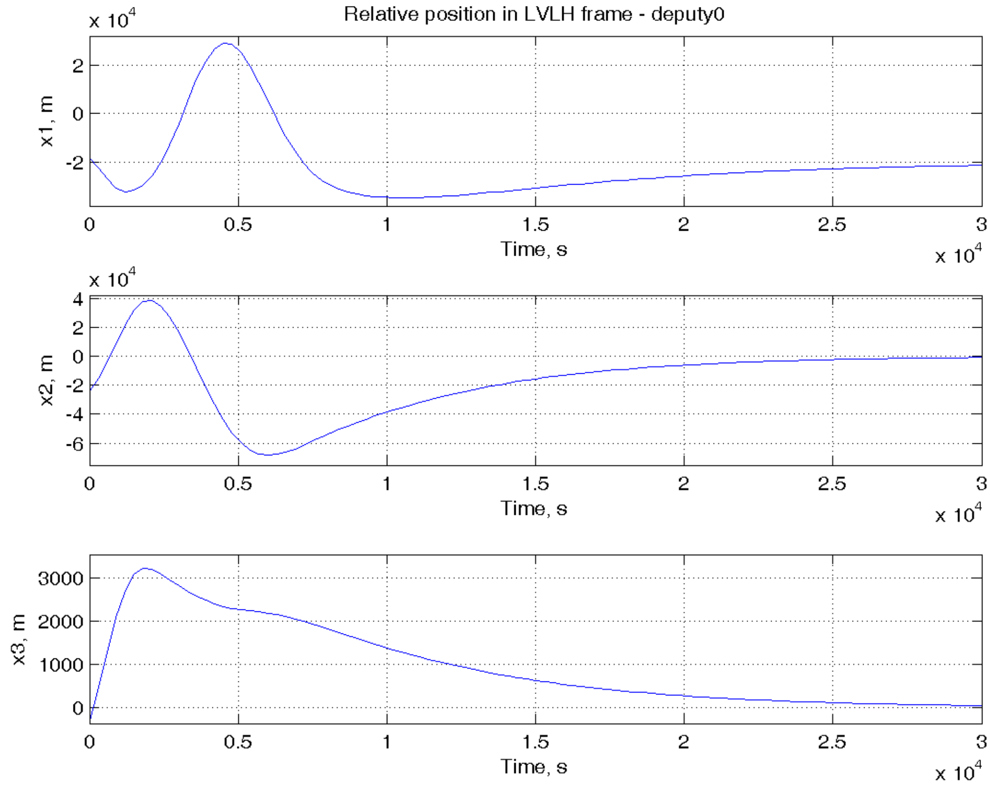

Fig. 4. Relative position trajectory of deputy 0

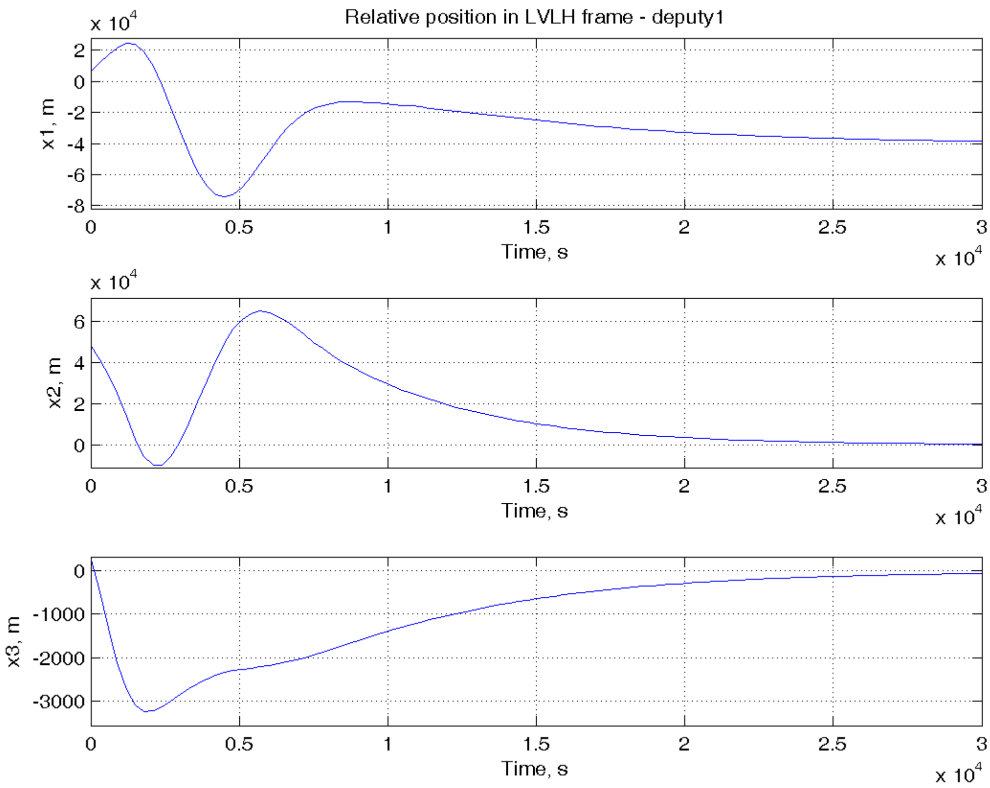

Fig. 5. Relative position trajectory of deputy 1

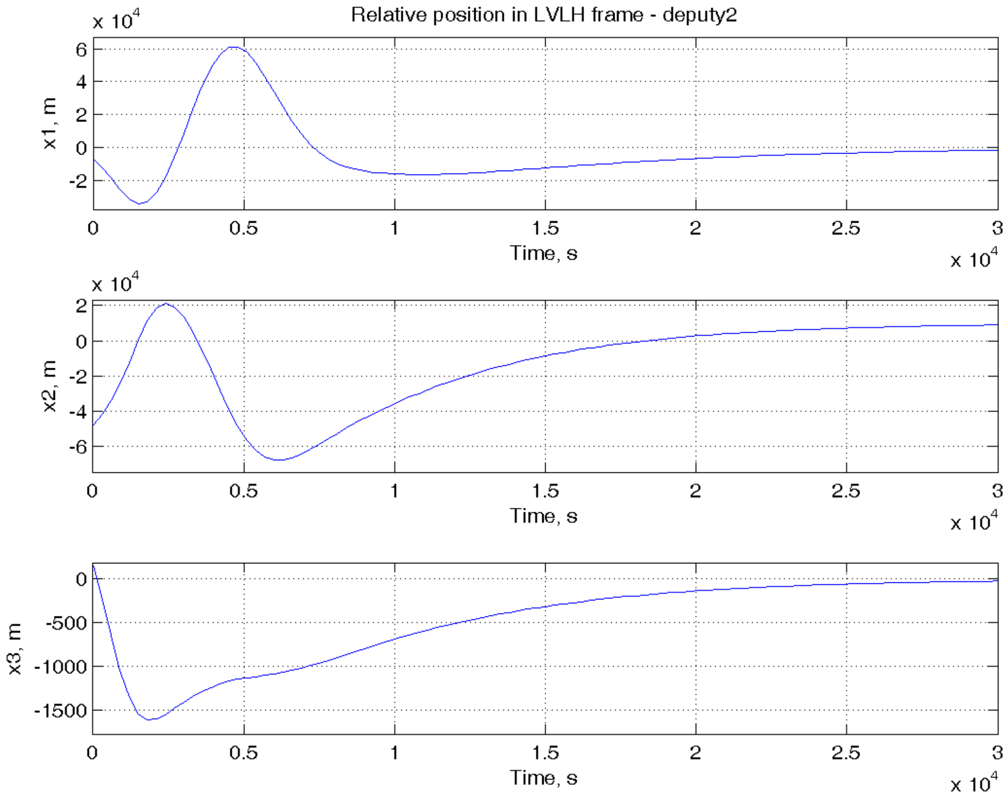

Fig. 6. Relative position trajectory of deputy 2

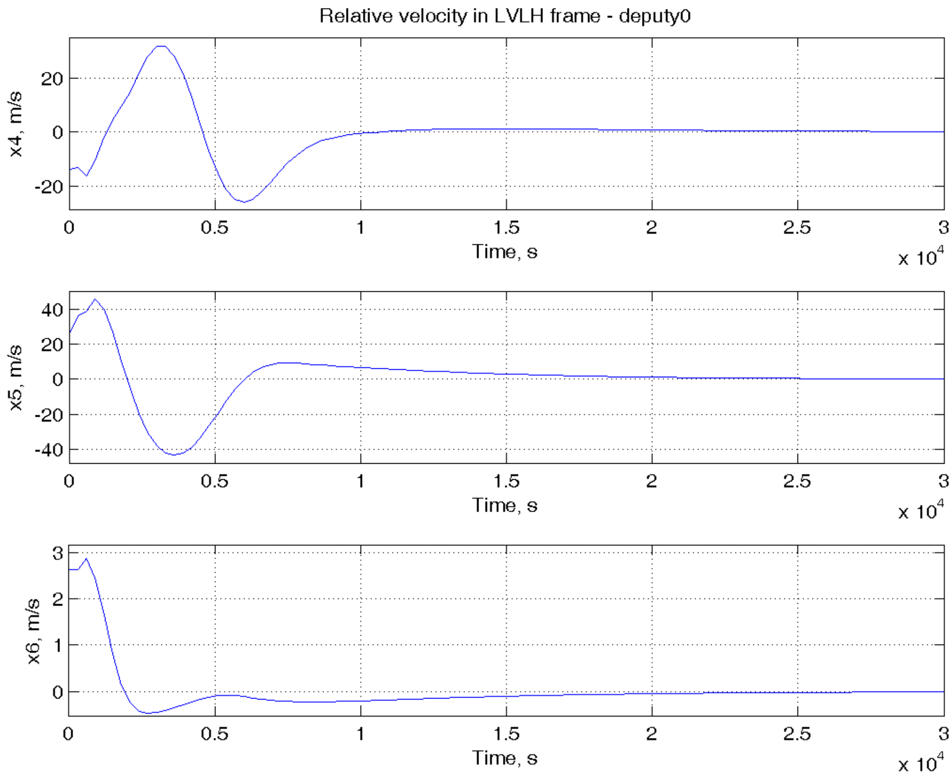

Fig. 7. Relative velocity trajectory of deputy 0

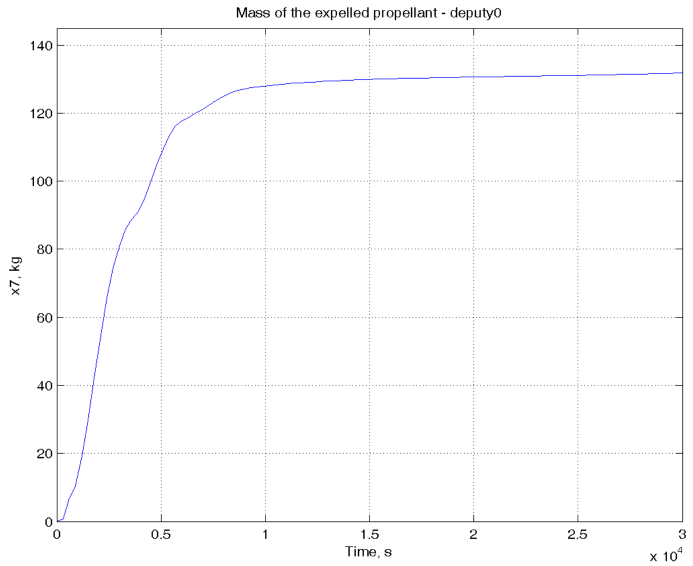

Fig. 8. Fuel mass expelled by deputy 0

In order to provide a complete and reliable control algorithm, able to safe constellation deployment and reorganization manoeuvres, next steps in this investigation should concern collision avoidance. We propose an approach commonly used in robotics: employment of a reference trajectory planner and application of the MPC algorithm proposed in this paper for controlling the relative motion of each deputy satellite according a planned reference trajectory. The planned reference trajectory could have a form of intermediate points, whereas the proposed MPC algorithm would find a quasi-optimal control strategy for steering the system between the planned points. Authors consider an application of genetic algorithm for the reference trajectory-planning task, where the constraints implied by the collision avoidance problem can be implemented. Additionally, for safety reasons, a set of time-variant constraints on state would be imposed in the presented MPC algorithm.

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