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OPTIMISATION OF THE SET OF SENSORS FOR DIAGNOSING THE ROTOR MACHINES' BLADES WITH THE BTT (BLADE TIP TIMING) METHOD

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Abstract

The article presents the issue related to optimisation of a set of sensors used for testing the dynamics of the rotor machines' blades with the BTT (Blade Tip Timing) method. The aim of optimisation is to determine a minimum set of sensors, which provides a collection of information that allows to reliably estimating the blades' vibration parameters for adopted vibration forms and the engine order (EO). The task of optimisation was presented in the form of a multidimensional and multivalued decision problem – discernibility of the engine order (EO) on the basis of the BTT signal implementation in selected points on the perimeter of the rotor machine's groove. For the purposes of its solution, the information theory methods with the use of combinatorial entropy and information were applied. A method, which allows determining sets of measurement points (sensor mounting areas) to ensure the discernibility of EO, taking into account the restrictions on permissible zones of mounting the sensors, was offered. In this article, a method, which allows to determine the sets of measurement points (sensor mounting areas) ensuring the discernibility of EO, taking into account the restrictions on permissible zones of mounting the sensors, was offered.

Keywords: *Blade Tip Timing, optimisation, information theory, combinatorial entropy, combinatorial information*

1. Introduction

The BTT (Blade Tip Timing) method was commonly applied in testing the dynamics and in the diagnostics of the rotor machines' blades [2-8]. One of the important issues is to determine an optimal set of sensors, which provides a collection of information that allows to reliably estimating the blades' vibration parameters for the adopted sets of vibration forms and EO.
The task of optimisation can be presented in the form of a multidimensional and multivalued decision problem – discernibility of EO on the basis of the BTT signal implementation in the selected points on the perimeter of the rotor machine's groove. With such an approach, it is convenient to apply an information theory method with the use of combinatorial entropy and information presented in [1].

In this article, a method, which allows to determine the sets of measurement points (sensor mounting areas) ensuring the discernibility of EO, taking into account the restrictions on permissible zones of mounting the sensors, was offered.

2. Decision problem model

The quantity determined with the use of a tip-timing sensor includes time of the blade reaching the conventional measurement point, specified by the mounting place and the sensor construction.

The measurement is performed once per rotation of the rotor, which results in the necessity to use several sensors arranged on the perimeter in order to collect the required quantity of information [4-8].

The considered signal is a function of the rotor's rotational speed, the blade's vibration forms and EO. In order to specify a set of measurement points (position of sensors), it is necessary to analyse the signal possible implementation in the expected scope of the rotor machine operation.

Due to the fact that collecting data from the experiments on a real object is hindered, in the first approximation, the signal models are used, e.g. [6]:

$$y(t) = b_0 + b_1 \sin(EO * t) + b_2 \cos(EO * t);\ B = \sqrt{b_1^2 + b_2^2};\quad \varphi = arctg(b_2 / b_1), \tag{1}$$

where:

- $b_0, b_1, b_2,$ – coefficients,
- EO – EO.

In order to analyse the signal during one rotation of the rotor, assuming the constant rotational speed, it is possible to use simplified models, e.g.:

$$y_k(t) = 1 + a_k \sin(\mathrm{e}_k\, t + \varphi_k)\,. \tag{2}$$

In the equation (2), the value of 1 corresponds to the rotational speed and e_k – to EO. For a set of analysed E engine orders, with a number n, it can be written as follows:

$$E = \begin{bmatrix} e_1 \\ e_2 \\ \ldots \\ e_n \end{bmatrix};\qquad \mathrm{Y(t)} = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \ldots \\ y_n(t) \end{bmatrix};\qquad A = \begin{bmatrix} a_1 \\ a_2 \\ \ldots \\ a_n \end{bmatrix};\qquad \Phi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \ldots \\ \varphi_n \end{bmatrix}. \tag{3}$$

The adopted model reflects knowledge about the tested object's behaviour (blade of a specific type) in the environment. On its basis, the movement trajectories of the blade's top in the potential positions of sensors, with different values of parameters, for the considered orders, are generated.

From the perspective of the diagnostics, these are not the exact trajectory values that are so important, but their belonging to ranges, which can be interpreted, in the considered set of orders. The determined trajectories can be written in the form of a multivalued code by assigning to the actual value one integer from the following set:

$$\Lambda = \{0, \ldots, \lambda - 1\}\,. \tag{4}$$

The code range width should be at least twice bigger than the measurement resolution. The number of λ code ranges is determined by dividing the maximum gap between the signal's trajectories by the code range width.

As a result, a decision-making model in the form of a multivalued matrix, the rows of which correspond to e_k orders, and columns – to encoded values of the trajectory in the specific points on the perimeter of the rotor machine's groove, is obtained.

Additionally, the restriction may be imposed on permissible zones of mounting the sensors

$$SMC = \left[SMC_1, SMC_2, \ldots, SMC_m\right], \tag{5}$$

$$SMC_i = \left[smca_i : smch_i : smcb_i\right], \tag{6}$$

where:

- $smca_i, smcb_i$ – respectively, the beginning and the end of $i-tej$ permissible zone of mounting the sensors,
- $smch_i$ – step of analysing the positions of sensors.

In this case, the positions, in which the assembly of sensors is prohibited, are removed from the decision-making model's original matrix.

The model should be analysed in order to test whether it contains enough information to discern EO, and if it is possible to reduce the set of the signal measurement points, which leads to minimisation of the set of sensors.

3. Entropy and decision information

The analysis of the decision-making model can be carried out with the use of information theory methods based on the concepts of entropy and information introduced by Shannon or combinatorial entropy and information offered in [1].

In this article, the entropy is interpreted as a measure of complexity/uncertainty of the decision problem.

The entropy $H(E)$ describes the initial uncertainty, before selecting any attribute. Assume that $d_r \in D$ attribute was selected as the first one, and its set of values has the following form:

$$\Lambda = \{0, \ldots, \lambda - 1\} . \tag{7}$$

Based on (8), in E set, it is possible to distinguish $E_j(d_r) \subset E$ subsets:

$$\underset{j=0,\cdots,\lambda-1}{\forall} E_j(d_r) = \{e_{i_j} : S(d_r / e_{i_j}) = j;\, i_j = 1,...,n_j,\ j \in \Lambda\} . \tag{9}$$

In order to simplify the markings of $E_j(d_r) \subset E$ subsets, the indexes consistent with relevant values of $d_r \in D$ attribute were assigned.

By assuming the unambiguity of $d_r \in D$ attribute, the following conditions are met:

$$\underset{\substack{j,l=0,\ldots,\lambda-1 \\ j \neq l}}{\forall} E_j(d_r) \cap E_l(d_r) = \varnothing,\ \bigcup_{j=0}^{\lambda-1} E_j(d_r) = E,\ \sum_{j=0}^{\lambda-1} n_j = n . \tag{10}$$

According to (11) relationship, it results that selected d_r attribute generates distribution of the set of orders, which is symbolically marked in the following way:

$$\{E_j(d_r)\} = \{E_0(d_r), \cdots, E_{\lambda-1}(d_r)\} . \tag{12}$$

The quantity of information, which is included in d_r attribute, is defined as the difference of entropy $H(E)$ and $H(E / d_r)$:

$$J(d_r) = H(E) - H(E / d_r) . \tag{13}$$

After selecting the first attribute, $H(E / d_r)$ entropy is generally greater than zero. Therefore, it is necessary to select the following attributes. The quantity of information contributed by the attribute selected in the k order is determined on the basis of the general relationship:

$$J(d_{(k)}) = H(E / D_{k-1}) - H(E / D_{k-1}, d_{(k)}) , \tag{14}$$

where:

$d_{(k)}$ – attribute selected in the *k* order,

D_{k-1} – set of $k-1$ attributes selected before $d_{(k)}$ attribute.

The selection of subsequent attributes is carried out in accordance with the determined rule, e.g. the main decrease:

$$J(d_{(k)} / D_{k-1}) = \max_{d_r \in D \setminus D_{k-1}} J(d_r / D_{k-1}) \wedge J(d_r / D_{k-1}) > 0 . \tag{15}$$

This process is completed, when information on each of the other attributes is equal to zero. The quantity of information obtained after selecting K attributes with non-zero information is:

$$J(D_K) = H(E) - H(E / D_K) . \tag{16}$$

$H(E / D_K)$ final entropy will reach the value of zero, if the distribution of the order set adopts the form of $\{\{e_i\}\}, i = 1,\dots n$ unit subsets. Otherwise, the final entropy is different from zero, and the attribute that allows for its reduction does not exist. It means that a set of attributes adopted in the model does not provide a distinction of all the orders.

The combinatorial entropy $H_C(E)$, offered in the article [1], with the following form

$$H_C(E) = \binom{n}{2} = 0,5n(n-1) , \tag{17}$$

has the following characteristics:

– is increasing n function:

$$H_C(E) = f(n) , \tag{18}$$

$$H_C(E) \geq H_C(E') \Leftrightarrow n \geq n', \ n = Card(E), \ \ n' = Card(E') , \tag{19}$$

– for $n = 1$, function adopts the value of zero:

$$H_C(E)\big|_{n=1} = 0 , \tag{20}$$

– for any $\{E_j\}, \ j = 1,...,m$ distribution, the following relationship is met:

$$H_C(E / \{E_j\}) = \sum_{j=1}^{m} H_C(E_j), \ Card(\{E_j\}) = m , \tag{21}$$

– if the distribution of the set of orders is given in the form of $\{\{e_i\}\}, i = 1,...,n$ unit subsets, the entropy is equal to zero:

$$H_C(E / \{\{e_i\}\}) = 0, \ i = 1,...,n . \tag{22}$$

The entropy $H_C(E)$ is equal to the number of unstructured pairs of orders, which should be discerned, in the decision-making process.

By using (15)-(20), it is possible to develop the relationship on combinatorial information of attributes. The initial entropy is equal to:

$$H_C(E) = 0,5n(n-1) . \tag{23}$$

If $d_r \in D$ attribute will be chosen as the first one, the entropy will be:

$$H_C(E / d_r) = 0,5\sum_{j=0}^{\lambda-1} n_j(n_j - 1) . \tag{24}$$

The combinatorial information of d_r attribute can be presented in the form of:

$$J_C(d_r) = H_C(E) - H_C(E / d_r) . \tag{25}$$

After substituting (21) and (22) to (23) and transformation, it is possible to obtain:

$$J_C(d_r) = 0,5\sum_{j=0}^{\lambda-1} n_j(n - n_j) . \tag{26}$$

The combinatorial information of $J_C(d_r)$ attribute specifies the number of order pairs discerned by d_r attribute:

$$J_C(d_r) = \sum_{j=0}^{\lambda-2} \sum_{l=j+1}^{\lambda-1} n_j n_l \ . \tag{27}$$

If $d_s \in D$ attribute will be chosen as the second one, the entropy after selecting both $d_r, d_s \in D$ attributes is equal to:

$$H_C(E / d_r, d_s) = 0,5 \sum_{j=0}^{\lambda-1} \sum_{l=0}^{\lambda-1} n_{jl}(n_{jl} - 1) \ . \tag{28}$$

The combinatorial information of d_s attribute, provided that d_r was chosen as the first one, has the following form:

$$J_C(d_s / d_r) = H_C(E / d_r) - H_C(E / d_r, d_s) \ . \tag{29}$$

By substituting (22) and (26) to (27), it is possible to obtain:

$$J_C(d_s / d_r) = 0,5 \sum_{j=0}^{\lambda-1} \sum_{l=0}^{\lambda-1} n_{jl}(n_j - n_{jl}) \ . \tag{30}$$

The information of d_s attribute provided that a set of D_k. attributes was previously chosen and entropy is greater than zero,

$$H_C(E / D_k) = 0,5 \sum_{j=0}^{m_k-1} n_j(n_j - 1) > 0 \ , \tag{31}$$

is determined in accordance to the following relationship:

$$J_C(d_s / D_k) = H_C(E / D_k) - H_C(E / D_k, d_s) \ , \tag{32}$$

where:

$$H_C(E / D_k, d_s) = 0,5 \sum_{j=0}^{m_k-1} \sum_{l=0}^{\lambda-1} n_{jl}(n_{jl} - 1) \ . \tag{33}$$

After substituting (29) and (31) to (30) and transformation, it is possible to obtain:

$$J_C(d_s / D_k) = 0,5 \sum_{j=0}^{m_k-1} \sum_{l=0}^{\lambda-1} n_{jl}(n_j - n_{jl}) \ . \tag{34}$$

The relationship includes the case when D_k set is unit. For $k = 1$ and $m_k = \lambda$, it adopts the same form as (28).

The information $J_C(D_k)$ of k set of $D_k \subset D$ attributes, where $k < card(D)$:

$$J_C(D_k) = H_C(E) - H_C(E / D_k) \ . \tag{35}$$

After substituting (21) and (29) to (33) and transformation, it is possible finally to obtain:

$$J_C(D_k) = 0,5 \sum_{j=0}^{m_k-1} n_j(n - n_j) \ . \tag{36}$$

According to the comparison of (24) and (34), it is clear that both relationships have the same form, when the set of attributes is unit.

If, after the selection of all attributes from D_k set, the uncertainty is greater than zero and $d_s \notin D_k$ attribute was chosen as the next one, in this case, the uncertainty is described by relationship, thus:

$$J_C(D_k, d_s) = H_C(E) - H_C(E / D_k, d_s). \tag{37}$$

After substituting (21) and (29), and transformation, (35) relationship takes the following form

$$J_C(D_k, d_s) = 0,5 \sum_{j=0}^{m_k - 1} \sum_{l=0}^{\lambda - 1} n_{jl}(n - n_{jl}). \tag{38}$$

The carried out considerations result in the following conclusions:

Conclusion 1

The complete information of the set of $D_k \subset D$ attributes and $d_s \notin D_k$ attribute is equal to the sum of $J_C(D_k)$ information and conditional information of d_s – $J_C(d_s / D_k)$ attribute:

$$J_C(D_k, d_s) = J_C(D_k) + J_C(d_s / D_k). \tag{39}$$

Conclusion 2

The information of the set of $D_K = \{d_{(k)}\}, k = 1, ..., K,\ D_K \subset D$ attributes is equal to the sum of conditional information of $d_{(k)} \in D_K$ attributes:

$$J_C(D_K) = \sum_{k=1}^{K} J_C(d_{(k)} / D_{k-1}). \tag{40}$$

It results that combinatorial information of attributes has the property of additivity, which provides its use in multi-stage procedures of optimisation.

4. Optimisation algorithm of the number and location of sensors

The algorithm of multi-stage optimisation of the number and position of sensors on the basis of a multivalued decision-making model with the use of combinatorial entropy and decision information was presented below.

1. Determination of combinatorial decision entropy:

$$H_C(E) = \binom{n}{2} = 0,5n(n-1); \tag{41}$$

2. Calculation of the information value of all the positions of sensors:

$$J_C(d_r) = 0,5 \sum_{j=0}^{\lambda - 1} n_j (n - n_j); \quad d_r \in D; \tag{42}$$

3. Determination of the sets of equivalent positions.

Two $d_r, d_s \in D$ positions are equivalent in terms of information if they generate the same distribution of a set of orders. It means that they have the same values of own information:

$$J_C(d_r) = J_C(d_s) \tag{43}$$

and zero values of mutual information:

$$J_C(d_r / d_s) = J_C(d_s / d_r) = 0. \tag{44}$$

1. Remove from the 1-1 model of positions among 1 of equivalent positions.
2. Determination of the order discernibility matrix defined as follows:

$$\begin{gathered} \Gamma(D) = \left[\gamma_{ij}\right], \quad \gamma_{ij} = Card(D_{ij}), \\ D_{ij} = \{d_r \in D : S(d_r / e_i) \neq S(d_r / e_j)\}. \end{gathered} \tag{45}$$

The element γ_{ij} specifies the number of attributes discerning $e_i, e_j \in E$ ordering. At the same time, the following conditions are fulfilled:

$$a)\ D_{ij} = D_{ji} \quad b)\ \gamma_{ij} = \gamma_{ji} \quad c)\ D_{ii} = \varnothing \quad d)\ \gamma_{ii} = 0\,. \tag{46}$$

In further considerations, two specific values of $\Gamma(D)$ matrix elements are important:

- $\gamma_{ij} = 0$ – there is no attribute discerning e_i and e_j ordering,
- $\gamma_{ij} = 1$ – there is only one attribute discerning e_i and e_j ordering.

In case of $\gamma_{ij} = 0$, it is necessary to modify a set of attributes – increasing their number or changing the encoding rules.

If $\gamma_{ij} = 1$, the attribute discerning e_i and e_j ordering must be obligatory used in the inferential process – it becomes an element of the core of the set of positions.

1. Determination of the core of the set of positions:

$$\tilde{D} = \left\{\tilde{d}_r \in D :\ \gamma_{ij} = 1 \wedge S\left(\tilde{d}_r / e_i\right) \neq S\left(\tilde{d}_r / e_j\right);\ \ e_i, e_j \in E,\ i \neq j\right\}\,; \tag{47}$$

2. Study of the existence of a solution for the core of the set of positions.
   If the complete information of the core's attributes is equal to the initial entropy:

$$J_c(\tilde{D}) = H_c(\mathrm{E})\,. \tag{48}$$

$\tilde{D}$ core constitutes the sought solution of the optimisation task

3. If the core of the set of positions does not constitute a solution – adding successive positions (according to the principle of the maximum of information) to achieve, the problem is zero entropy or zero information values of all the remaining positions.
4. If the entropy does not reach zero – it is important to modify the decision-making model.

5. Example of optimising the distribution of sensors

The example of optimising the distribution of sensors on the perimeter with one zone of permissible positions from p5 to p30 was presented below.

Table 1 contains the encoded values of a signal for the ordering from 1 to 12 and combinatorial information values of selected positions from the range of p5-p30.

Tab. 1. Decision-making model example

	Positions of sensors on the perimeter						
E	p29	p7	p14	p16	p17	p18	p19
e9	0	10	11	10	9	8	7
e10	0	11	10	8	8	7	5
e8	1	10	11	11	10	10	9
e11	1	11	9	7	6	5	4
E12	3	11	8	5	4	3	2
e7	4	10	11	11	11	11	10
e6	7	9	11	11	11	11	11
e1	8	6	7	7	7	7	7
e5	9	8	11	11	11	11	11
e2	10	7	8	8	9	9	9
e3	11	7	9	10	10	10	10
e4	11	8	10	11	11	11	11
J(p#i)	126	116	106	106	116	116	120

The ordering, for which different encoding values of a signal are discernible in the position of p29. For the purposes of discerning, there are three pairs: <e9, e10>, <e8, e11> and <e3, e4>. For this purpose, the second sensor placed in any of the positions {p7, p14, p16, p17, p18, p19} is sufficient.

The example demonstrates the possibility of obtaining, thanks to the offered method, a minimum variant solution of the optimisation problem, which is crucial from the perspective of the technical implementation.

6. Conclusion

The use of multivalued encoding of the movement trajectory of the blades top in the rotor machine results in the multivalued decision matrix, which can be processed with the use of information theory methods.

The optimisation method presented in this article uses the concepts of combinatorial entropy and information of a multidimensional and multivalued decision problem – discernibility of EO on the basis of the BTT signal implementation in the selected points on the perimeter of the rotor machine's groove.

The method can be applied to any models of the observed BTT signals resulting from individual properties of the analysed rotor machines. It leads to an optimal solution (if it exists) in the form of a minimum set of points for mounting the sensors, which provides the EO discernibility.

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