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NUMERICAL ANALYSIS OF TEMPERATURE INFLUENCE ON DEFORMATION AND EFFORT OF BIMETALLIC ELEMENT

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Abstract

Bimetallic elements consist of at least two, connected together, material layers. The materials have different coefficients of thermal expansion. An active layer of bimetallic elements has a greater thermal expansion coefficient, whereas a passive layer has a lower one. Therefore, deformation of bimetallic elements occurs due to a change in temperature. The simplest case of bimetallic elements is a strip, which bends in the shape of an arc. As a consequence of elements deformation, stresses appear in them. An important problem is optimization of each layer of the element to achieve certain characteristics of the system. The elements should work in an elastic domain in the entire operating range. The aim of the paper is to present both numerical and analytical results for three different bimetallic elements. The following pairs of materials are considered: invar – incoloy, steel – brass and aluminium – molybdenum subjected to uniform heating. An influence of temperature and active layer dimensions of the sample on deformation and stresses of each layer was developed. It was found that the stress value is close to zero, on the boundary of each layer, for characteristic deflections of the considered bimetallic elements. The results of numerical analyses were compared with a theoretical solution presented by Timoshenko. Numerical analyses were performed using the Finite Element Method implemented in the commercial software LS-Dyna.

Keywords: *finite elements method, bimetallic elements, thermoelasticity*

1. Introduction

Bimetallic elements are widely used in the industry, electronics and in everyday items. They are composed of two permanently connected layers of materials characterized by a different coefficient of thermal expansion (CTE). This feature influences their properties by dint of which they undergo different forms of deformation than in the case of a single material, due to a change in temperature.

On the grounds of their reliability resulting from operation based not on electrical phenomena but on thermo-mechanical ones, bimetallic elements are often used in a wide range of heat regulators e.g., electric kettles (water boilers). Nowadays, the requirements imposed for modern bimetallic components should fulfil the specific requirements at the possible lowest costs of production, design and development. In the case of bimetallic elements production, it is only about a choice of proper materials and their geometry to set work within their elastic range as well as provide them with clearly defined parameters at the possible lowest use of materials for production.

In this paper, numerical analyses of bimetallic strips subjected to uniform heating were performed and their deflection was measured. Numerical tests were executed for three material compositions currently used for bimetals production: Incoloy 800 H – Invar, Brass (MB30) – Steel and Molybdenum – Aluminium [1-4]. Additionally, an influence of passive layer geometric parameters on element deflection and stress distribution over a cross-section was investigated. It allowed choosing the most optimal solution. The numerical results were compared with the analytical solution, proposed by Timoshenko in 1925, for bimetallic strips characterized by the same width [6].

Numerical analyses were performed using a finite element method implemented in LS-Dyna commercial code, which allows, among others, accomplishment of calculations for coupled mechanical and thermal phenomena [5]. This program allows solving numerical equations of equilibrium with the use of finite element method with an implicit or explicit integration scheme.

2. Numerical model

A finite element model of an investigated bimetallic component was composed of 64000 fully integrated solid elements. It consists of two permanently connected beams with a length equal to 200 mm and a cross section characterized by dimensions of h_1 x d_1 and h_2 x d_2 for an active and a passive layer, respectively. The values of height and width used in simulations for each layer are presented in Tab. 1.

A bimetallic strip was constrained on the opposite edges using a movable and hinged support according to assumptions presented by Timoshenko in the analytical solution.

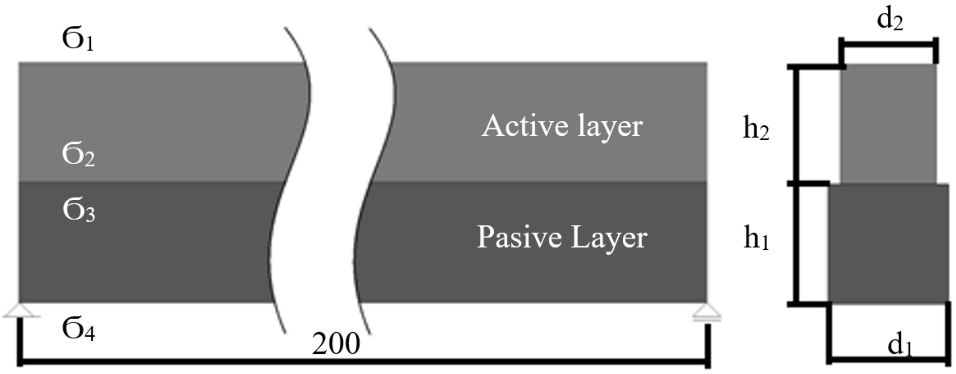

Fig. 1. Numerical model of bimetallic strip [mm]

Tab. 1. Cases of geometry according to Fig. 1

Case	h_1[mm]	h_2 [mm]	d_1 [mm]	d_2 [mm]
I	5	6	5	4
II		5		6
III		5		4
IV		5		3
V		5		2.5
VI		5		2
VII		4		4
VIII		3		4
IX		2		4
X		1		4
XI		0.5		4
XII		0.25		4

Mechanical and thermal properties of materials used for numerical analyses were taken from literature [1-4] and are shown in Tab. 2. In the table, the first material from each group constitutes an active layer, whereas, the second one is a passive layer. Simplifications, in the form of a constant coefficient of thermal expansion and Young modulus as a function of temperature, were adopted to analyses.

The simulated sample was uniformly heated over the entire volume within the range of temperature from 0 to 100 Celsius degrees.

Due to the stationary nature of the phenomenon, an implicit integration method was applied in calculations.

Tab. 2. Mechanical and thermal properties of the used materials [1-4]

Material	Density [kg/m^3]	Young modulus [GPa]	Poisson ratio [−]	Coefficient of thermal expansion [10^{-6}/K]
Incoloy 800 H	7940	197	0.3	14.4
Invar	2710	69	0.3	2.0
Brass (MB30)	8500	115	0.35	19.6
Steel	7940	205	0.29	12.0
Aluminium	2720	69	0.345	23.6
Molybdenum	10800	325	0.293	4.9

3. Numerical and analytical analyses

The results of the preliminary research related to model validation along with the adopted simplifications are shown in Fig. 2. This graph compares the maximum deflections of a bimetallic strip, obtained from numerical simulations and analytical solution proposed by Timoshenko [6], subjected to heating from 0 to 100 Celsius degrees. According to this theory, the maximum deflection of bimetals is described by the formula (1):

$$\delta = \frac{l^2}{8}(\alpha_2 - \alpha_1)(T - T_0)\left[\frac{h}{2} + \frac{2(E_1 I_1 + E_2 I_2)}{h}\left(\frac{1}{E_1 a_1} + \frac{1}{E_2 a_2}\right)\right]^{-1}, \tag{1}$$

whereas, the maximum stress is given by:

$$\sigma_{\max} = \frac{1}{\rho}\left(\frac{2}{h a_1}(E_1 I_1 + E_2 I_2) + \frac{a_1 E_1}{2}\right), \tag{2}$$

where:

l – beam length [m],
α_1, α_2 – coefficients of thermal expansion for each layer [1/K],
$T - T_0$ – Temperature difference [K],
h – beam height [m],
E_1, E_2 – elastic limits for each layer [Pa],
I_1, I_2 – moments of inertia [m^4],
a_1, a_2 – height of upper and lower layer [m],
$\frac{1}{\rho}$ – inverse of deflection radius [1/m].

The most significant error between analytical and numerical results was obtained for a bimetallic element made of Brass (MB30) – Steel. The value of this error was less than 2.5%. An average error for the temperature of 100^0 for all the investigated systems of materials was equal to 1.8%. Additionally, the results of stress distribution in the upper layer of the passive material close to its contact with the active layer received from numerical simulations and analytical equation (2) are shown in Tab. 3. This stress was measured in the middle part of the sample to eliminate an influence of the constrained on the obtained results. In this case, the biggest difference was found for a strip made of molybdenum and aluminium.

Tab. 3. Values of maximum stress in the upper layer of the passive material

Material composition	Theoretical [MPa]	FEM [MPa]	Error [%]
Incoloy – Invar	-104.1	-104.0	0.1
Brass – Steel	-47	-49.8	5.62
Aluminium – Molybdenum	-79.9	-86.2	7.31

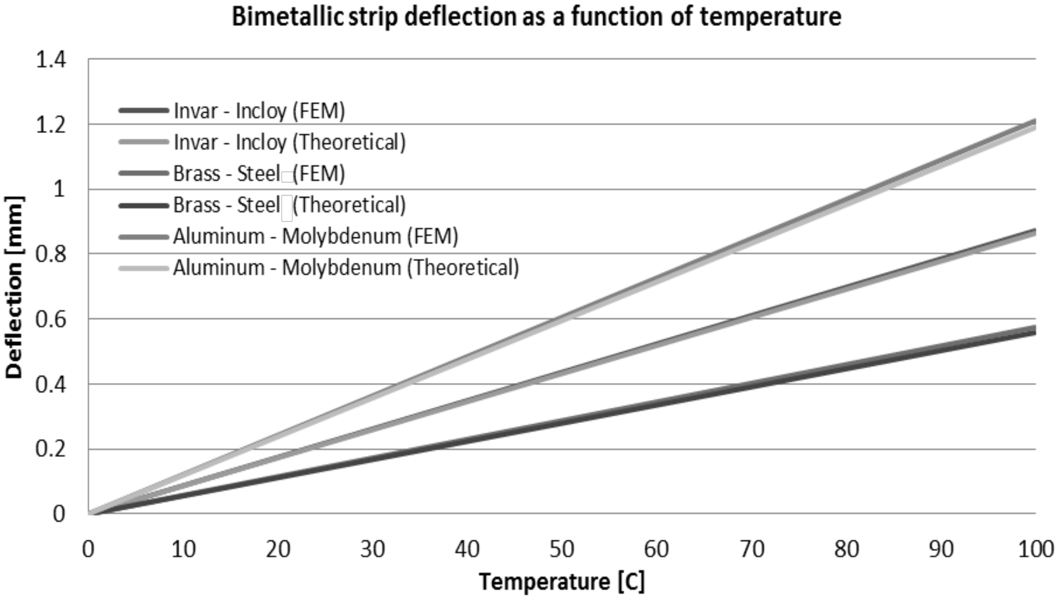

Fig. 2. Deflection of bimetallic strip as a function of temperature

Figure 3 presents a dependence of maximum deflection for a bimetallic strip as a function of the upper layer height. A gradient of deflection value proves that a height of active layer significantly influences displacement and sensitiveness of strip. It can be observed that if elastic modulus E for an active layer is greater than for a passive layer (for example in the case of Incoloy – Invar), deflection increases significantly initially until it reaches a certain threshold value, whereupon it decreases. For other material compositions, the deflection increases monotonically up to a maximum value. The deflection is almost constant (Aluminium – Molybdenum) or decreases slowly (Brass – Steel) for the higher active layer.

Similar results related to an influence of an active layer width are presented in Fig. 4, which illustrates maximum deflection curves for a bimetallic strip subjected to the temperature of 100 Celsius degrees for a width range changing from 2 mm to 6 mm, considering constant passive layer dimensions. In this case, there can be observed a dependence, related to elastic limit, different to the one described above. Compositions, wherein the value of E is higher in the active layer than in the passive layer are characterized by a slow decrease of the deflection with an increase in the width of the active layer. The deflections of other materials compositions do not decreases but slowly increases. For composition of molybdenum and aluminium, deflection for both a minimum and maximum value of the active layer width increases to approximately 33%, whereas for Incoloy – Invar, the deflection decreases approximately to 11.5%.

For each analysed geometry, stress distribution in boundary elements of both active and passive layers was determined (Tab. 4). Stress σ_1 and σ_2 correspond to the external points of the active layer, whereas, σ_3 and σ_4 correspond to the same points of the passive layer (Fig. 1). Therefore, values σ_2 and σ_3 correspond to stress on the contact border between the material layers analysed in the bimetallic strip. The stress range was measured in the middle part of the sample.

The differences in axial stress for each upper layer dimensions are caused by shift of strip cross section neutral axis. Moreover, for each composition of the investigated materials, there can be distinguished such geometry of the active region that stresses occurring in the sample oscillates around zero (bolded in Tab. 4). It was found that a change of active layer width insignificantly influences stress distribution in the sample, which is associated with a small change of the form. The highest value of stress occurred, as expected, for cases, wherein deflection reaches the highest value. The only exception is Incoloy – Invar composition, in which 92 MPa is obtained for maximum deflection of 1.26 mm, whereas, for minimum deflection of 0.601 mm, the obtained value of axial stress was equal to -166 MPa.

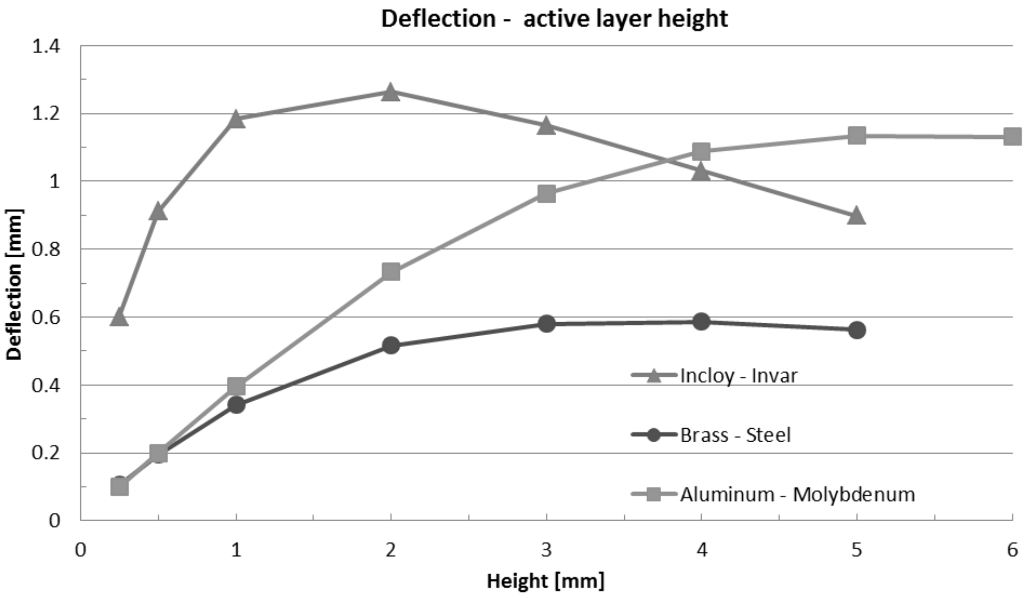

Fig. 3. Deflection curves of bimetallic strip for active layer height

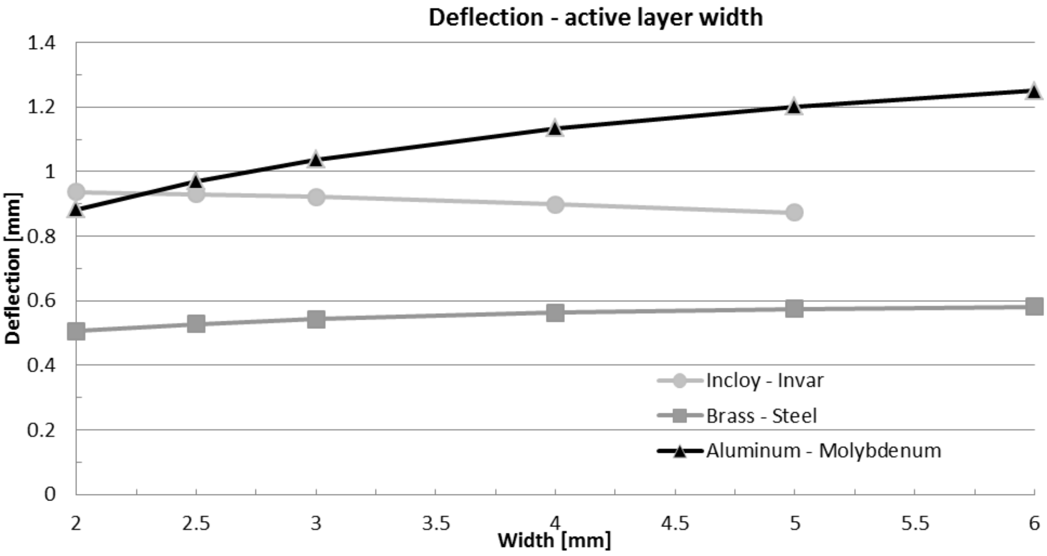

Fig. 4. Deflection curves of bimetallic strip for active layer width

Tab. 4. Axial stress for four selected points in bimetal layers

Case	б$_1$			б$_2$			б$_3$			б$_4$		
	I	**II**	**III**	**I**	**II**	**III**	**I**	**II**	**III**	**I**	**II**	**III**
I	-	-	7.51	-	-	-91.1	-	-	204	-	-	-137
II	-	17.5	5.79	-	-40	-62.6	-	80	259	-	-40	-149
III	62.4	13	-7.32	-116	-53.4	-90.9	41.3	64.6	205	-12.4	-39.8	-138
IV	61.8	8.83	-18.1	-120	-55.3	-95.2	40.6	61.5	186	-15.7	-39.3	-127
V	60.7	5.67	-25.5	-122	-56.5	-98.1	30	59.3	174	-17.4	-38.6	-119
VI	58.5	1.26	-39.4	-124	-58.2	-102	39.4	56.3	158	-19	-37.4	-109
VII	53.6	3.92	-26.7	-110	-52	-92.6	43.1	-66.8	198	-19.4	-42.3	-131
VIII	37.2	-9.58	-51	-101	-52.2	-97.4	45.8	66.4	177	-25.6	-42.1	-114
XI	6.89	-29.2	**0.68**	-92	-56.3	**-29**	48.5	59.8	**-8.52**	-29.5	-36.6	**0.007**
X	-53	-55.6	-106	-100	-67.9	-120	45.9	-39.9	74	-27.3	-22.9	-42.6
XI	**7.62**	-70.6	-117	**-23.9**	-76.6	-125	**-8.04**	22.6	33.7	**0.007**	-12.4	-20.6
XII	-166	**-4.2**	-127	-166	**-4.2**	-127	24.2	**-2.08**	18.8	-13.1	**0.002**	-10

4. Conclusions

On the grounds of numerical analyses, it was found that a change of geometry and mechanical properties of bimetallic materials significantly influences both characteristics deflection as a function of temperature and effort of material individual layers. The numerical results for the simplest case were compared to the analytical solution with accuracy at the level of a few percent.

It was also found that for every material composition it is possible to determine such geometry of layers cross sections for which the maximum stress value on both an active and a passive layer will be insignificant, despite the significant element deflection. It was proved that in the case of different geometry of the individual bimetallic layers, Young modulus significantly influences maximum deflection of the strip against the Timoshenko solution, according to which Young modulus for the same geometries of both active and passive layers has almost negligible influence on the results.

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