

# MODELLING OF THERMODYNAMICS, MASS EXCHANGE AND HEAT TRANSFER OF PLASMATRON PLASMA REACTOR FOR RECYCLING OF USED ELECTRIC AND ELECTRONIC COMPONENTS

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## **Abstract**

*New technologies and systems require precise insight of their parameters, which are in some cases difficult to measure. One of technique for obtaining of such data is simulation and modelling of objects, processes and their parameters. Application of mathematical modelling for thermodynamics, mass exchange and heat transfer calculation is presented in this article. Mathematical model of the plasmatron plasma reactor was developed and allows simulation of reactor and process parameters including: heat release during combustion, heat transfer from combustion process and exhaust gases to the reactors walls, temperature calculation inside the chamber, surface film conductance by convection and radiation, heat carried out with fumes, and other.*

*The model was implemented in OCTAVE software and it can be calculated for relatively short time on single PC computer. Model of plasmatron reactor chamber allows quantitative simulation of the phenomena occurring in the exanimated process. Moreover, model allows answering how to control the process, and identification of correlations with sensitivity analysis.*

*The article presents results of modelling of key process parameters of the plasma reactor essential for developed process understanding and analysis. Calculations results are interesting and important for further technology development for example, change of heat carried with fumes, or surface film conductance, which are hard to measure.*

*Developed model offers good accuracy of calculations and correlation to measurements on experimental setup, and allows further optimization, scaling and development.*

**Keywords:** *model, simulation, recycling, energy recovery, combustion, environmental protection, electric and electronic waste*

## **1. Introduction**

The Industrial Research Institute for Automation and Measurements in Warsaw has developed a new plasma technology, for recovery of metals from printed circuit boards. The technology includes a plasma reactor in which the processes of smelting and incineration of waste printed circuit boards takes place. In order to understand the processes occurring in the plasma reactor, as well as enabling optimization of developed technology, we have developed a mathematical model of the reactor and of the developed process.

This mathematical model of the process enables the simulation of quantitative key phenomena occurring in the reactor and sensitivity testing of process parameters necessary for its optimization. The model of plasma technology for smelting metals from printed circuit boards, maps occurring thermodynamic processes, and exchange of heat and mass. The model allows answering the

question of how to control the process to minimize energy consumption and maximize throughput. In addition, it is used in the development of the control system and selection of its settings and control parameters. Using the model it is possible for example to calculate the mass of waste portions and their feeding frequency, to control the temperature of the process. Based on the implemented equations, it is possible, to determine the heat carried out with the flue gas, and both heat transfer coefficients convective and radiative for the reactor wall.

## 2. Description of modelled system

The process model is implemented using Octave [1] software and consists of two sub programs:

- Model of heating of the plasma reactor to operating temperature.
- Model of combustion and smelting of metal and slag from the processed portion of the waste.

Due to the limitation of the volume of this work, only the selected topics and equations have been presented from the first part of the model.

Model of reactor heating comprises, a heating of its chamber by providing heat in the form of three streams of plasma generated in the three plasmatrons. The model takes into account heat transfer through the walls, carrying out of the heat by the exhaust gases, heat transfer coefficients, momentary temperature inside the reactor, and many more related phenomena necessary for the mathematical description of the modelled process.

Figure 1 presents a block diagram of the modelled plasma reactor with its inputs and outputs.

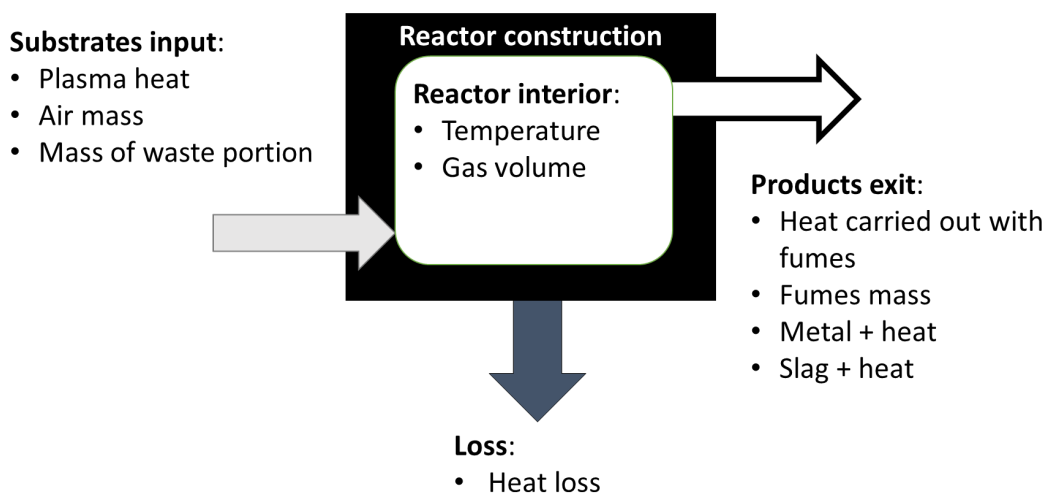


Fig. 1. Block diagram of the plasma reactor, presenting inputs and outputs

## 3. Model equations

The model calculates heating up of the reactor, basing on the 27 equations. All values are calculated as the instantaneous values. Simulation step was 1/8 second. Due to limited volume of this publication, only selected equations are presented. Each value in the model is calculated for momentary temperature inside the reactor:  $\rho_{Pow}$  – air density,  $C_{WPow}$  – specific heat of air,  $m_v$  – gas mass in the chamber,  $E_v$  – the heat accumulated in the gas inside the reactor chamber,  $m_{pp}$  – mass of air supplied with the plasma,  $E$  – the energy supplied with the plasma,  $P_{strat}$  – energy dissipated as a result of reactor loss,  $C_{wb}$  – specific heat of the concrete,  $C_{wi}$  – specific heat of the isolation,  $\mu$  – kinematic viscosity,  $T_p$  – the gas temperature in the reaction chamber,  $T_{s(lwarstw)}$  – the temperature distribution in the reactor wall for a given number of points on its thickness,  $dE_{r(1)}$  – heat change in the wall over time for a given number of layers [4],  $dE_{s izol}$  – heat change in insulation layer over time, etc.

Furthermore, certain variables such as the tabulated ones for example: heat conduction coefficient  $\gamma$ , based on the table data [7], is approximated by polynomial, and is being calculated for the instantaneous temperature in the reactor from the equation (1):

$$\gamma(t) = (6 \cdot 10^{-10} \cdot T_p^3 - 2 \cdot 10^{-6} \cdot T_p^2 + 0.0069 \cdot T_p + 2.3948) \cdot 10^{-2} \left( \frac{\text{W}}{\text{m} \cdot \text{K}} \right), \quad (1)$$

where  $T_p$  is the gas temperature in the reactor chamber ( $^{\circ}\text{C}$ ).

Below selected model equations are presented, on which calculation of process simulation is being carried out.

The thermal energy absorbed by the reactor wall was calculated by the equation:

$$dE_k = [\alpha_{1T}(T_p) \cdot S_p \cdot (T_p - T_{s(1)}) \cdot dt] + [\alpha_4 \cdot S_p \cdot (T_{plazmy} - T_{s(1)}) \cdot dt] \quad (\text{J}), \quad (2)$$

where:

$\alpha_1$  – convective,

$\alpha_4$  – radiative,

$S_p$  – internal surface area of the reactor walls ( $\text{m}^2$ ),

$T_{s(1)}$  – the temperature of the first layer of the inner reactor wall, the mean value ( $^{\circ}\text{C}$ ).

The mass of the exhaust gas is calculated from:

$$m_s = \dot{m}_{pp} + V_r \cdot [\rho_{Pow}(T_p + dT_p) - \rho_{Pow}(T_p)] \quad (\text{kg}), \quad (3)$$

where:

$\dot{m}_{pp}$  - the stream of air supplied to the reactor with plasma (kg/h).

$V_r$  - the internal volume of the reactor chamber ( $\text{m}^3$ ).

The momentary heat carried with flue gas  $dt$ :

$$dE_{spal} = C W_{Pow}(T_p) \cdot dm_s \cdot T_p \quad (\text{J}). \quad (4)$$

Computational temperature of the plasma is determined from the dependence:

$$T_{plazmy} = \frac{P_p}{\dot{m}_{pp} \cdot C W_{Pow}(T_{plazmy})} \quad (\text{K}), \quad (5)$$

where  $P_p$  is thermal power of the plasmatrons released into the reactor, including their efficiency (kW).

Change of energy in the reactor cavity after time – energy balance of the reactor chamber [6]:

$$dE_v = dE - dE_k - dE_{spal} - (P_{strat} \cdot dt) \quad (\text{J}), \quad (6)$$

where:

$P_{strat}$  – the parameter describes heat losses of the reactor as a function of temperature (cooled components: camera, feeding system, plasmatrons, and cold air infiltration) (J),

$dE$  – the momentary energy delivered by the plasma,

$dE_{spal}$  – the momentary heat carried with flue gas.

Convective heat transfer coefficient (surface film conductance) for the heat from gas transferred into the reactor walls:

$$\alpha_{1T}(T_p) = \frac{\text{Nu} \cdot \gamma}{d} \left( \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \right), \quad (7)$$

where  $d$  is equivalent diameter (m).

Nusselt number was calculated according to the formula, as in the case of a flat plate, based on the values calculated for the instantaneous temperature in the reactor [7]:

$$\text{Nu}(t) = 0.664 \cdot (\text{Re}^{0.5} \cdot \text{Pr}^{0.33}) \cdot 1 \cdot \left( \frac{\text{Pr}_f}{\text{Pr}_w} \right)^{0.25} \quad (\text{Dimensionless}), \quad (8)$$

where Pr is Prandtl number.

Reynolds number is calculated from the equation [3]:

$$\text{Re}(t) = \frac{v \cdot L}{\mu} \quad (\text{Dimensionless}), \quad (9)$$

where:

$v$  – velocity.

$L$  – equivalent diameter.

In addition, the radiative heat transfer coefficient is calculated for the internal reactor chamber walls, of fading out plasma heat radiation:

$$\alpha_4 = \frac{(P_p \cdot dt) - dE_{spal}}{(T_{plazmy} - T_{s(1)}) \cdot S_p} \quad \left( \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \right), \quad (10)$$

#### 4. Simulations results

The simulations carried out using the developed mathematical model of plasma reactor, enabled identification of its key process parameters. Simulations on one hand reflect some of the measured physical values such as temperature, flow, which measurement is easy. On the other hand, they calculate the parameters whose values are not available for direct measurement, or an error of their measurement is significant. Below selection of modelled operating parameters of the plasma reactor, are presented.

Figure 2 presents calculated results of the temperature in the plasma reactor chamber during its warming up, together with measured temperature profile during one of the experiments. Constant power fed to reactor was assumed  $E$ . Initially, the temperature rises at high speed. Then the temperature, increase slows down, as it is limited by the amount of heat that conduct through the reactor walls to the environment and amount of heat being carried out with the flue gas. At the end, sudden temperature drop is presented when the plasmatrons power was switched off.

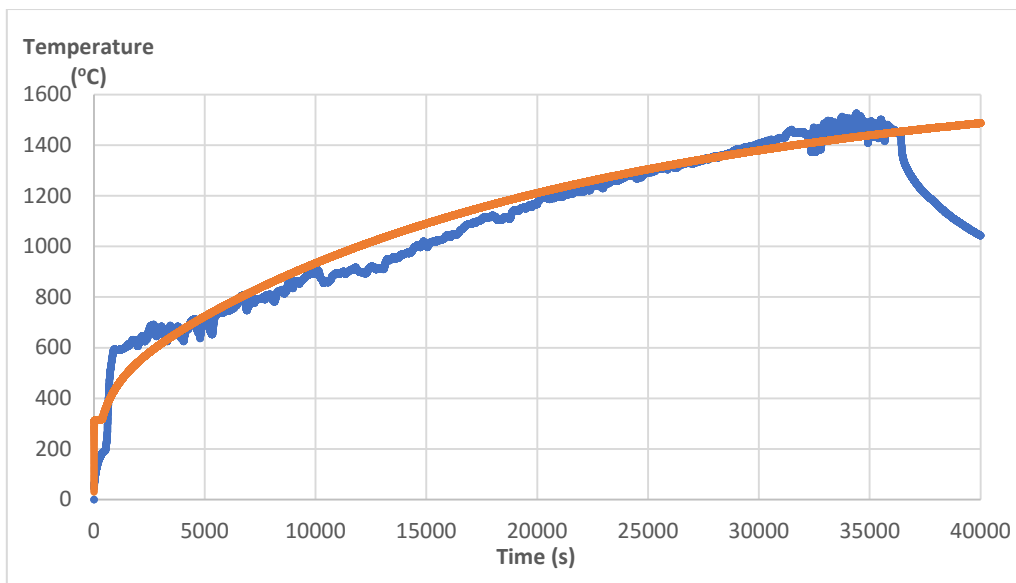


Fig. 2. Result of modelling of the temperature inside the plasma reactor during heating up (orange line), together with the measured temperature during one of the experiments (blue line)

Presented in Fig. 2 graph confirms that the simulation result accurately reproduces the recorded temperature during heating up of the reactor, and the temperature of some 1400°C is reached in both cases after about 30,000 seconds.

Next in Fig. 3 are presented, the results of time-course modelling of heat carried out with the exhaust gases from the reactor during its heating to the operating temperature. Slow increase of the amount of heat carried out with the flue gas is visible, in proportion to the temperature of the reactor chamber. The simulation result is correct and in line with expectations.

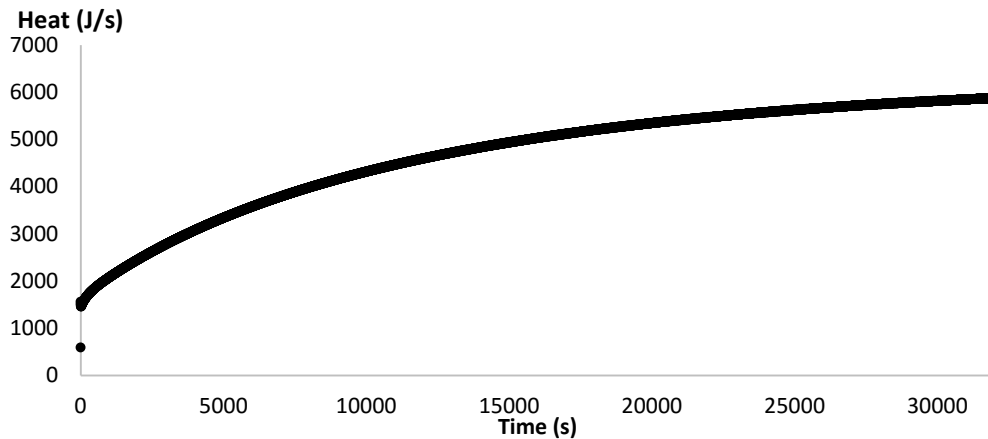


Fig. 3. Result of modelling of the heat carried out from the reactor by the fumes, during heating up of the plasma reactor

Figure 4 shows the simulation of the coefficients of heat transfer (surface film conductance) to the walls of the reactor chamber as a function of temperature. The obtained result is interesting in this type of application, because the measurement of those coefficients is associated with many difficulties, and considerable error of measurement. The calculated convective heat transfer coefficient  $\alpha_{konwekcji}$ , initially has the value close to 6 W/(m<sup>2</sup>·K), and with the increase of temperature, decrease of gas density, its value decreases to about 5 W/(m<sup>2</sup>·K). On the other hand  $\alpha_{promieniowania}$ , in investigated case is at a similar level of about 2.3 W/(m<sup>2</sup>·K). These results were obtained based on data on mass flow, given geometry and calculations of Reynolds (9) and Nusselt numbers (8), as well as with approximated polynomial coefficients occurring in these equations, inter alia equations (1), (7) and (10).

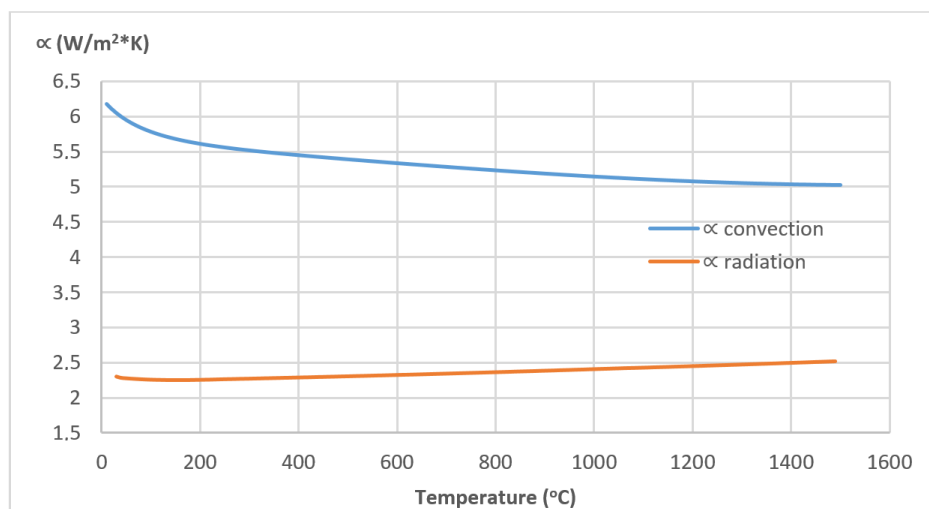


Fig. 4 Modelling results of radiative heat transfer coefficient (orange) and convective heat transfer coefficient (blue) inside the plasmatron plasma reactor in function of temperature

## 5. Results discussion

In the examined case, the heat transfer in the reactor chamber between the gas and the reactor walls takes place through convection and radiation. Therefore, the total heat transfer coefficient is the sum of convection and radiation coefficients:

$$\alpha_{calc} = \alpha_{konwekcji} + \alpha_{promieniowania} \left( \frac{W}{m^2 \cdot K} \right), \quad (11)$$

where  $\alpha_{konwekcji}$  is calculated from equation (7). As it results from the calculation of  $\alpha_{konwekcji}$  its value ranges in 5-6 W/(m<sup>2</sup>·K). As to the  $\alpha_{promieniowania}$ , in examined case its calculated value is about 2.3 W/(m<sup>2</sup>·K).

For the calculation of the radiative heat transfer coefficient, it has been assumed that the energy from the plasma stream is radiated after leaving the plasmatron into the chamber. Radiation of the plasma energy is irreversible and is absorbed by the walls of the reactor chamber. It was assumed that the air in the reactor does not participate in the exchange of heat by radiation. This simplification is justified, because of the closed construction of the reactor chamber, as any of the reflected radiation is absorbed by the opposite walls. Assumed mechanisms for modelling calculations are based on observed phenomena. Hot plasma stream of temperature of about 11,000 K flows into the reactor, emitting strong thermal radiation including in the visible spectrum. Approximately in 25 cm from the outlet of the plasmatron plasma “fades out”, and becomes invisible due to the “cooling” by radiation and convection, transmitting its energy to the reactor chamber. “Fading out” of plasma stream is a result of the drop of its energy (temperature) to a level which no longer emits visible radiations – the ionized gas (plasma) becomes “normal gas” that had returned to its initial state prior to excitation [5]. The radiative energy is only emitted out from the plasma stream, and it does not excite other particles, in a degree that would justify taking into account also secondary ionization phenomenon in the model. Based on the above assumptions and the equation (10) it was calculated that the amount of energy radiated from the plasma to the walls equals of an average 35 kW. This energy is absorbed by the inner surface of the reactor chamber, which is 1.81 m<sup>2</sup>. It should be noted that the nominal low radiative heat transfer coefficient (2.3 W/(m<sup>2</sup>·K)), results in a major part of heat that is transmitted due to huge difference in temperature. Computational temperature of the plasma is about 11,000 K, and the wall temperature is 20-1500°C. This means that the energy transferred to the wall is about 35 kW. For comparison, the convective heat transfer coefficient for heat transfer from gas to the reactor wall, is about 5-6 W/(m<sup>2</sup>·K). For given temperature difference between gas and wall, calculated convective heat transfer rate is 3440 W at the beginning of the heating of the reactor chamber, and 2680 W at the reactor temperature in operating parameters.

The carried out and presented simulations result are the interesting because they reveal that in the examined case, the main way of heat transfer from the plasma to the walls of the reactor is through the radiation.

## 6. Conclusions

The presented results of the modelling of the main operating parameters of the plasmatron plasma reactor, allows to learn the process and to analyse the developed technology. The achieved accuracy of the simulations and accuracy of mapping of the measured values is sufficient and can be used for optimization of the developed process, and to scaling it up as well as for further development in a technical scale.

The model not only reproduces the measured in experiments parameters of the process, but also allows an insight and understanding of the phenomena which measurement is difficult.

Reassuming the model allows simulations of:

- heat transfer coefficients for heat being transferred to the walls of the reactor,
- change of the mass of the exhaust fumes, and the amount of heat carried out with the flue gas, as well as flue gas momentary temperature,
- the temperature inside the reactor and the temperature of the reactor wall, including temperature of the thermal insulation,
- energy balance of the reactor,
- plasma temperature.

The data obtained from the model for example the exhaust gas mass, the amount of heat carried away with fumes, are interesting and important for the further development of this technology. Those values are essential for the development of the test stand and its extension by an additional afterburner – combustion chamber for exhaust gases, and next for the heat recovery. Based on the modelling results it is possible to calculate the parameters of these components in relation to the assumed throughput and load of those facilities.

Next to the presented selected elements of the model, further works planned will include additional modules and functions in developed model, among others planned are:

- the complete combustion stoichiometry on the basis of the waste feedstock chemical composition [2]. This issue is crucial for the modelling parameters of the afterburner chamber and flue gas cleaning processes,
- modelling of the reactions occurring within the molten metal and slag, including construction changes in the test stand to enable the retention of the molten metal and slag in the reactor. This will allow simulations of oxidation and reduction reactions in smelted metals and the molten slag from the processed waste. This topic is necessary for the analysis of the feasibility of the initial metals converting in the plasma reactor utilizing a plasma stream. This operation will allow precise control of the final composition of the produced alloy, which is produced in the reactor.

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