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PARAMETRIC AND NONPARAMETRIC DIAGNOSTIC MODELS FOR BLADES IN THE ROTATING MACHINERY WITH ENVIRONMENT ELIMINATION

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Abstract

The article presents an analysis of the technical condition of the two blades with different technical conditions (for medium and low wear). It is shown that the evaluation of the blades basis only on the endoscopic research does not provide full information about the technical condition (known only the external condition of the blades).

Proposed supplement endoscopic examinations of is the use of two parametric methods (the blades are tested during operation of rotating machine) based on the auto and cross power spectral density functions for the signal measured as displacement of the rotor blade tip y(t). As result of these methods is the portrait of the blades was obtained (in the form of clear colour images providing a degree of a wear), which also includes the internal condition of the blades.

The additional complement to parametric methods may be nonparametric method. Auto and cross power spectral density functions (obtained from measurement of the blade tip movement) is a basis to determine the characteristics (abstract step and impulse responses) of each blade. Both parametric methods which nonparametric method clearly confirmed that the technical condition of first considered blade (no. 2) differs from the technical condition of the second blade (no. 22).

Keywords: diagnostic model, phase shift, amplitude gain, power spectral transfer function

1. Introduction

Fault diagnosis of the blades in the rotating machine during its operation is usually based on the measuring of the time intervals between the following blades tip passing by the sensor, known as Non-contact Stress Measurement or blade tip-timing. Physical quantity directly measured is the time of blades arrival (TOA). This signal allows detection and location of potential cracks or its condition close before fracture. [2, 4, 11-13, 16, 17, 19-23]

Parametric method described in the Patent Office Bulletin No. 18(957) 2010, No. A1(21) 387596(22) 2009.02.20 "A method of technical condition changes monitoring in the turbine rotor blades based on the parameters quotient of the diagnostic signals amplitude gains" – model based on auto power densities. Model uses a signal from the blade working machine in two consecutive periods T_{01} (time approaching the blade to the sensor) and T_{12} (01 (time recede the blade to the sensor) [13]. Signal powers are determined for the time T_{01} and T_{12} , which are used to calculate amplitude gain A^2_{T01} and A^2_{T12} and their quotient A^2_{T12T01} – the parametric diagnostic model. This

model has been verified in the laboratory.

Next parametric method was described in the Patent Office Bulletin No. 25(1016) 2012, No. A1(21) 394970(22) 2011.05.23 "A method of technical condition changes monitoring in the turbine rotor blades during its work, based on the diagnostic signals phase shift difference", using a model based on cross power densities. In this method blades environment is described by the distribution $\delta(t, \tau)$, such that the signal observation time described the distribution was equal to the observation of the measured signal (the same number of samples in the signal), and the power of the measured signal diagnostic of power established distribution was adequate. This allows obtaining cross power density blades displacement signal y(t) and distribution $\delta(t, \tau)$ (environment) in two sub-periods of blades time observing T_{01} and T_{12} and next the difference of the phase shifts φ_{T01} and φ_{T12} and finally their quotient φ_{T12T01} , which is a parametric diagnostic model. This model also has been verified in the laboratory. [4, 11, 12]

Above models are complex variable "s" with a form referring to complex transfer functions, which can be transformed to a spectral transfer function variable $j\omega$. The real part of the power spectral transfer function $P(\omega)$ can be used to determine the impulse g(t) and step h(t) response of the blades in the rotating machinery during its operation. [3, 15]

These characteristics are the non-parametric models of the diagnostic blades. Changes of the characteristics may be related with the changes of the blades technical condition.

2. Description of the object. Diagnostic signal. Time observation object T01-T12

The test object was blades of first compressor stage of the turbine engine SO-3. Examination of the blades was realized on the engines test bench in the Air Force Institute of Technology in Warsaw. Contactless inductive sensor was permanently installed in turbine engine hull in order to measure temporary placement of blades tips in operating compressor.



Fig. 1. Inductive sensor signal: T_{02d} , T_{02k} – respectively long and short observation period of blade being in sensor area; T_0 , T_1 , T_2 – characteristic observation moments of blade tip below the sensor; T_{01} , T_{12} – observation sub periods of blade tip for T_d and T_k respectively; mV – blade tip translocation signal, μS – blade translocation time [12]

The strain of tip of each blade is contained in the signal measured by the sensor (Fig. 1 – the first tip – blade no. 1, the second tip – the blade no. 2 etc.), as a signal shift in time relative to the expected position. Extracting the information from the sampled signal with a suitable resolution, allows completing the tasks of measurement displacements of the blades tips of the axial compressor first stage [1, 7, 8, 9, 14, 16-18].

The research was performed for rotational velocity of 6900 rpm. Recorded time and the signal value of the instantaneous position of the tops of the compressor blades during operation of the engine when the blade is approaching and receding from the sensor. Figure 1 shows the registered signal displacement of four consecutive tip of blades (of total 28) [12].

Registered signal of the movement blade tip is a complex signal that contains information about the operational condition of blades y(t) in the environment x(t) [10, 12].

The diagnostic signal y(t) can be described:

$$y(t) = f(Y_w, Y_g, Y_s, Y_f, Y_c),$$
 (1)

where:

 Y_w – strain of the longitudinal blades,

 Y_g – strain of the bending blades,

Y_s – angle of torsional blades,

Y_f – signal of various forms of vibration (bending, torsional, longitudinal),

Y_c – thermal strain of the uneven temperature distribution

and the environment signal x(t):

$$\mathbf{x}(t) = \mathbf{f}(\mathbf{n}, \mathbf{F}_0, \mathbf{P}_z, \mathbf{P}_x, \mathbf{P}_1, \mathbf{P}_2, \mathbf{f}, \mathbf{f}_{ob}, \mathbf{c}),$$
(2)

where:

n – rotational speed,

F₀ – centrifugal force,

 P_z – aerodynamic lift blades,

 P_x – resistance force,

 P_1 – the pressure of gas at the inlet rim of the rotor,

 P_2 – the gas pressure at the outlet rim of the rotor,

f - vibration signal,

 f_{ob} – casing vibrations,

c – signal of temperature distribution.

They depend on many different, often unidentified factors.

It can be concluded that the problem of blade diagnostics during turbomachine operation is very complex in its nature as in order to complete a process of blade diagnostics only two signals can be used: the first one is the measurable but interfered signal y(t) and the second one is the environment signal x(t) which is practically immeasurable (except signal n).

To realize the diagnostic process must be subjected special processing signal y(t). In the recorded signal y(t) comprising a many passes of the all 28 blades in the first stage compressor of the engine-SO 3 – is isolated repetitive part of the signal containing multiple passes of one selected blades at the face of the sensor (from the time T_0 to T_2) (Fig. 1). Having the full set signal of the blades passing under the sensor T_{02} , it can be divided into two sub periods: approaching blades to the sensor T_{01} and T_{12} recede blade from the sensor. Moment T_1 corresponds to the situation when the tip of the blades is directly under the sensor.

Having thus separated times (T_{01} and T_{12}) approaching and recede the blade tip to the sensor can be used two parametric methods described in the Patent Bulletin and articles [2-6, 10-13].

3. Parametric diagnostic models

Described parametric diagnostic models are based on auto and cross power spectral density functions displacement blades y(t) at the time when blades recede from the sensor blades T_{12} and approaching the blades to the sensor T_{01} and the environment x(t).

Initially, it is assumed that operation signals y(t) and environment x(t) are temporal, stochastic and interfered with functions autocorrelation $R_{xx}(\tau)$ and $R_{yy}(\tau)$ and the cross-correlation $R_{xy}(\tau)$. Afterwards estimates of autocorrelation function R_{yy}^{*T01} and R_{yy}^{*T12} which are determined for

Afterwards estimates of autocorrelation function R_{yy}^{*101} and R_{yy}^{*112} which are determined for y(t) translocation in observation periods T_{01} and T_{12} , which are then approximated analytical expressions providing alignment above 0.997.

$$R_{vv}(\tau) = a_n \tau^n + \dots + a_4 \tau^4 + a_3 \tau^3 + a_2 \tau^2 + a_1 \tau + a_0,$$
(3)

where $a_1, a_2, \ldots a_n$ – parameters of analytic form of a autocorrelation function.

Using Fourier transform of the analytic forms of autocorrelation functions the corresponding auto power spectral density functions $S_{yy}^{T01}(j\omega)$ and $S_{yy}^{T12}(j\omega)$, were determined:

$$S_{yy}^{T01}(j\omega) = F(R_{yy}^{T01}(\tau)),$$
(4)

$$S_{vv}^{T12}(j\omega) = F(R_{vv}^{T12}(\tau)).$$
(5)

Furthermore can be made assumption that signal observation period T_{12} occurs shortly (ms) after signal observation time T_{01} . Therefore $S_{xx}^{T01}(j\omega)$ and $S_{xx}^{T12}(j\omega)$ are assumed to be almost equal. Since the time of observation are very close to each other it can be assumed that the environment at that time has no changed – so $S_{xx}^{T01}(j\omega) \cong S_{xx}^{T12}(j\omega)$. Then can determine the first parametric diagnostic model as the quotient of auto power spectral density functions for eliminating immeasurable environment:

$$\frac{S_{yy}^{TZ}}{S_{xx}^{T01}} \xrightarrow{S_{xx}^{T12} \equiv S_{xx}^{T01}} \xrightarrow{S_{yy}^{T12} \equiv S_{xy}^{T01}} \xrightarrow{S_{yy}^{T12}} \frac{S_{yy}^{T12}}{S_{yy}^{T01}} = \frac{M_0^* + M_1^* s + M_2^* s^2 + \dots + M_n^* s^n}{1 + L_1^* s + L_2^* s^2 + \dots + L_n^* s^n},$$
(6)

where:

s - complex variable,

 $s = j\omega; M_i^*,$

i = 0, 1, ..., n - numerator estimates parameters,

 L_i^* , i = 0, 1, ..., n – denominator estimates parameters.

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The calculated parameter values for the blade no. 2 for model based on auto power densities are shown in Tab. 1. One cycle is the one transition of all 28 blades under the sensor, so the cycle 10 is understood as a 10 times passage of all 28 blades under the sensor.

Tab. 1. Parameters of the numerator and denominator model based on the auto power density

	Blade no. 2											
	L	L ₁	L_2	L ₃	L ₄	L ₅	M ₀	M ₁	M ₂	M ₃	M 4	M ₅
Cycle 1	0	0.695	1.081	1.326	1.141	1.757	3.365	2.590	2.215	2.015	2.181	1.748
Cycle 10	0	0.862	1.341	1.646	1.410	2.171	3.405	2.815	2.528	2.373	2.508	2.162
Cycle 20	0	0.738	1.154	1.415	1.217	1.877	3.688	2.827	2.407	2.178	2.377	1.867
Cycle 30	0	0.686	1.067	1.313	1.128	1.736	2.722	2.233	2.002	1.876	1.967	1.730
Cycle 40	0	0.745	1.159	1.422	1.223	1.881	2.706	2.290	2.094	1.988	2.055	1.874

For each column, appointed the value of the average μ and standard deviation σ . On their basis was appointed the diagnostic thresholds as $\mu + \sigma$, $\mu + 2\sigma$, $\mu + 3\sigma$.

Then for easier analysis turned them into colours:

- green normal wear for a value not exceeding the threshold $\mu + \sigma$,
- yellow average wear for a value not exceeding the threshold $\mu + 2\sigma$,
- red increased wear for a value not exceeding the threshold $\mu + 3\sigma$,
- black high wear values above the threshold $\mu + 3\sigma$.

In this way were created portrait blades showing its overall condition.

In Fig. 2. summarizes the two blades of a worse technical condition (blade no. 2) and a much better technical condition (blade no. 22) for parametric diagnostic model based on the quotient of the auto power density.

The second parametric diagnostic model bases on cross-power spectral density functions of signals y and x can be written as [2, 4, 11, 12]:

$$\frac{Arg \frac{S_{xy}^{T12}}{S_{xx}^{T01}}}{Arg \frac{S_{xy}^{T01}}{S_{xy}^{T01}}} \xrightarrow{S_{xx}^{T12} \equiv S_{xx}^{T01}} Arg \frac{S_{xy}^{T12}}{S_{xy}^{T01}} = \frac{B_0^* + B_1^* s + B_2^* s^2 + \dots + B_n^* s^n}{1 + A_1^* s + A_1^* s^2 + \dots + A_1^* s^n},$$
(7)

where:

Arg - argument of the main function,

 B_i^* , i = 0, 1, ..., n – numerator estimates parameters,

 A_i^* , i = 0, 1, ..., n – denominator estimates parameters.

	Portrait of blade no. 2											
	LO	L1	L2	L3	L4	L5	M0	M1	M2	M3	M4	M5
Cycle 1												
Cycle 10												
Cycle 20												
Cycle 30												
Cycle 40												
						Portrait of b	plade no. 22	2				
	LO	L1	L2	L3	L4	L5	M0	M1	M2	M3	M4	M5
Cycle 1												
Cycle 10												
Cycle 20												
Cycle 30												
Cycle 40												

Fig. 2. Portrait of the blades using a model based on auto power spectral densities a) blade no. 2 with increased wear; b) blade no. 22 with normal wear

The signal y is measurable, signal x can be accepted as any distribution of $x=\delta(t,\tau)$ [2, 4, 11, 12]. Same as for the model based on the auto power spectral densities, were conducted the same research for model based on cross power spectral densities.

	Blade no. 2											
	A ₀	A ₁	A ₂	A3	A ₄	A5	B ₀	B ₁	B ₂	B ₃	B 4	B 5
Cycle 1	0	0.322	0.580	0.792	1.026	1.100	1.478	1.472	1.484	1.550	2.779	1.123
Cycle 10	0	0.346	0.625	0.845	0.980	1.215	1.394	1.429	1.471	1.542	2.546	1.232
Cycle 20	0	0.443	0.803	1.112	1.857	1.398	1.679	1.678	1.697	1.753	2.707	1.430
Cycle 30	0	0.342	0.618	0.836	0.931	1.210	1.583	1.586	1.606	1.683	2.998	1.236
Cycle 40	0	0.398	0.713	0.989	1.460	1.305	1.804	1.779	1.775	1.832	3.175	1.343

Tab. 2. Parameters of the numerator and denominator model based on the cross power density

	Portrait of blade no. 2											
	LO	L1	L2	L3	L4	L5	M0	M1	M2	M3	M4	M5
Cycle 1												
Cycle 10												
Cycle 20												
Cycle 30												
Cycle 40												
						Portrait of I	olade no. 22	2				
	LO	L1	L2	L3	L4	Portrait of I	olade no. 22 MO	2 M1	M2	M3	M4	M5
Cycle 1	LO	L1	L2	L3	L4	Portrait of I	olade no. 22 MO	M1	M2	M3	M4	M5
Cycle 1 Cycle 10	LO	L1	L2	L3	L4	Portrait of I	olade no. 22 MO	2 M1	M2	M3	M4	M5
Cycle 1 Cycle 10 Cycle 20	LO	L1	L2	L3	L4	Portrait of I	olade no. 22 MO	2 M1	M2	M3	M4	M5
Cycle 1 Cycle 10 Cycle 20 Cycle 30	LO		L2	L3	L4	Portrait of I	olade no. 22 MO	2 M1	M2	M3	M4	M5

Fig. 3. Portrait of the blades using a model based on cross power spectral densities a) blade no. 2 with increased wear; b) blade no. 22 with normal wear

Based on the above portraits of the blades, can be expected that the changes parameters of the numerator and denominator of each diagnostic models will correspond to change the design features of feather blades (cracks, dents, strain, pitting).

For this purpose, research was conducted by direct methods (endoscopic) set of blades. Of all the captured images were selected, one for each blade – most enhanced surface change.



Fig. 4. Selected view of the leading edge of the lock to the top a) blade no. 2; b) blade no. 22 [12]

In Fig. 4 presents a selected view of the leading edge of blades to the tip of the lock blades no. 2 (a) and blade no. 22 (b). From the presented images can be seen that on the blade 2 occurs greater loss of enamels and its dripstone. This is the information may tell that the technical condition of blades no. 2 differs from the technical condition of blades 22. Thus, we can conclude that received large differences in portraits may indicate a different internal condition of blades (exploitation). Captured images do not exhibit the need for disassembly, replacement or repair of blades.

4. Non-parametric diagnostic models

Formulas (6) and (7) may be considered as a blades transfer function H(s). The transfer function input signal is the blade movement during air inflow on blade (time T_{01}) and the output signal is the blade movement during air trailing from blade (time T_{12}). Therefore, to the formulas (6) and (7) for "s" should be insert "j ω ". The result is a spectral transfer function $H(j\omega)$. From this transfer function can be determined its real part $P(\omega)$. On the basis of $P(\omega)$ can be determined the blades step response h(t) and the blades impulse response g(t) [3, 15].

$$g(t) = \frac{2}{\pi} \int_{0}^{\infty} P(\omega) \cdot \omega \frac{\cos \omega t}{\omega} d\omega,$$
(8)

$$h(t) = \frac{2}{\pi} \int_{0}^{\infty} P(\omega) \frac{\sin \omega t}{\omega} d\omega.$$
(9)

Step and impulse responses for blade no. 2 and no. 22 for the model based on auto power density functions shows Fig. 6 and for model based on cross power density functions shows Fig. 7.



Fig. 6. Impulse and step responses for blades no. 2 and no. 22 for five different cycles



Fig. 7. Impulse and step responses for blades no. 2 and no. 22 for five different cycles

According to the principles of automation (small overshoot, short adjustment time) we can see that the blade 22 is correct, and the blade 2 defective (long adjustment).

5. Conclusions

Examined blades were mounted on a technically efficient, continuously operated engine. By using endoscopy can be assessed only the external condition (cannot investigate the internal condition). Only a significant change in the quality of the blades surface can give diagnosticians information to decide whether to replace or repair (by welding and grinding) the blade. Endoscopy researches do not show that the internal condition of the blade no. 2 may differ from the blade no. 22.

Proposed parametric methods can fill diagnostician knowledge gaps about the internal condition of blades (through its analysis in the course of work – portraits of blades in each cycle). Used methods clearly showed that the condition of the blade no. 2 differs from the technical condition of the blade no. 22.

It is worth noting that parametric methods should not be used in case of a small amount of data, because they require the diagnostic thresholds determined based on its statistical operations. They can be used only if there is the possibility to register at least a dozen blades passing the sensor (determined diagnostic threshold has to be reliable).

Additional supporting to determine blades condition may be non-parametric methods that do not require the determination of diagnostic thresholds. These methods are based on impulse and step responses of the each blade.

Comparing the waveforms for the step and impulse responses the differences in performance for each cycle of operation of the each blade can be seen. By examining the step and impulse responses, quality indicators for each cycle of the blades move their operating status can be assess and based on it their condition.

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