

# ROBUST FUZZY MODEL PREDICTIVE CONTROL OF AN OVERHEAD CRANE

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## **Abstract**

*The method of controlling an overhead crane with respect to the variation of operating conditions and control constraints is developed using a model predictive control (MPC) and fuzzy interpolation applied in linear parameter varying (LPV) approach to crane dynamic modelling. The proposed control approach is based on the assumption that operating conditions vary within the known range of scheduling variables, and the parameters of a crane dynamic model can be interpolated by a quasi-linear fuzzy model designed through utilizing the PI-TS fuzzy theory. Hence, a crane dynamic is approximated through interpolation between a set of local linear models determined through identification experiments at the local operating points selected within the bounded intervals of scheduling variables. For the modelling assumptions, the control algorithm is developed based on a generalized predictive control (GPC) procedure taking into consideration the constraints on sway angle of a payload and control signal. Feasibility and applicability of the proposed control technique were confirmed during experiments carried out on a laboratory-scaled overhead crane. The results of experiments are presented and compared with performances of a fuzzy logic-based scheduling control scheme.*

**Keywords:** *overhead crane, model predictive control, linear parameter varying model, fuzzy interpolation*

## **1. Introduction**

The control of the load transfer by a crane is a problem that requires developing solution, which should satisfy the requirements of control system performance frequently over a wide range of operating conditions variation. Thus, the fast and precise transfer of goods with suppressing the residual vibration when a crane arrives at the expected destination involves implementation of robust control techniques taking into consideration the variation of rope length and mass of a payload.

Different types of constructional and technological problems, as well as problems related to automation of crane operations are a serious concern of numerous researchers [10, 11, 16]. Many solutions are developed using an input-shaping method, which may demonstrate the robustness in the presence of varying natural frequency of an oscillatory system [17]. Feedback control methods are adapted to ensure the robustness to external disturbances and model uncertainty in feed forward or optimal control strategies [9, 15], and developed using gain-scheduling [19], sliding mode control [14], or fuzzy logic technique [9, 12].

The rigorous requirements for safety and efficiency of cranes operations require also taking into consideration the constraints on the sway angle of a payload and the speed of crane motion mechanisms. The optimal motion planning for overhead crane with respect to the control constraints such as maximum payload swing is reported in [18]. In addition, the idea of MPC, which enables to optimize prediction of process behaviour with respect to constraints of process variables, has been recently applied in different crane control approaches. The MPC-based control scheme is developed for hydraulic forestry crane [5], boom crane [1], and laboratory models of a gantry crane [13] and overhead crane [6].

In this paper, the GPC-based technique coupled with fuzzy interpolation of crane dynamic model parameters within the known range of varying operating condition is presented. The robust control technique is developed with respect to the constraints on sway angle of a payload and control signal. The P1-TS theory proposed in [7] is applied to approximate the parameters of a crane discrete-time dynamic model within the range of rope length and mass of a payload changes.

The paper is organized as follows. Section two introduces the planar model of a crane approximated using a discrete-time LPV model with parameters interpolated by a P1-TS fuzzy system. Section three describes GPC-based approach developed based on the modelling assumptions. The results of experiments carried out on a laboratory scaled overhead crane are presented in section four. Section five delivers the final conclusions.

## 2. Modelling assumptions

Consider a planar model of a crane transferring a payload (Fig. 1), which is assumed a point-mass suspended at the end of a massless rigid cable.

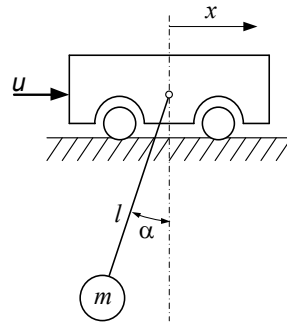


Fig. 1. Planar model of a crane, where  $m$ ,  $l$ ,  $u$  and  $\alpha$  are, respectively, mass of a payload, rope length, controlling signal corresponding to control force acting on a crane, and sway angle of a payload

The dynamic system is approximated using the second- and first-order discrete-time linear parameter varying models representing relations between sway angle of a payload and crane speed, and between crane speed and input function, respectively:

$$\alpha(t) = \frac{B(z^{-1})}{A(z^{-1})} \dot{x}(t-1) + \tilde{e}_1, \quad (1)$$

$$\dot{x}(t) = \frac{D(z^{-1})}{C(z^{-1})} u(t-1) + \tilde{e}_2, \quad (2)$$

where  $z^{-1}$  is a time shift operator,  $\tilde{e}_1$  and  $\tilde{e}_2$  represent un-modelled dynamic and disturbances and  $A(z^{-1})$ ,  $B(z^{-1})$ ,  $C(z^{-1})$ ,  $D(z^{-1})$  are polynomials with the parameters varying in relation to the rope length and mass of a payload:

$$A(z^{-1}) = 1 + a_1(l, m)z^{-1} + a_2(l, m)z^{-2},$$

$$B(z^{-1}) = b_0(l, m) + b_1(l, m)z^{-1},$$

$$C(z^{-1}) = 1 + c_1(l, m)z^{-1},$$

$$D(z^{-1}) = d_0(l, m).$$

Assuming, that operating conditions vary within the known range of scheduling variables, the parameters of a crane dynamic model can be interpolated by a quasi-linear fuzzy model. Hence, a crane dynamic is approximated through interpolation between a set of local linear models

determined through identification experiments at the local operating points selected within the bounded intervals of scheduling variables  $w_i \in [w_i^-, w_i^+]$  (where  $i = 1, 2$ , and  $w_1 = l$ ,  $w_2 = m$ ). Applying the P1-TS fuzzy theory proposed in [7], a fuzzy quasi-linear interpolator can be developed by dividing an each interval  $[w_i^-, w_i^+]$  into  $n_i$  subintervals  $[\beta_{i,j}, \beta_{i,j+1}]$  (where  $\beta_{i,j} < \beta_{i,j+1}$ , and  $j = 1, 2, \dots, n_i$ ), that leads to obtain  $n_1 \cdot n_2$  fuzzy interpolation regions. For each interval, the linear membership functions (Fig. 2) are defined as follows:

$$N_{i,j}(w_i) = \frac{\beta_{i,j+1} - w_i}{\beta_{i,j+1} - \beta_{i,j}}, \quad P_{i,j}(w_i) = 1 - N_{i,j}(w_i). \quad (3)$$

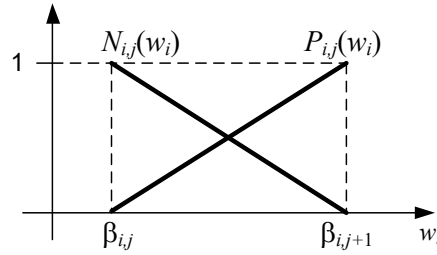


Fig. 2. Linear membership functions specified for the interval  $[\beta_{i,j}, \beta_{i,j+1}]$  of scheduling variable  $w_i$  (where  $i = 1, 2$ , and  $w_1 = l$ ,  $w_2 = m$ )

The output vector of interpolated parameters  $\mathbf{y} = [a_1, a_2, b_0, b_1, c_1, d_0]^T$  is determined according to the function:

$$\mathbf{y} = \mathbf{g}^T \mathbf{\Omega} \mathbf{Q}_k, \quad (4)$$

where  $\mathbf{g}$  and  $\mathbf{\Omega}$  are called generator vector and fundamental matrix, respectively, which can be determined recursively for  $i = 1, 2$  according to (5) starting from the initial generator  $\mathbf{g}_0 = 1$  and fundamental matrix  $\mathbf{\Omega}_0 = 1$ :

$$\begin{aligned} \mathbf{g}_i &= \begin{bmatrix} 1 \\ w_i \end{bmatrix} \otimes \mathbf{g}_{i-1}, \\ \mathbf{\Omega}_i &= \frac{1}{\beta_{i,j+1} - \beta_{i,j}} \begin{bmatrix} \beta_{i,j+1} & -\beta_{i,j} \\ -1 & 1 \end{bmatrix} \otimes \mathbf{\Omega}_{i-1}, \end{aligned} \quad (5)$$

where  $\otimes$  denotes the Kronecker product, and  $\mathbf{Q}_k$  (where  $k = 1, 2, \dots, n_1 \cdot n_2$ ) is the matrix containing in consecutive rows the model's parameters determined through identification experiments conducted at operating points  $\{\beta_{1,j}, \beta_{2,j}\}$ ,  $\{\beta_{1,j+1}, \beta_{2,j}\}$ ,  $\{\beta_{1,j}, \beta_{2,j+1}\}$  and  $\{\beta_{1,j+1}, \beta_{2,j+1}\}$ , respectively.

### 3. Generalized predictive control scheme

Based on the discrete-time models (1) and (2), the two CARIMA (Controlled Auto-Regressive and Moving-Average) models representing relations between sway angle of a payload and input function, and between crane position and input function are proposed as follows

$$A(z^{-1})C(z^{-1})\alpha(t) = z^{-1}D(z^{-1})B(z^{-1})u(t-1) + \xi_1(t) / \Delta, \quad (6)$$

$$C(z^{-1})\Delta x(t) = T_s D(z^{-1})u(t-1) + \xi_2(t) / \Delta, \quad (7)$$

where  $\Delta x(t) = T_s \dot{x}(t)$  is an increment of crane position (where  $\Delta = 1 - z^{-1}$ ),  $T_s$  is a sample time, and  $\xi_1$  and  $\xi_2$  are the uncorrelated random sequences. According to the procedure introduced in [2],

and taking into consideration that one-step ahead prediction of sway angle of a payload  $\hat{\alpha}(t+1)$  can be given as

$$\hat{\alpha}(t+1) = \frac{B(z^{-1})}{A(z^{-1})} \dot{x}(t), \quad (8)$$

the two-step predictor of sway angle of a payload and one-step predictor of crane position can be written as

$$\hat{\alpha}(t+2) = G_1(z^{-1})\Delta u(t) + F_1(z^{-1})\hat{\alpha}(t+1), \quad (9)$$

$$\hat{x}(t+1) = G_2(z^{-1})\Delta u(t) + F_2(z^{-1})x(t), \quad (10)$$

where  $G_1(z^{-1})$ ,  $G_2(z^{-1})$ ,  $F_1(z^{-1})$  and  $F_2(z^{-1})$  are the polynomials recalculated through recursion of the Diophantine equation:

$$G_1(z^{-1}) = \sum_{i=0}^1 g_{1i}z^{-i} = b_0d_0 + b_1d_0z^{-1},$$

$$F_1(z^{-1}) = \sum_{i=0}^3 f_{1i}z^{-i} = 1 - a_1 - c_1 + (a_1 + c_1 - a_2 - a_1c_1)z^{-1} + (a_2 + a_1c_1 - a_2c_1)z^{-2} + a_2c_1z^{-3},$$

$$G_2(z^{-1}) = g_{20} = T_s d_0,$$

$$F_2(z^{-1}) = \sum_{i=0}^2 f_{2i}z^{-i} = 2 - c_1 + (2c_1 - 1)z^{-1} - c_1z^{-2}.$$

Formulating the constraints for control signal and sway angle of a payload in the defined  $j$ -step prediction horizon:

$$\begin{aligned} u_{\min} &\leq u(t) \leq u_{\max}, \\ \alpha(t+j) &\leq |\alpha_{\max}|, \end{aligned} \quad (11)$$

the control increment is constrained as follows:

$$\Delta u_{\min}(t) \leq \Delta u(t) \leq \Delta u_{\max}(t), \quad (12)$$

where:

$$\Delta u_{\min}(t) = \max \left( u_{\min} - u(t-1), - \left( \alpha_{\max} + g_{11}\Delta u(t-1) + F_1(z^{-1})\hat{\alpha}(t+1) \right) g_{10}^{-1} \right),$$

$$\Delta u_{\max}(t) = \min \left( u_{\max} - u(t-1), \left( \alpha_{\max} - g_{11}\Delta u(t-1) - F_1(z^{-1})\hat{\alpha}(t+1) \right) g_{10}^{-1} \right).$$

Taking into consideration the constraints limiting the control signal and transient payload oscillations, and assuming that the reference signal  $x_r$  is a constant value, the cost function is proposed as a quadratic function of a distance between one-step ahead predicted crane position and reference position, and a quadratic function of the two-step ahead predicted payload deflection simplified to the product of rope length and sway angle of a payload  $l\hat{\alpha}(t+2)$ , plus a quadratic function measuring the control effort:

$$\begin{aligned} J = & \left( G_2(z^{-1})\Delta u(t) + F_2(z^{-1})x(t) - x_r \right)^2 + \lambda_1 l^2 \left( G_1(z^{-1})\Delta u(t) + F_1(z^{-1})\hat{\alpha}(t+1) \right)^2 + \\ & + \lambda_2 (\Delta u(t))^2 + \nu_1 \left( \Delta u_{\max}(t) - \Delta u(t) \right) + \nu_2 \left( \Delta u(t) - \Delta u_{\min}(t) \right), \end{aligned} \quad (13)$$

where  $v_1$  and  $v_2$  are the Lagrangian multipliers. Considering the Kuhn-Tucker conditions

$$\begin{cases} \nabla J(\Delta u(t)) = 0, \\ v_1(\Delta u_{\max}(t) - \Delta u(t)) = 0, \\ v_2(\Delta u(t) - \Delta u_{\min}(t)) = 0, \\ v_1, v_2 \geq 0, \end{cases} \quad (14)$$

the function (13) has the three solutions:

$$\begin{cases} \Delta u_1(t) = \frac{g_{20} \left( x_r - F_2(z^{-1})x(t) \right) - \lambda_1 l^2 g_{10} \left( g_{11} \Delta u(t-1) + F_1(z^{-1})\hat{a}(t+1) \right)}{g_{20}^2 + \lambda_1 l^2 g_{10}^2 + \lambda_2}, \text{ for } v_1 = v_2 = 0, \\ \Delta u_2(t) = \Delta u_{\min}(t), \text{ for } v_1 = 0, v_2 > 0, \\ \Delta u_3(t) = \Delta u_{\max}(t), \text{ for } v_1 > 0, v_2 = 0. \end{cases} \quad (15)$$

Consequently the current control can be formulated as

$$u(t) = u(t-1) + \max(\min(\Delta u_1(t), \Delta u_3(t)), \Delta u_2(t)). \quad (16)$$

#### 4. Experiments on a laboratory stand

The proposed control techniques was tested on a laboratory scaled overhead crane equipped with DC motors, and incremental encoders used for sensing the position of crane and sway angle of a payload. Moreover, the vision-base measurement techniques were developed and tested on the laboratory crane [3, 4]. The control algorithm was implemented using structured text on the RX3i controller, and the measurement system was completed with the PC equipped with PLC1710HG I/O board.

The GPC strategy was tested for experimentally selected weighting coefficients  $\lambda_1 = 2.4$  and  $\lambda_2 = 0.3$  specified in the cost function (13), sample time  $T_s = 0.1$  s, and control signal range  $-10 \leq u(t) \leq 10$ . The objective of the control was positioning a crane to  $x_r = 1$  m and reducing the payload deflection within tolerance  $\pm 0.02$  m. The GPC scheme was compared with the P1-TS fuzzy-based scheduling control scheme reported in [12]. The fuzzy scheduling control system was designed using interval analysis of closed-loop system characteristic polynomial coefficients. The P1-TS fuzzy interpolator obtained during designing the fuzzy scheduler of the linear controllers [12] was adopted to approximate the parameters of a crane dynamic model within the range of scheduling variables  $l = [1.0, 2.2]$  m and  $m = [10, 90]$  kg, and implemented in the GPC algorithm. The P1-TS fuzzy interpolator was designed with intervals  $[\beta_{1,1}, \beta_{1,2}] = [1.0, 1.6]$  m,  $[\beta_{1,2}, \beta_{1,3}] = [1.6, 2.2]$  m and  $[\beta_{2,1}, \beta_{2,2}] = [10, 90]$  kg specified for input variables  $l$  and  $m$  respectively (Fig. 3).

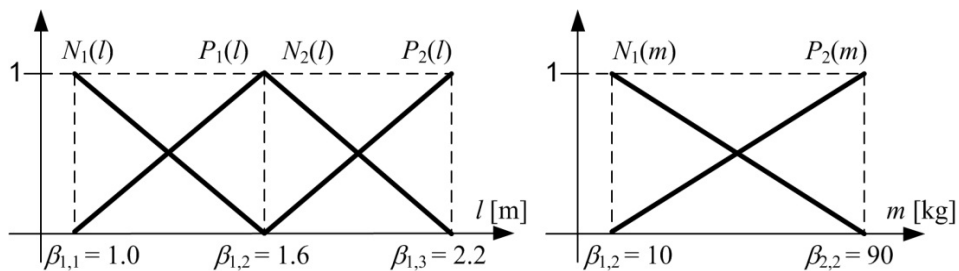


Fig. 3. Linear membership functions specified for the intervals of input variables  $l$  and  $m$

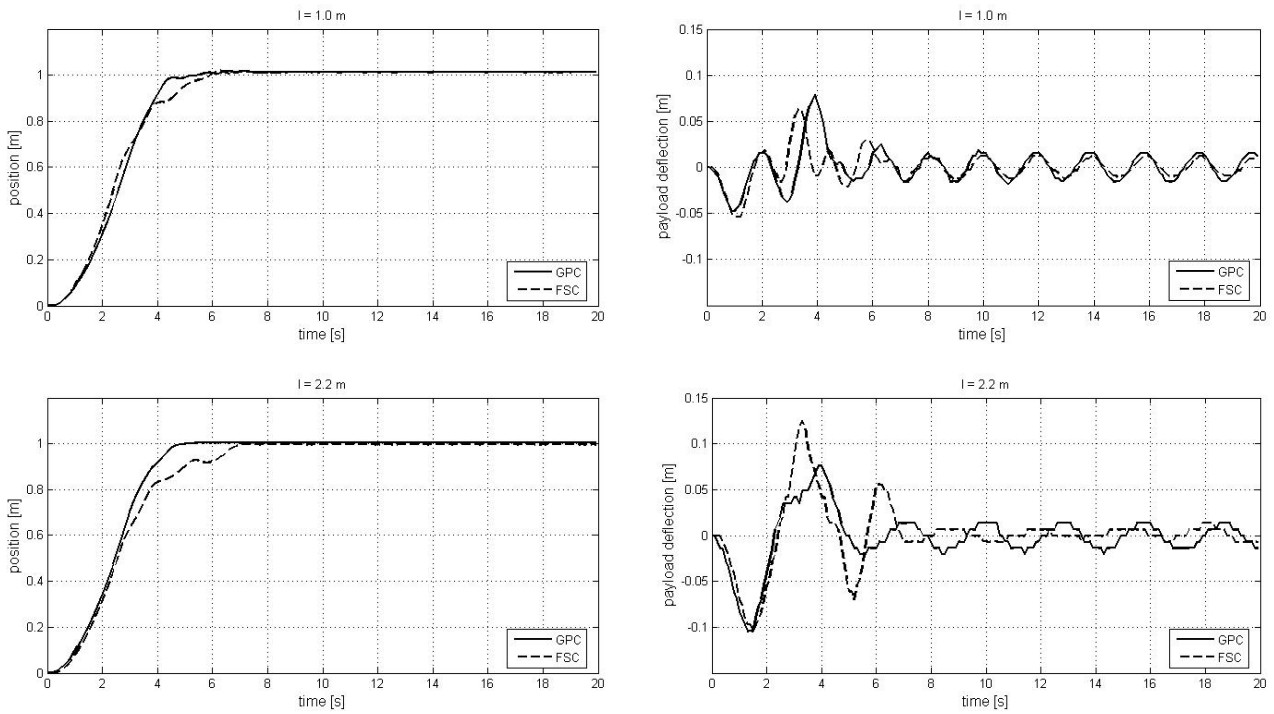


Fig. 4. Crane position and payload deflection – experiments for  $l = \{1.0, 2.2\}$  m and  $m = 10$  kg – comparison between GPC and FSC

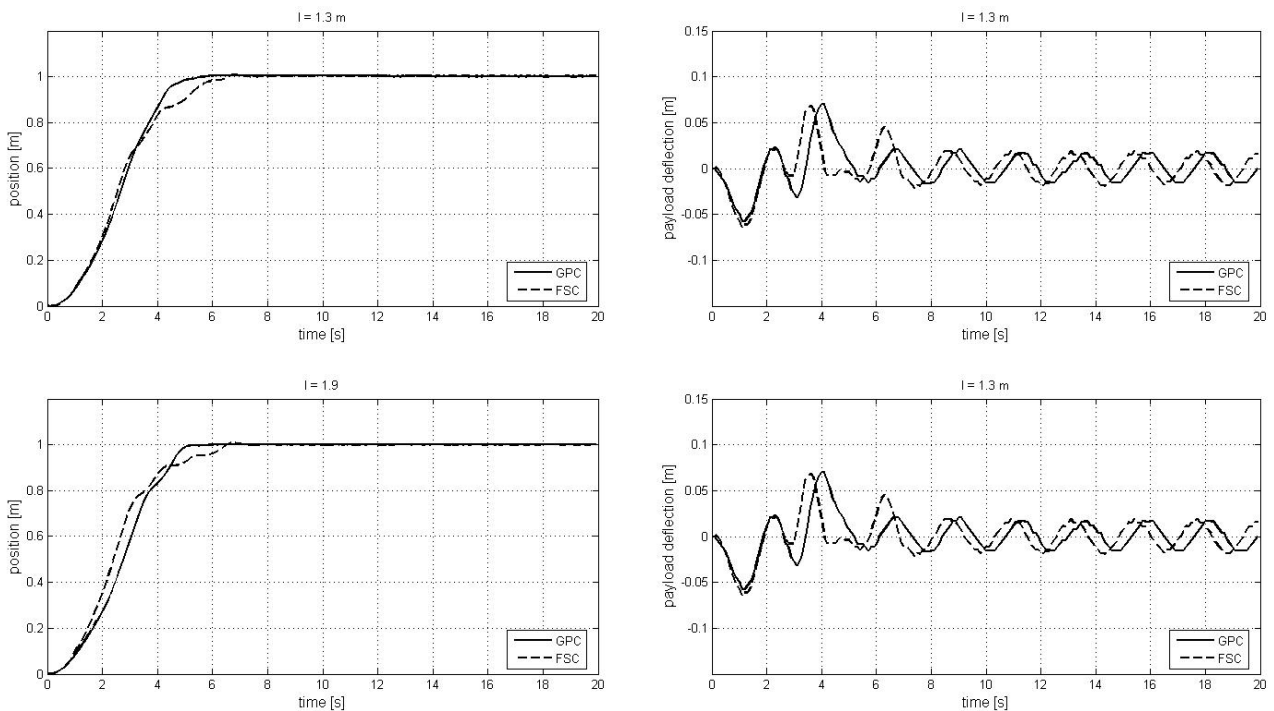


Fig. 5. Crane position and payload deflection – experiments for  $l = \{1.3, 1.9\}$  m and  $m = 50$  kg – comparison between GPC and FSC

Figures 4 and 5 present comparison of experiments carried out using GPC technique and fuzzy scheduling control (FSC) scheme. The both techniques were tested without feedback from sway measurement system, which was used only in identification experiments to determine parameters of a model (1) at operating points corresponded to bounds of the intervals  $[\beta_{i,j}, \beta_{i,j+1}]$  (Fig. 3). Hence, the both control techniques were experimentally validated using model-estimated feedback based on the discrete-time model (1). The experiments were conducted for rope length  $l = \{1.0,$

2.2} m and mass of a payload  $m = 10$  kg (Fig. 4), and for mass of a payload  $m = 50$  kg and rope length  $l = \{1.3, 1.9\}$  m (Fig. 5). The both techniques proved robustness against variation of rope length and mass of a payload; however, the GPC technique showed definitely better performances than the fuzzy scheduling control scheme. The settle time was within [4.4, 5.3] s for the GPC system, while using the FSC a desired position of a payload was obtained within [5.5, 6.7] s. The GPC scheme ensures fast positioning of a payload and sway suppressing within the assumed tolerance of a payload deflection  $\pm 0.02$  m.

## 5. Conclusions

The MPC-based control scheme coupled with fuzzy interpolation of an LPV discrete-time crane dynamic model's parameters is developed in the paper using the GPC procedure. The robust control technique is developed with respect to the constraints on sway angle of a payload and control signal. The P1-TS fuzzy interpolator is applied to approximate the parameters of a crane discrete-time dynamic model within the range of rope length and mass of a payload changes. The experiments carried out on a laboratory stand confirmed effectiveness and feasibility of the proposed solution, which was realized using PAC system with RX3i controller. The GPC scheme ensures fast and precise positioning of a payload transferred by a crane with suppressing the residual vibration within the assumed tolerance, and limiting the payload deflection in transient state. The series of experiments carried out for different rope lengths and masses of a payload proved robustness of the presented control approach, and showed also better performances in comparison to the scheduling control approach developed using a P1-TS fuzzy interpolator.

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## References

- [1] Arnold, E., Sawodny, O., Neupert, J., Schneider, K., *Anti-sway system for boom cranes based on a model predictive control approach*, Proceedings of IEEE International Conference on Mechatronics and Automation, pp. 1533-1538, Niagara Falls, Canada 2005.
- [2] Clarke, D.W., Mohtadi, C., Tuffs, P. S., *Generalized predictive control – Part I. The basic algorithm*, Automatica, Vol. 23 (2), pp. 137-148, 1987.
- [3] Hyla, P., Szpytko, J., *Vision method for rope angle swing measurement for overhead travelling crane – validation approach*, Activities of Transport Telematics: Communications in Computer and Information Science, Vol. 395, pp. 370-377, 2013.
- [4] Hyla, P., Szpytko, J., *Crane payload position measurement vision-based system dedicated for anti-sway solutions*, Telematics – Support for Transport: Communications in Computer and Information Science, Vol. 471, pp. 404-413, 2014.
- [5] Kalmari, J., Backman, J., Visala, A., *Nonlinear model predictive control of hydraulic forestry crane with automatic sway damping*, Computers and Electronics in Agriculture, Vol. 109, pp. 36-45, 2014.
- [6] Kapernick, B., Graichen, K., *Model predictive control of an overhead crane using constraint substitution*, Proceedings of American Control Conference, pp. 3973-3978, Washington, DC, USA 2013.
- [7] Kluska, J., *Analytical methods in fuzzy modeling and control*, Studies in Fuzziness and Soft Computing 241, Springer, Heidelberg 2009.
- [8] Kłosiński, J., *Swing-free stop control of the slewing motion of a mobile crane*, Control Engineering Practice, Vol. 13, pp. 451-460, 2005.

- [9] Kłosiński, J., Janusz, J., Nycz, R., *The impact of the FLC controller's setting on the precision of the positioning of a payload transferred by a mobile crane*, Acta Mechanica et Automatica, Vol. 8, No. 4, pp. 181-184, 2014.
- [10] Sioma, A., Tytko, A., *Vision methods for assessing the geometrical parameters of steel ropes*, Acta Mechanica et Automatica, Vol. 6, No. 1, pp. 63-67, 2012.
- [11] Smalko, Z., Szpytko, J., *Safety in engineering practice*, Proceedings of 17th European Safety and Reliability Conference ESREL, pp. 1231-1237, Valencia, Spain 2009.
- [12] Smoczek, J., *PI-TS fuzzy scheduling control system design using local pole placement and interval analysis*, Bulletin of the Polish Academy of Sciences – Technical Sciences, Vol. 62, No. 3, pp. 455-464, 2014.
- [13] Su, S. W., Nguyen, H., Jarman, R., Zhu, J., Lowe, D., McLean, P., Huang, S., Nguyen, N. T., Nicholson, R., Weng, K., *Model predictive control of gantry crane with input nonlinearity compensation*, World Academy of Science, Engineering and Technology, Vol. 3, No. 2, pp. 899-903, 2009.
- [14] Sun, N., Fang, Y., Chen, H., *A new antiswing control method for underactuated cranes with unmodeled uncertainties: theoretical design and hardware experiments*, IEEE Transactions on Industrial Electronics, Vol. 62, No. 1, pp. 453-465, 2015.
- [15] Tomczyk, J., Cink, J., Kosucki, A., *Dynamics of an overhead crane under a wind disturbance condition*, Automation in Construction, Vol. 42, pp. 100-111, 2014.
- [16] Trąbka, A., *The impact of the support system's kinematic structure on selected kinematic and dynamic quantities of an experimental crane*, Acta Mechanica et Automatica, Vol. 8, No. 4, pp. 189-193, 2014.
- [17] Vaughan, J., Yano, A., Singhose, W., *Comparison of robust input shapers*, Journal of Sound and Vibration, Vol. 315, pp. 797-815, 2015.
- [18] Wu, Z., Xia, X., *Optimal motion planning for overhead cranes*, IET Control Theory and Applications, Vol. 8, No. 17, pp. 1833-1842, 2014.
- [19] Zavari, K., Pipeleers, G., Swevers, J., *Scheduled controller design: illustration on an overhead crane*, IEEE Transactions on Industrial Electronics, Vol. 61, No. 7, pp. 3713-3718, 2014.