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WEAR PROCESS IN SUCCESSIVE TIME UNITS DESCRIBED BY FIRST ORDER RECURRENCES

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Abstract

The numerous tribology problems occurring in power-train and transport industry lead to wear bearing determination solutions. Especially the project designer demand the more and more information referring the slide bearing wear anticipation in succeeding years of machine operations. In this paper a perspective will be provided on what is known about various types of influences which are caused wear effects. Some machinery eventually fails or becomes uneconomical to operate because of single causes for example types of wear, but most mechanical devices succumb to combinations of causes. A direct parallel is seems in the human machine. Therefore this paper presents the methods of solutions of some specific class of ordinary non-homogeneous recurrence equations of first order with variable coefficients occurring in hydrodynamic theory of bearing wear problems. The various coefficients occurring in considered recurrent equations are determined from experimental measurements where the influence of various operating parameters on the wear effects is taking into account. The influence of numerous operating parameters on the wear effects is experimentally determined in the case if mentioned parameters are independent as well if are mutually connected. Moreover in this paper the theorems will be presented of the existence, determination and an algorithm construction of discrete solutions of non-homogeneous, linear, first order recurrent equations with variable coefficients. The Lemmas and Theorem are formulated and proved by means of the Unified Operator of Summation (UOS operator) with a unitary translation operator, where the operator properties and features are taking into account. In the case of the space of solution functions, the above mentioned wear problems determination are attributed to practical applications related to the non-homogeneous, linear, first order differential equations. The examples presented in this paper for various variable coefficients i.e. for various operating conditions determine the wear values of micro- bearing system during the indicated time units of operating time.

Keywords: *wear determination during slide bearings operating process, first order recurrence equation, class of variable coefficients*

1. Initial information about micro-bearing wear prognosis

The paper presents the new calculation method and a new solutions of abrasive wear prognosis of two cooperating surfaces in micro and nano scale, especially wear occurring in HDD micro-bearing during the operating time.

The wear prognosis of two cooperating HDD micro-bearing surfaces during the operation time has very important meaning in contemporary software technological processes [1-4]. Solution of presented problem can be possible on the basis of recently obtained measurements of micro-bearing wear during the first periods (may be month or years) of exploitation and regarding the parameters describing the random wear effects completed by the dimensional standard deviation [6, 9, 11, 16]. After experimental AFM measurements follows, that discrete wear values f_{n+1} of the sequence $\{f_n\}$ for n=1,2,3,… i.e. the wear values increases in mm^3 or μm^3 of HDD micro-bearing journal and sleeve equal to sum $(A_n f_n+B_n)$ of wear in foregoing successive time units (may be months) where addends are multiplied by dimensionless variable stochastic wear coefficient A_n plus some dimensional, variable stochastic values function of materials B_n [16]. Variable coefficients A_n, B_n depend on the obtained in experimental way dimensional wear standard deviation of micro-bearing material, the journal angular velocity and the frequencies of vibrations [6,9]. Variable n is numbered by natural numbers 1, 2, 3, … .

The dimensionless n dependent variable random parameter A_n, denotes any coefficients which changes the wear in foregoing time units. Standard deviation dependent term B_n, and term values ($A_n f_n$) of foregoing wear, make up the sequence values of real wear in succeeding time unit. In this case wear of HDD micro-bearing can be described by the following recurrence equation [5,6,7]:

$$f_{n+1} = A_n f_n + B_n \quad \text{for} \quad n = 1,2,3,.... \tag{1}$$

If we know dimensionless values A_n [1], and dimensional value B_n in mm^3 or μm^3, then recurrent equation (1) determines analytical sequence $\{f_n\}$ presenting dimensional wear values in particular time units numbered by n=1,2,3,… . To solve mentioned problem it is necessary to add the boundary conditions [10,12]. Hence we assume that in first time unit (may be month), the wear obtained from experiments attains dimensional values W_1 expressed in mm^3 or μm^3.

2. Lemmas of recurrences equations for wear process identifications

To recognize the wear process value we show in this intersection at first the lemmas of the existence, determination and an algorithm construction of discrete solutions of non-homogeneous, linear, first order recurrent wear equations (1) with variable coefficients [13-15]. The Lemmas and Theorem are formulated and proved by means of the UOS operator. In the case of the space of continuous functions, the above mentioned problems are attributed to non-homogeneous, linear, first order differential equations [8].

LEMMA 1

Linear, homogeneous, first order recurrent wear equations with experimental defined variable dimensionless coefficient $a_{n:}$

$$f_{n+1} + f_n a_n = 0, \tag{2}$$

has the following general solution:

$$f_1 = W, \quad f_n = W \cdot (-1)^{n-1} \cdot \prod_{j=1}^{n-1} a_j \quad n = 2,3,4,..., \tag{3}$$

where:

f_n – unknown dimensional wear function,

W – arbitrary dimensional summation constant as the wear after first time unit , whereas index j changes values from 1 to n−1.

PROOF OF LEMMA 1

From (3) follows:

$$f_{n+1} = W \cdot (-1)^n \cdot \prod_{j=1}^{n} a_j. \tag{4}$$

Putting (3) , (4) into equation (2), we obtain:

$$\underbrace{W \cdot (-1)^n \cdot \prod_{j=1}^{n} a_j}_{f_{n+1}} + a_n \cdot \underbrace{W \cdot (-1)^{n-1} \cdot \prod_{j=1}^{n-1} a_j}_{f_n} \stackrel{?}{=} 0. \tag{5}$$

It is easy to see, that the following equalities are true:

$$(-1)^n = -(-1)^{n-1}, \quad a_n \cdot W \cdot \prod_{j=1}^{n-1} a_j = W \cdot \prod_{j=1}^{n} a_j. \tag{6}$$

Utilizing the identities (6) in equation (5), we have:

$$\underbrace{-W\cdot(-1)^{n-1}\cdot\prod_{j=1}^{n}a_j}_{f_{n+1}}+\underbrace{W\cdot(-1)^{n-1}\cdot\prod_{j=1}^{n}a_j}_{a_n\cdot f_n}=0. \tag{7}$$

It is easy to see that the l.h.s. of equation (7) equals zero, hence Lemma 1 is completed.

LEMMA 2

Linear, non-homogeneous, first order recurrent equations with constant dimensionless coefficient a and variable dimensional free term b_n:

$$f_{n+1}+a\,f_n=b_n \tag{8}$$

have the following general solution:

$$f_n=W\cdot(-1)^{n-1}\cdot a^{n-1}+f_n^b,\quad f_n^b=\sum_{k=1}^{n-1}(-1)^{n-k-1}\cdot a^{n-k-1}\cdot b_k,\quad for\ \ n=2,3,\ldots, \tag{9}$$

where:

f_1=W, f_n – an unknown function,

W – an arbitrary dimensional summation constant as the wear after first time unit, whereas index k belong to set 1,2,…, n–1 for n=2,3,4,…

PROOF OF LEMMA 2

Translating the lower index in formula (9) we obtain the following two expressions [13]:

$$f_{n+1}=W\cdot(-1)^n\cdot a^n+\sum_{k=1}^{n}(-1)^{n-k}\cdot a^{n-k}\cdot b_k=W\cdot(-1)^n\cdot a^n+\sum_{k=1}^{n-1}(-1)^{n-k}\cdot a^{n-k}\cdot b_k+b_n, \tag{10}$$

$$af_n=-W\cdot(-1)^n\cdot a^n-\sum_{k=1}^{n-1}(-1)^{n-k}\cdot a^{n-k}\cdot b_k, \tag{11}$$

If we add mutually equations (11) + (10) then we easily get the satisfying of the non-homogeneous recurrent equation (9) and the proof of Lemma 2 is completed.

3. Theorem of recurrences equations for wear process identifications

In this section we shall be concerned the solutions of linear, non-homogeneous, first order recurrent equations with variable coefficient and variable free term function. Hence we formulate at first the following.

THEOREM 1

Linear, non-homogeneous, first order recurrent wear equations with experimental defined variable dimensionless coefficient a_n and variable dimensional free term b_n:

$$f_{n+1}+a_n\,f_n=b_n, \tag{12}$$

have the following general dimensional solution [13, 14]:

$$f_n = (-1)^{n-1} \cdot \prod_{j=1}^{n-1} a_j \left\{ W + \sum_{k=1}^{n-1} \left[\frac{b_k}{(-1)^k \prod_{s=1}^{k} (a_s)} \right] \right\} \quad for \quad n = 2,3,4,..., \tag{13}$$

where:

$f_1=W$, f_n – an unknown dimensional discrete function,

W – an arbitrary dimensional constant as the wear after first time unit, indexes j and k belong to set 1,2,…, n–1 whereas s=1, 2, …, k.

ANALYTICAL PROOF OF THEOREM 1

If we divide both sides of equation (12) by the product $a_1, a_2, a_3, \cdots, a_n,$ and after reducing fractions by factor a_n, we obtain following implication [13,14]:

$$\frac{f_{n+1}}{\prod_{j=1}^{n} a_j} + \frac{a_n \ f_n}{\prod_{j=1}^{n} a_j} = \frac{b_n}{\prod_{j=1}^{n} a_j} \Rightarrow \frac{f_{n+1}}{\prod_{j=1}^{n} a_j} + \frac{f_n}{\prod_{j=1}^{n-1} a_j} = \frac{b_n}{\prod_{j=1}^{n} a_j}, \quad n = 1,2,3,.... \tag{14}$$

Now we take into account the following assumption [13,14]:

$$g_n \equiv \frac{f_n}{\prod_{j=1}^{n-1} a_j}, n > 1 \Rightarrow g_{n+1} \equiv \frac{f_{n+1}}{\prod_{j=1}^{n} a_j}, \quad n = 2,3,..., \tag{15}$$

$$g_1 \equiv f_1, \tag{16}$$

$$B_n \equiv \frac{b_n}{\prod_{j=1}^{n} a_j} \Rightarrow B_k \equiv \frac{b_k}{\prod_{j=1}^{k} a_j}, \quad n = 1,2,3,.... \tag{17}$$

The recurrent equation in r.h.s. of implication (14) we can write by using formulae (15), (16), (17) (14) in following form:

$$g_{n+1} + g_n = B_n \quad \text{for} \quad n = 1,2,3,... \tag{18}$$

Comparison of Lemma 2 solution (9) for equation (8) with recurrent equation (18) for constant coefficient a=1, the general solution of recurrence equation (18) has the following form [13,14]:

$$g_n = W \cdot (-1)^{n-1} + \sum_{k=1}^{n-1} (-1)^{n-k-1} \cdot B_k, \quad n = 2,3,.., \tag{19}$$

where W is the arbitrary constant. If we replace B_k in Eq. (19) with expression (17), we obtain:

$$g_1 = W, \quad g_n = W \cdot (-1)^{n-1} + \sum_{k=1}^{n-1} (-1)^{n-k-1} \cdot \frac{b_k}{\prod_{j=1}^{k} a_j}, \ for \ n = 2,3,4,.... \tag{20}$$

If we replace g_n in the l.h.s. of Eq. (20) with expressions (15), (16), we have:

$$f_1 = W, \quad \frac{f_n}{\prod_{j=1}^{n-1} a_j} = W \cdot (-1)^{n-1} + \sum_{k=1}^{n-1} (-1)^{n-k-1} \cdot \frac{b_k}{\prod_{j=1}^{k} a_j}, \ for \quad n = 2,3,4,\ldots . \tag{21}$$

It is easy to see that after conformal transformations the obtained formula (21) is identical with the general solution presented in the formula (13) what completes Theorem 1.

COROLLARY 1

Solution (3) of homogeneous recurrence equation (2) with variable coefficient $a_n(n) \neq 0$ is the particular case of solution (13) for b_k=0 [13], [14].

The solution (9) of non-homogeneous recurrence equation (8) with a constant coefficient and arbitrary variable free term is a particular case of solution (13) for $b_n(n) \neq 0, a_n \equiv a = \text{const.}$

COROLLARY 2

The solution of the linear, non-homogeneous first order wear recurrence equation:

$$f_{n+1} + a_n f_n = b, \quad n = 1,2,3,\ldots . \tag{22}$$

with variable dimensionless coefficient $a_n(n) \neq 0$ and constant dimensional free term $b_n \equiv b = \text{const.}$ by virtue of solution (13), leads to the following form [13,14]:

$$f_1 = W, \quad f_n = (-1)^{n-1} \cdot \prod_{j=1}^{n-1} a_j \left\{ W + b \sum_{k=1}^{n-1} \left[\frac{1}{(-1)^k \prod_{s=1}^{k} (a_s)} \right] \right\} \ for \quad n = 2,3,4,\ldots . \tag{23}$$

PROOF OF COROLLARY 2

The proof of Corollary 2 follows from Lemma 1 described by the expression (3) and follows from the Theorem 1 presented by the formulae (13) and moreover follows from the linearity of the recurrence equation. The second term multiplied by factor b occurring in above solution (23), denotes the particular solution of the non-homogeneous following recurrence equation (22).

COROLLARY 3

On the basis of Theorem 1 we show, that the following recurrence presenting wear equations:

$$f_{n+1} - a_n f_n = b_n \quad for \quad n = 1,2,3,\ldots \tag{24}$$

has the following general solution:

$$f_n = \frac{W}{a_n} \prod_{j=1}^{n} a_j - \frac{b_n}{a_n} + \left(\frac{1}{a_n} \prod_{j=1}^{n} a_j \right) \cdot \sum_{k=1}^{n} \left(\frac{b_k}{\prod_{s=1}^{k} a_s} \right) \quad \text{for} \ \ n = 1,2,3,\ldots \tag{25}$$

and W an arbitrary constant denotes the wear value after first time unit.

PROOF OF COROLLARY 3

The expression (13) presents the general solution of the recurrence equations (12). The recurrence equations (24) and (12) differ only in sign of the variable coefficient a_n. Hence in general solution (12) we replace coefficient (a_n) by $(-a_n)$ and after sign conversion and transformations in Eq. (13) we get:

$$f_1 = W, \quad f_n = \prod_{j=1}^{n-1} a_j \left\{ W + \sum_{k=1}^{n-1} \left[\frac{b_k}{\prod_{s=1}^{k} (a_s)} \right] \right\} \quad for \quad n = 2,3,4,.... . \tag{26}$$

The following expressions are true:

$$\prod_{j=1}^{n-1} a_j = \frac{1}{a_n} \left(\prod_{j=1}^{n} a_j \right), \quad n = 2,3,... , \tag{27}$$

$$\left(\prod_{j=1}^{n-1} a_j \right) \left(\frac{b_k}{\prod_{s=1}^{k} a_s} \right) = \prod_{j=1}^{n} \left(\frac{b_k}{\prod_{s=1}^{k} (a_s)} \right) - \frac{b_n}{\prod_{s=1}^{n} a_s}, \quad n = 2,3,..., \tag{28}$$

$$\frac{1}{a_n} \left(\prod_{j=1}^{n} a_j \right) \cdot \frac{b_n}{\prod_{s=1}^{n} a_s} = \frac{b_n}{a_n}, \quad n = 1,2,3,... . \tag{29}$$

Putting conversions (27), (28), (29) in formula (26), we obtain the desired general solution (25). This fact completes the proof.

4. Example of wear determination

After first time units, slide-bearing journal attains wear decreases W_1 μm^3. Determine the wear in successive n time units. For considered sliding nod, regard to the material properties, environmental conditions, operation mode, obtained parameters in experimental way have following variable dimensionless $A_n=a_n(n)$ and dimensional $B_n=b_n(n)$ form. By virtue of solutions (25), (26) the wear values in successive time units have the form:

$$\begin{aligned}
&f_1 = W, \\
&f_2 = Wa_1 + b_1, \\
&f_3 = Wa_1 a_2 + b_1 a_2 + b_2, \\
&f_4 = Wa_1 a_2 a_3 + b_1 a_2 a_3 + b_2 a_3 + b_3, \\
&f_5 = Wa_1 a_2 a_3 a_4 + b_1 a_2 a_3 a_4 + b_2 a_3 a_4 + b_3 a_4 + b_4, \\
&f_6 = Wa_1 a_2 a_3 a_4 a_5 + b_1 a_2 a_3 a_4 a_5 + b_2 a_3 a_4 a_5 + b_3 a_4 a_5 + b_4 a_5 + b_5, \\
&\text{..} \\
&f_n = Wa_1 \cdots a_{n-1} + b_1 a_2 \cdots a_{n-1} + b_2 a_3 \cdots a_{n-1} + \cdots + b_{n-3} a_{n-2} a_{n-1} + b_{n-2} a_{n-1} + b_{n-1} . \qquad (30) \\
&\text{..}
\end{aligned}$$

In particular case for constant parameter values i.e. for $a_n \equiv a$ [1], $b_n \equiv b$ μm^3, the dimensional wear values (30) in successive time units have the following form:

$$f_1 = W,$$
$$f_2 = Wa + b,$$
$$f_3 = Wa^2 + b(1 + a),$$
$$f_4 = Wa^3 + b\left(1 + a + a^2\right),$$
$$f_5 = Wa^4 + b\left(1 + a + a^2 + a^3\right),$$
....
$$f_n = Wa^{n-1} + b\left(1 + a + a^2 + a^3 + \cdots + a^{n-3} + a^{n-2}\right). \tag{31}$$
....

For 0<a<1 the dimensional wear during the time unit n has the form:

$$f_n = Wa^{n-1} + b\frac{1-a^{n-1}}{1-a} = \left(W - \frac{b}{1-a}\right)a^{n-1} + \frac{b}{1-a}, \text{ for } n=1,2,\ldots,N. \tag{32}$$

The wear after N time units has the following form:

$$\sum_{n=1}^{N} f_n = \left(W - \frac{b}{1-a}\right)\sum_{n=1}^{N} a^{n-1} + \sum_{n=1}^{N}\frac{b}{1-a} = \left(W - \frac{b}{1-a}\right)\frac{1-a^N}{1-a} + \frac{bN}{1-a}. \tag{33}$$

Presented wear process (33) is divergent if N tends to infinity.

5. Conclusions

1. The bearing wear prognosis presented in an analytical recurrent form was performed by virtue of hypothesis examined in an experimental manner, where the wear after sufficiently numerous time units of the exploitation process depends on the wear value in the previous time units multiplied by the experimental determined function A_n plus correction function B_n.
2. This paper presents the influence of bearing and bio-bearing material properties, journal vibrations and revolution, the standard deviation values of bearing surface deformations on the values of the bearing wear process in the succeeding time periods of the operation.
3. The application of the presented theory may be need in the preparation of control methods for the wear slide bearing divergence and convergence course in particular time units of the operating time.
4. The analytical tools for mentioned thesis examination are integrated by the method of solution of the first order homogeneous recurrence equations with variable coefficients, which are determined from the experiments.

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