

## ANALYSIS OF THE CHANGE OF QUALITATIVE STATES OF TRANSPORTATION SYSTEM OPERATION

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### **Abstract**

*This paper deals with the problems connected with evaluation of a technical system and its being later used for decision making in order to provide complex technical systems with appropriate operation quality. The research object is a real municipal transportation system. The assessment quality of the systems operation is based on the changes of the systems features values. The features describe the operation of the systems operators, controlled technical objects and the environment influence. The assessment and the support the demand quality of the systems operation from the safety, efficiency, economic and reliability point of view is the basic factor of the executed exploitation process. The notion of a system operation quality has been defined, a scheme of an assessment model has been presented and a random process, providing the basis for evaluation of a technical system, has been developed. On the basis of carried out experimental tests there have been distinguished four states of a system operation quality reflecting intervals of values of grades that were characteristic for given time moments. A model of system operation qualitative state changes, based on Markov chain, has been considered in this paper. A system of equations has been determined for a distinguished Markov chain on the basis of which a mathematical model for assessment and control of selected types of transportation systems will be developed.*

**Keywords:** *transport, system operation quality, model, Markov chain, semi-Markov process, expected coast*

### **1. Introduction**

The entire study is related to the research on operation quality of complex transport systems. The analysed objects belong to the group of real systems with an intended set of applications. These are socio-technical objects of the type <H-M-E> (human-machine–environment), where their operation quality depends on quality changes of characteristic features describing actions of operators, operation of technical objects controlled by them and the impact of the environment.

The considered issues concern interdisciplinary areas and the analysed problems can be placed in the field of science known as ontology, which is a theory of reality, but its goal is to cover all different walks of life and the relations occurring between them [5]. Complexity of the presented

issues results from the complicated character of the operation quality: actions of operators, functioning of transport means and the environmental impact. This, in turn, involves the necessity to refer to different fields of science, such as psychology, metrology, and widely understood mechanical sciences.

In pursuit of the paper goal, was made an analysis of several definitions of the concept of quality, the most adequate was found to be as follows: the system operation quality is a set of features expressed by means of numerical values, in a given moment  $t$ , determining the degree to which the set requirements have been met [7].

In connection with the above, the research main goal is to determine assessment criteria meeting the set requirements and to identify and choose features to be used for operation quality assessment.

A feature is a characteristic or an attribute of the analysis subject. Property is referred to such a feature common for all the subjects which is expressed as a physical quantity, a feature which lets us distinguish some objects which do not have these features is called quality.

It should be noted that the features determined for assessment of the transport system operation quality should bear signs of independence, essentiality, variability and measurability. Independence of the features is necessary those ones, which provide the same information on the research, object have to be eliminated. In the resultant model, there should be distinguished features that are of biggest significance from the point of view of the carried out research. Whereas, features of little significance, due to their slight influence on the research results, should be neglected. Their variability conditions purposefulness of acceptance of a given feature as a feature whose values do not undergo changes in a considered period of time, does not provide any information on the system state and causes redundancy of the considered set. Measurability of the features, according to the accepted definition of quality, is the basis for quality assessment, and it must be remembered that the set of features accepted for the examined transport system operation quality description consists of two subsets: measurable features (e.g. costs) and immeasurable ones (e.g. esthetics) [3].

## **2. The research object**

All the research is connected with operation quality of complex transport systems, especially those ones that carry out transport of passengers and freight by water, land or air. The main goal of such systems operation is to provide transport services with the use of technical objects, in a given environment, quantity, time and under the influence of given environmental factors. Thus, providing the object with the required operation quality and its assessment in terms of safety, efficiency reliability, availability, including the economic factor are of key importance for the operation process. The studied transport systems belong to a group of sociotechnical systems of the type H-M-E (human- machine- environment) in which their operation quality assessment is made depending on changes of values of features describing actions of operators, technical objects controlled by them, and the impact of the environment.

On the basis of an identification and analysis of real transport, systems it was established that at particular levels of their decomposition, there could be distinguished the following subsystems [8]:

- logistic, including actions connected with the system management, information flow and processing are performed as well as maintenance serviceability of transport means used in the system and this subsystem consists of:
  - decision making subsystem,
  - traffic continuity maintenance subsystem,
  - information subsystem,
- executive, whose main goal is, to provide transport services, environment – a synergy subsystem.

### 3. The system operation quality

This section contains a description of rules, on the basis of which the system operation quality assessment method has been formulated with special emphasis on municipal systems of public transport.

On the basis of literature and the author's own research it has been defined that: the system operation quality is a set of features expressed by means of their numerical values in given time  $t$ , defining the fulfilment degree of the set requirements [7].

It was assumed that the evaluator establishes a set of criteria for assessment of system  $K$  operation quality. Next, the research object is identified and on this basis, with reference to the established criteria, a set of features -  $X$ , describing the system in terms of its operation quality, is determined.

Assessment made in such a way involves determination of criteria, that is, requirements set by outside observers (users, decision makers, operators, maintenance workers), with the assumption that it makes sense when:

$$K_1(t) \cup K_2(t) \cup \dots \cup K_{n-1}(t) \in \text{True}, \quad (1)$$

where:

$K_i(t)$  – logical variable:

0 – if the  $i$ -th criterion has not been accepted,

1 – if it has been accepted.

The assessment process involves monitoring whether and to what degree particular features fulfil the established  $K$  criteria. Evaluation is performed on the basis of the features values measured in time  $t$  (measurable features) or states in which they are in a given time  $t$  (immeasurable features), through assigning appropriate identifiers to them. In connection with this, the level of the system operation quality in given time  $t$  determines a set of values of significant features  $\{X_i\}$   $i=1, 2, \dots, p$ , accepted for its description, from an established point of view.

The system model signifies such a system which when devised or implemented will reflect or reconstruct the research object in such a way that it can be replaced it in such a way that upon being examined it provides new information on this object [3, 4]. It is assumed that the model should aim at distinguishing significant, variable features of the examined phenomena and processes, neglecting others. Division into significant and insignificant variables depends largely on the researchers, their knowledge, possibilities of calculation and measurement and the accepted by them methods, tools and research techniques.

Defining the fulfilment degree of the set requirements-criteria provides the basis for evaluation of a given transport system operation quality. Condition justifying acceptance of a given criterion is dependent on whether its fulfilment degree can be checked by at least one of the describing it (significant, variable, measurable, non-correlated) features. Thus, the general, criteria based assessment model is described by dependence 2:

$$\begin{aligned} K_1(t) &= \langle X_1(t), X_2(t), \dots, X_{K_1}(t) \rangle, \\ K_2(t) &= \langle X_{K_1+1}(t), X_{K_1+2}(t), \dots, X_{K_2}(t) \rangle, \\ &\dots \\ K_i(t) &= \langle X_{K_{i-1}+1}(t), X_{K_{i-1}+2}(t), \dots, X_{K_i}(t) \rangle, \\ &\dots \\ K_{n-1}(t) &= \langle X_{K_{n-2}+1}(t), X_{K_{n-2}+2}(t), \dots, X_n(t) \rangle. \end{aligned} \quad (2)$$

Thus, for a random  $i$ -th criterion the condition of non-emptiness needs to be satisfied – condition of existence of a set of criteria described by dependence 3:

$$\bigwedge_{i \in \{1, 2, \dots, n-1\}} K_{i+1} - K_i \geq 1. \quad (3)$$

On the basis of literature analysis and the author's own research, a method and a resultant model for transport systems operation quality assessment have been developed. The developed method enables assessment and comparison of operation quality of different transport systems of the same type. For this purpose, metrics on the basis of which the transport system operation quality assessment was made were developed. Values of the metrics described by dependence (1), is determined basing on the values of significant features describing the system, accounting for values of weights attributed to particular features. For the analysed system, a random process is defined for the analysed system, reflecting the system operation quality, in the form:

$$Z_x(t) = \sum_{i=1}^p \alpha_i X_i(t), \quad (4)$$

$$\alpha_i \geq 0, \sum_{i=1}^p \alpha_i = 1.$$

It should be emphasized that for determination of a given system operation quality assessment it is important to specify a set of the most significant assessment criteria and basing on them choose significant features conforming with the accepted criteria and establish their significance.

#### **4. Description of a system operation quality states**

Since the resultant model of a system quality assessment also includes features for which the most desirable state is reflected by their lowest values, and their variability intervals are different, therefore for the purpose of interpretation unambiguity of the obtained tests results, they are being recoded onto the range <0 – 10> according to dependence (5).

$$\text{Range} = 10 * (X_i - X_{\min}) / (X_{\max} - X_{\min}), \quad (5)$$

where:

$$X_{\min} = \text{Min} \{X_i\}, \quad X_{\max} = \text{Max} \{X_i\}.$$

On this basis, knowing the value of a system operation in interval <0-10> in given times t, the system operation quality state is determined [4, 7].

There have been distinguished 4 states, which reflect the values from the accepted range:

- I state – ‘desirable’ – reflects values from interval: <8, 10>. This state is an equivalent to the system operation model or the required quality,
- II state – ‘acceptable’: <5.5, 8) is a state of the system proper operation which does not require to take actions necessary to reach state I, whereas transition to this state is connected with undertaking a strategic decision e.g. exchange of transport means of their operators whose actions have an adverse influence on its level.
- III state – ‘limited’: <4, 5.5). This state reflects a boundary level of the system operation quality. In this state provided services are not performed in a proper way, inconsistently with the requirements, and intervention is necessary (replacement of operator, recovering of the technical object serviceability, reduction or elimination of the negative impact of external factors) in order to reach the system operation quality state I or II,
- IV state - ‘critical’: – <0, 4) – in this state the system is not able to accomplish its tasks. It requires be renovating, replacing or modernizing in order to provide at least II level of its operation quality.

#### **5. Semi – Markov model of process changes of qualitative state**

The semi – Markov process is a generalization of a discrete – time Markov chain where the times between transitions are allowed to be random variables, which depend on the current and possibility the next state [1, 2].

A natural way to approach semi – Markov process is through renewal theory, where inter-arrival times between events do not need to be exponentials distributed. For this purpose, it is helpful to define a Markov renewal sequence as a sequence of a bivariate random variable first. The two elements of this bivariate random variable are the observation time  $S_n$  of the  $n$ -th transition and the corresponding  $n$ -th observation  $Y_n, n \geq 0, Y_n \in \{0,1,2, \dots, n\}$ .

The joint probability of observing  $Y_{n+1} = j$  in an inter – arrival time of  $S_{n+1} - S_n \leq x$  conditioned on observation history, satisfies the Markov property:

$$\begin{aligned} P\{Y_{n+1} = j, S_{n+1} - S_n \leq x \mid Y_n = i, S_n, Y_{n-1}, S_{n-1}, \dots, Y_0, 0\} = \\ = P\{Y_{n+1} = j, S_{n+1} - S_n \leq x \mid Y_n = i\} \end{aligned} \tag{6}$$

and let:

$$P\{Y_{n+1} = j, S_{n+1} - S_n \leq x \mid Y_n = i\} = G_{ij}(x). \tag{7}$$

Finally, a semi – Markov process is a stochastic process that the records the state of the renewal process at each point in time.

More formal, let  $\{(Y_n, S_n), n \geq 0\}$  be a Markov renewal sequence. Let be the state with the last completed state spell before  $t, N(t) = \sup \{n \geq 0: S_n \leq t\}$ , and let  $X(t) = Y_{N(t)}$ . Then the stochastic process  $\{X(t), t \geq 0\}$  is denoted as a semi – Markov process. The process  $Y_n$  is called embedded Markov chain of  $X(t)$ .

The matrix  $G(x) = [G_{ij}(x)]$  as defined in equation (7) is called the kernel of the semi – Markov process.

Next, we discuss some properties of the semi – Markov process, which help to classify them. A semi – Markov process is time – homogenous if just the interval until the next transition matters for the probability not when this interval is started, or more specific.

$$P\{Y_{n+1} = j, S_{n+1} - S_n \leq x \mid Y_n = i\} = P\{Y_1 = j, S_1 \leq x \mid Y_0 = i\}. \tag{8}$$

A semi-Markov process is called regular if there is only a finite time period. The semi – Markov process is irreducible if each state can be reached from any other state. The state are said to communicate with each other in this case. A state  $j$  is called recurrent, if the process returns to state  $j$  in a spell less than infinity and it is called transient otherwise (if it never returns). A state is denoted as positive recurrent if it is recurrent and the expected returning time to state  $i$ , given the process started in  $i$ , is less than infinity. For the semi – Markov process, a recurrent state  $i$  is called aperiodic if it is possible to visit this anytime [1].

The initial distribution of states  $a = [a_{ij}]$  reports the probability that the state of system is  $i$  at beginning,  $a_j = P\{X(0) = i\}$ . Finally, a regular semi – Markov process is fully specified by initial distribution of states  $a$  and the kernel  $G(x) = [G_{ij}(x)]$ .

For positive recurrent, irreducible and aperiodic a semi-Markov process, the limiting probability of being in state  $j$  when starting in state  $i$  is independent of  $i$ :

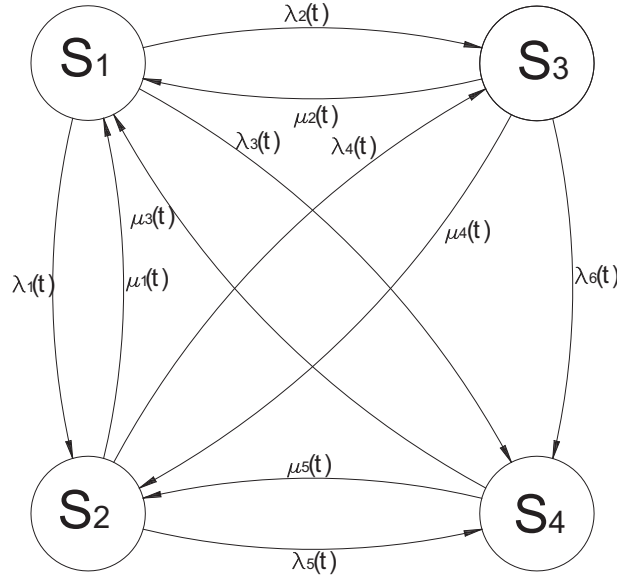
$$p_j = \lim P\{X(t) = j \mid X(0) = i\} = \Pi_j ET_j / \sum \Pi_i ET_i, \tag{9}$$

where:

$\Pi_i, i = 1, 2, \dots, n$  is the limit probability of the embedded Markov chain and  $ET_i$  is the expected value of duration of state  $i, i = 1, 2, \dots, n$ .

## 6. Numerical example of qualitative changes states changes

A graph and a matrix of a system operation qualitative state transitions have been built for the considered object. A graph reflecting possible transitions between the four distinguished states is presented in Fig. 1.



*Fig. 1. Graph of changes*

Basing on this, a matrix of transition intensity has been developed:

$$P = \begin{bmatrix} 0 & p_{12} & p_{13} & p_{14} \\ p_{21} & 0 & p_{23} & p_{24} \\ p_{31} & p_{32} & 0 & p_{34} \\ p_{41} & p_{42} & 0 & 0 \end{bmatrix}. \quad (10)$$

It has been established that for the analysed research object this matrix has the following form:

$$P = \begin{bmatrix} 0 & 0.6 & 0.3 & 0.1 \\ 0.1 & 0 & 0.65 & 0.25 \\ 0.2 & 0.7 & 0 & 0.1 \\ 0.15 & 0.85 & 0 & 0 \end{bmatrix}. \quad (11)$$

For probability transition matrix  $P$ , we have calculated limit probabilities of the embedded Markov chain:

$$\begin{aligned} \Pi_1 &= 0.126, \\ \Pi_2 &= 0.417, \\ \Pi_3 &= 0.309, \\ \Pi_4 &= 0.148. \end{aligned}$$

Basing on experimental tests, there have been calculated mean values of the objects presence in the following states:

$$\begin{aligned} ET_1 &= 2, \\ ET_2 &= 6, \\ ET_3 &= 0.5, \\ ET_4 &= 1.5. \end{aligned}$$

For the limit probabilities of semi – Markov process, we have:

$$\begin{aligned} P_1 &= 0.193, \\ P_2 &= 0.533, \\ P_3 &= 0.237, \\ P_4 &= 0.038. \end{aligned}$$

## 7. Analysis the expected cost

For our model, let  $U_i$  denote the length of the  $i$  – th successive the operational cycle for  $i=1,2,3,\dots$ : and let  $V_i$  denote the operational cost over the renewal interval  $U_i$ . Thus,  $\{(U_i, V_i)\}$  constitutes a renewal reward process. If  $D(t)$  denotes the expected cost of operating the system over time interval  $[0, t]$ , then it is well known that [6].

$$\lim_{n \rightarrow \infty} \frac{D(t)}{t} = \frac{EV_i}{EU_i}. \quad (12)$$

Therefore, the long-run expected cost per unit time for the operating system is given by

$$C = EV_i / EU_i. \quad (13)$$

We assume that  $c_i$ ,  $i = 1,2,3,4$  are the cost of the state  $S_i$ . The long-run expected cost per unit time for the operating system is given by:

$$C = \sum_{i=1}^4 c_i P_i E T_i / \sum_{i=1}^4 P_i E T_i. \quad (14)$$

## 8. Summary

The article deals with the problems connected with evaluation of complex technical systems operation on the basis of the considered research object. Construction of a model for a transport system qualitative state transition changes basing on Markov chain embedded in a certain semi-Markov process has been discussed. According to experimental tests, performed in a given time, a boundary distribution of semi-Markov process has been determined. This distribution reflects the system behaviour during sufficiently long operation time. A general form of a model for assessment of anticipated costs was developed for the analysed process. The considered issue is significant from the point of view of the decision process intended to shape the system operation quality level, especially the necessity and advisability of bearing costs connected with the system operation quality improvement – transition to the desirable, accepted qualitative state. Numerical simulation tests of the system can provide the basis for finding a way to modify the model in order to increase its operation quality and this is supposed to be the subject of further research.

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