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SENSITIVITY ANALYSIS FOR DYNAMIC MODELS OF AERIAL MUNITIONS

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Abstract

While investigating dynamic properties of aerial bombs, both the analytical and experimental outcome often find their applications. Analysis of how a series of changes in flight parameters affect flight dynamics is probably one of the most essential efforts throughout this study. Numerical solutions of equations of motion deliver no information on the effect of changes in the aerial bomb's design parameters on flight characteristics thereof. Therefore, the theory of sensitivity has been used in the paper to investigate the effect(s) of external disturbances, drop/release conditions and technical parameters on the aerial bomb's range and flight path.

Among other, the elements of the theory of sensitivity, equations of motion, numerical analysis of sensitivity of the bomb-describing mathematical model to changes in some selected parameters, numerical analysis of sensitivity of the bomb-describing mathematical model to changes in some selected parameters, analysis of the effect of changes in drop/release conditions on the aerial bomb's flight path, analysis of the effect of changes in engineering (design) parameters of the aerial bomb on the trajectory thereof, effect of drop conditions on the aerial bomb's range, effect of drop conditions on the side deflection of the aerial bomb's trajectory are presented in the paper.

Keywords*: aircraft, air combat means, dynamic properties of aerial bombs, conditions on the aerial bomb's flight path, trajectory of aerial bombs*

1. Introduction

Dynamic characteristics of aerial warfare agents are inputs for aircraft's attack avionics to compute the instant of drop, release, firing them. Therefore, precise determination – with required accuracy - has become an essential demand. Examination of dynamic properties can be carried out either with experimental, analytical or mixed methods [11, 12]. Of significance is also analysis of how changes to a series of design parameters of aerial warfare agents affect their dynamics of flight. Numerical solution of both non-linear and linearised equations of motion supplies no information on the effect of these changes on properties of motion. Any change to any parameter requires the equations of motion to be re-integrated, which makes analysis of the effect(s) of these changes extremely expensive. To reduce the amount of necessary numerical computations, one can use the theory of sensitivity. It enables finding directions of changes in the solutions to equations of motion, effected by changes of design (or other) parameters, within some selected time interval. The paper has been intended to investigate into effects of external disturbances, drop/release conditions, and technical parameters affecting the bomb range and flight path.

2. The elements of the theory of sensitivity

Analysis of sensitivity has been intended to investigate effects of changes to the parameters on dynamic properties of a moving body. Parameters that change most often are as follows: initial conditions, non-variable with time factors/coefficients, factors/coefficients that depend on time, parameters that change the order of a system of equations, frequency characteristics, design parameters, etc. [17, 19].

The analysis of sensitivity requires the so-called matrix of sensitivity to be defined.

If the vector $\mathbf{x}(\mathbf{t}, \mathbf{a}, \mathbf{x_0})$ is a solution of the following equation:

$$\dot{\mathbf{x}} = \mathrm{f}(t, \mathbf{a}, \mathbf{x}_0), \text{ for } \mathbf{x}(0) = \mathbf{x}_0, \tag{1}$$

and the vector $\mathbf{a}=[\alpha_1, \alpha_2, \ldots, \alpha_n]^T$ is that of n parameters against which the sensitivity is examined, then the matrix of sensitivity can be defined in the following way:

$$\mathbf{v} = [v_{ij}];_{i=1,2,\ldots\ldots g;\ j=1,2,\ldots\ldots n}, \tag{1a}$$

where:
i – number of the equations of motion,
j – number of parameters;
with v_{ij} given with the following relationships:

$$v_{ij} = v_{ij}(t, a_j) = \lim_{\Delta a_j \to 0} \frac{x_i(t, a_1, a_2, \ldots. a_j + \Delta a_j, \ldots a_n) - x_i(t, a_1, a_2, \ldots. a_j, \ldots a_n)}{\Delta a_j}. \tag{2}$$

Elements of the matrix of sensitivity are called functions of sensitivity. Numerical solution of equations of motion (1) does not allow the function of sensitivity to be found directly from the definition (2). These functions are to be found from the equations of sensitivity[18]:

$$\dot{\mathbf{v}}_j = \mathbf{D}\mathbf{v}_j + \mathbf{w}_j, \tag{3}$$

where matrix $\boldsymbol{D}$ is exactly the same for each and all equations of sensitivity, and takes the following form: $\mathbf{D} = \frac{\partial \mathrm{f}(t, \mathbf{a}, \mathbf{x})}{\partial \mathbf{x}}$; the vector $\boldsymbol{w_j}$ is found for each parameter according to the following relationship:

$$\mathbf{w}_j = \frac{\partial \mathrm{f}}{\partial a_j}, j = 1, \ldots, n. \tag{3a}$$

The right members of eq. (1) need to be continuous and differentiable against some selected parameters. In the case values of functions of sensitivity within some selected time interval are low, one can find functions of sensitivity of higher order.

When the α parameter depends on time, eq. (1) is expressed as:

$$\dot{\mathbf{x}} = \mathrm{f}\left[\left(t, \mathbf{x}, \mathbf{a}(t)\right)\right]. \tag{4}$$

An altered form of the time-dependent parameter can be presented as (1), then functions of sensitivity are defined in the following way:

$$v_i = \lim_{\varepsilon \to 0} \frac{x_i\left(t, a_i + \varepsilon\lambda(t)\right) - x_i\left(t, a_i\right)}{\varepsilon}. \tag{5}$$

Such being the case, a vector of right members of eq. (5) can be developed into the Taylor series:

$$\mathrm{f}(t, \mathbf{x}, \mathbf{a} + \varepsilon\lambda) = \mathrm{f}(t, \mathbf{x}, \mathbf{a}) + \varepsilon\, \mathrm{g}(t, \mathbf{x}, \mathbf{a})\lambda. \tag{6}$$

Functions of sensitivity are found by means of solving the following equation [18]:

$$\dot{\mathbf{v}} = \mathbf{D}\mathbf{v} + \mathrm{g}\left(t, \mathbf{x}, \mathbf{a}\right)\lambda, \ \mathbf{v}(0) = \mathbf{0}. \tag{7}$$

Equation (7) can be applied to examine the impact of changes in aerodynamic derivatives on co-ordinates of the state vector $\mathbf{x}$. If we want to check the sensitivity of solutions of the system of equations (4) to time-independent measuring error of aerodynamic characteristics, the λ function equals unity.

3. Equations of motion

Equations of motion show the following form:

$$\dot{\mathbf{x}} = \mathrm{f}(t, \mathbf{a}, \mathbf{x}_0). \tag{8}$$

In the bounded system, for any rigid body in 3-D motion, the state vector takes the following form [5-7]:

$$\mathbf{x} = [U, V, W, P, Q, R, \Theta, \Phi, \Psi, h]^{\mathrm{T}}, \tag{9}$$

where: P, Q, R are co-ordinates of the angular-velocity vector Ω; Θ, Ψ, Φ are Euler angles; h is flight altitude.

Functions f_n are expressed with the following relationships [9, 10]:

$$f_1 = -QW + RV + \frac{X}{m}, \quad f_2 = -RU + PW + \frac{Y}{m},$$

$$f_3 = -PW + QU + \frac{Z}{m}, f_4 = \left(\frac{J_x - J_z}{J_x} - \frac{J^2{}_{xz}}{J_x J_z}\right)\frac{QR}{D} + \left(1 - \frac{J_y - J_x}{J_z}\right)\frac{J_{xz}}{J_x}\frac{PQ}{D} + \frac{L}{J_x D} + \frac{J_{xz} N}{J_x J_z D}, \tag{10}$$

$$f_5 = \frac{M}{J_y} + \frac{J_z - J_x}{J_y} QP + \frac{J_{xz}}{J_y}\left(R^2 - P^2\right),$$

$$f_6 = \left(\frac{J^2{}_{xz}}{J_x J_z} - \frac{J_y - J_x}{J_z}\right)\frac{PQ}{D} + \left(\frac{J_y - J_z}{J_x} - 1\right)\frac{J_{xz} QR}{J_z D} + \frac{J_{xz} L}{J_x J_z} + \frac{N}{J_z D},$$

$$\begin{bmatrix} f_7 \\ f_8 \\ f_9 \end{bmatrix} = \begin{bmatrix} 1 & \sin\Phi\tan\Theta & \cos\Phi\tan\Theta \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \sin\Phi\sec\Theta & \cos\Phi\sec\Theta \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix},$$

$$f_{10} = U\sin\Theta - V\sin\Phi\cos\Theta - W\cos\Phi\cos\Theta.$$

where:

$$D = 1 - \frac{J^2{}_{xz}}{J_x J_z}, \quad X = F_x - mg\sin\Theta + T_x, \quad Y = F_y + mg\cos\Theta\sin\Phi,$$

$$Z = F_z + mg\cos\Theta\cos\Phi + T_z, \tag{10a}$$

where:

$\bar{F}_a$ – vector of aerodynamic forces,

$M_\alpha = [L, M, N]^T$ – aerodynamic moment,

m – bomb weight,

g – acceleration of gravity,

J_X, J_Y, J_Z –moments of inertia.

Finding the aerodynamic forces and moments that affect a bomb is a problem of significance, since they depend on the bomb's shape, angles of attack and slip, angular velocity, velocity of flow (Mach number [1-4, 8, 13-16]. However, the methods developed until now (both analytical and experimental ones) do not define these forces precisely for any 3-D motion of the body. In the case of a manoeuvring flight, for Mach numbers < 0.5, the method of superposition is used, according to which force X can be presented as:

$$X = -mg\sin\Theta + X_s(\alpha,\beta) + \sum_j X_{\delta_j}(\alpha,\beta)\,\delta_j + X_P(\alpha,\beta)\,P +$$

$$+ X_Q(\alpha,\beta)Q + X_R(\alpha,\beta)R + X(\alpha,\beta)_{PQ}PQ + X(\alpha,\beta)_{PP}P^2 + .. + T_x, \tag{11}$$

where:

$$X_s = \frac{1}{2}\rho V_0^2\, S\, C_{x0}\left(\alpha,\beta\right),\quad X_P = \frac{1}{4}\rho V_0 SbC_{xp}\left(\alpha,\beta\right),\quad X_Q = \frac{1}{4}\rho V_0 ScC_{xq}\left(\alpha,\beta\right), \tag{11a}$$

with:
ρ – air density,
S – wing area,
b – wing span,
c – total length of the bomb,
$C_{xi}(\alpha, \beta)$ – non-dimensional aerodynamic coefficients.

The force defined with eq. (11) depends in a linear way on angles of displacements of angular surfaces, and in a non-linear way - on co-ordinates of the state vector. It is a traditional simplification of the equations of motion although, as it has already been shown, there are higher derivatives of aerodynamic forces computed in relation to angles of control-areas displacements.

The above-presented model of the 3-D motion of any body is subject to further simplifications, which in turn depend on the way of synthesising control laws or identification methods accepted. One of frequently applied simplifications is that polynomial forms of aerodynamic coefficients are accepted:

$$C\left(\alpha,\beta\right) = C_0 + \sum_i C_{\alpha^i}\alpha^i + \sum_j C_{\beta^j}\beta^j + \sum_i\sum_j C_{\alpha^i\beta^j}\alpha^i\beta^j\,, \tag{12}$$

with: C_0, C_α', being constants. In the case of 3-D motion, the number of parameters rapidly increases with the degree of the polynomial. Therefore, it is advisable to examine the effect of some selected parameters on the simulated - with the above-accepted model - pod motion. To meet this goal, application of the theory of sensitivity is suggested.

4. Numerical analysis of sensitivity of the bomb-describing mathematical model to changes in some selected parameters

Exemplary computations that reflect suitability of the analysis of sensitivity to evaluate changes in parameters of the aerial bomb's trajectory have been made for several instances. For the range of some selected parameters, the effect of drop/release conditions and design characteristics on the bomb's range and side deflection of the bomb's flight path has been analysed.

4.1. Analysis of the effect of changes in drop/release conditions on the aerial bomb's flight path

Simulations have been conducted for several instances of computations, with the following assumptions taken into account:
- horizontal-flight drop,
- drop altitude H = 800 m,
- nominal velocity V = 230 m/s,
- nominal angle of attack $\alpha = 4^0$,
- nominal flight-path angle $\gamma = 0^0$.

Figures 1 and 2 show computational results gained. Fig. 1 illustrates the effect of small changes in aircraft's angle of attack, flight-path angle and flight speed on the range of a bomb dropped from this aircraft. Tangents of respective curve-slope angles are elements of matrix of sensitivity given with relationship (2). Fig. 2 illustrates the effect of small changes in aircraft's angle of attack, flight-path angle and flight speed on the side deflection of the aerial bomb's flight path. Tangents of respective curve-slope angles are elements of matrix of sensitivity given with relationship (2). What could easily be found is that both the bomb carrier's initial velocity and initial flight-path angle are of the greatest effect on the bomb's trajectory.

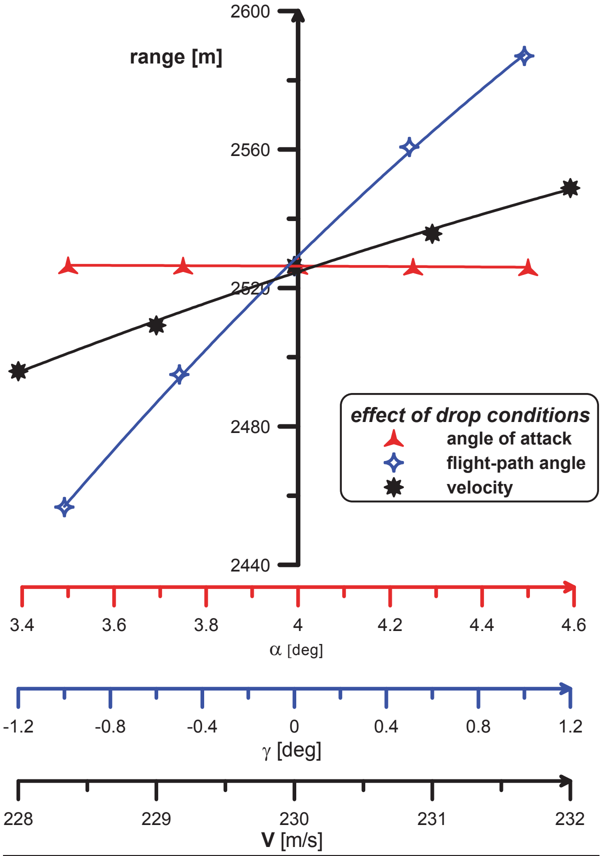

Fig. 1. Effect of drop conditions on the aerial bomb's range

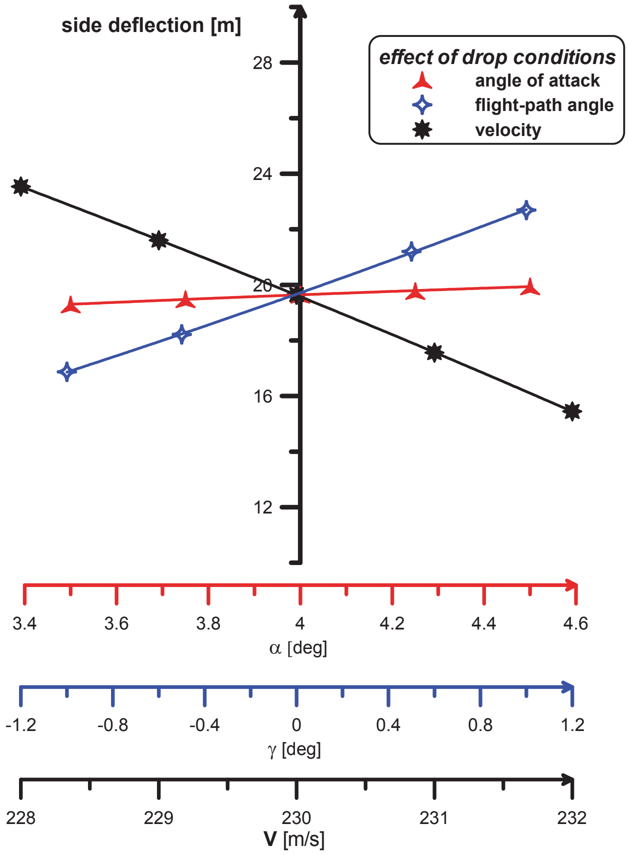

Fig. 2. Effect of drop conditions on the side deflection of the aerial bomb's trajectory

4.2. Analysis of the effect of changes in engineering (design) parameters of the aerial bomb on the trajectory thereof

Results of numerical analysis of sensitivity have been presented in Fig. 3. Investigation was conducted with the following assumptions made:

– the bomb was dropped from the horizontal flight,
– drop altitude H = 800 m,
– velocity V = 230 m/s,
– nominal weight of the aerial bomb m = 222.5 kg,
– nominal angle of fin setting $\delta = 0^0$.

Analysis of the plot (Fig. 3) encourages the following statement: from among many and various design parameters examined, the angle of fin setting is of the greatest effect on the bomb's trajectory. Tangents of respective curve-slope angles are elements of matrix of sensitivity given with relationship (2).

5. Conclusions

Sensitivity of the model to changes in some selected parameters thereof has been investigated. Effects of drop conditions (Fig. 1 and 2) and technical parameters (Fig. 3) have been analysed.

The analysis of effects of drop conditions on the bomb's flight path has proved what follows: a change of initial velocity by 4 m/s results in changes of both the bomb's range by approximately 55 m and side deflection by approximately 9 m; a change in angle of attack by 1^0 has proved to have no effect on the bomb's range; a change in the bomb's flight-path angle by 2^0 results in the change of range by approximately 130 m.

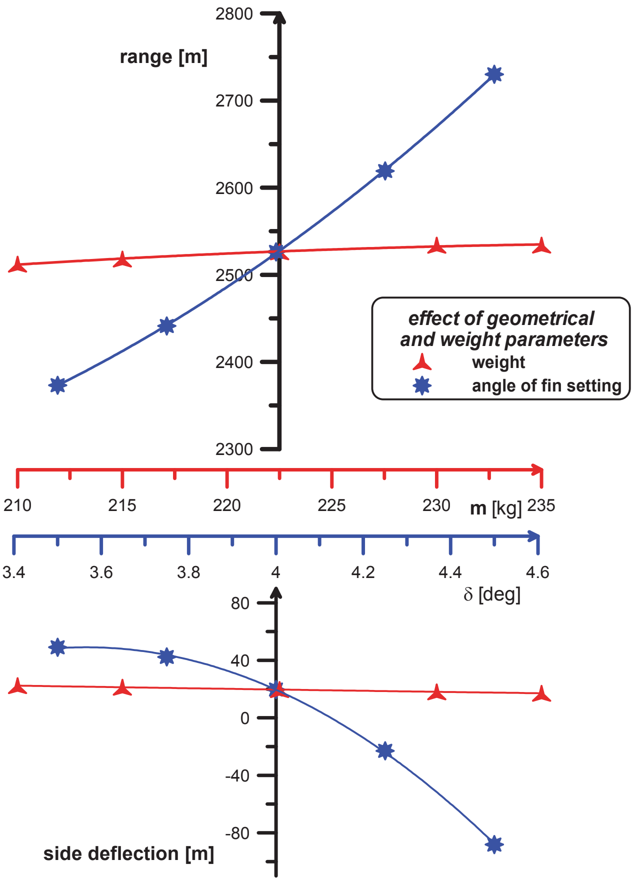

Fig. 3. Effect of technical parameters on the aerial bomb's range and side deflection of its trajectory

The effects of technical parameters on those of the bomb's flight-path elements are as follows: a change in weight by 25 kg proves to have practically no effect on the bomb's range and side deflection; the angle of fin setting proves to be of considerable effect: for one-degree (1^0) change the bomb's range changes by approximately 400 m, whereas the side deflection – by approximately 130 m; any increase in the angle of fin setting results in the reversal of sign and some change in side deflection.

On the grounds of the above-presented analyses one can find that the theory of sensitivity delivers effective tools to evaluate the effect of changes of some selected parameters on the aerial bomb's trajectory and hence, on the bombing accuracy. Information resulting from such analyses provides good grounds for the estimation of errors in data introduced in the aircraft's air attack system [20, 11].

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