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EXAMPLE OF APPLYING MARKOV DECISION PROCESS O MODEL VEHICLE MAINTENANCE PROCESS

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Abstract

The subject of this paper are selected issues connected with modelling, prognosis and control of the process of operation and maintenance of a certain category of technical objects. The research object is a system of transport means operation. Assumptions for building a model of the operation process state changes of objects used in a complex system of operation and maintenance have been presented. The operation process model was built on the basis of an analysis of the space of states and events concerning technical objects, which are used in the analysed real transportation system. Source data necessary to make assumptions for the model development and its initial verification, has been obtained by means of the passive experiment method from a real system of transport means operation.

An example of Markov decision process use for modelling and analysis of a public city transportation system operation process has been presented. Determination of values of indices characterizing the analysed process has been carried out basing on computer simulation of Markov decision process, being a mathematical model of technical objects operation process. A computing model has illustrated the entire study. Values of the model parameters have been estimated basing on the results of preliminary tests conducted in a real system of transport means operation.

The model presented in this work has been built in such a way that it can be used for a wide category of problems connected with technical objects operation.

Keywords: *transport system, operation process, Markov decision process, urban transportation, state of object*

1. Introduction

Real processes of operation and maintenance of technical objects are characterized by non-uniformity in terms of stochastic and the so-called secondary effect, that is, identified states of a process and analysed operation states of objects depend not only on their current state but also on their previous states or other factors. Despite the fact that mathematical models of real processes oversimplify their actual state, it seems that for a certain category of operation and maintenance processes and systems, omission of this type of dependence would be s simplification, which goes too far. Results obtained from oversimplified models can be burdened by a significant error and their analysis can lead to erroneous conclusions.

The authors of this paper present, a method for building a model of technical objects operation whose sequence of successive operation states, their duration times, incomes and costs connected with the objects being in these states and the way a given state is reached depend not only on the object current state but also on other factors. A simplified computing model has been presented in order to illustrate the discussion.

The model of operation process was built basing on the analysis of spaces of states and operation events concerning technical objects used in a real transportation system. In result of identification of the analysed system and its multi-state process of technical objects operation, operation states and possible transitions between these states, significant for this research, have been determined.

The main information source, allowing obtaining objective results of quantitative analysis of technical objects failures and their operation states is the statistical data collected during experimental tests.

Sets of source data indispensable for the model assessment and its initial verification were obtained on the basis of experimental tests with the use of passive experiment method from a real research object.

It seems that simulation of changes of technical objects operation states and using it for prognosis of the operation system behaviour as well as determination and analysis of the values of selected decision indices can enable control of a complex operation system [1, 2, 6, 11, 12]. This type of indices include indices concerning the subsystem providing vehicles with serviceability, specifically the number of repairs, repair labour consumption, demand for specialist equipment, etc. [6, 7, 8, 12]. Prognosis of changes in the analysed indices values are also of importance.

Due to the character of this paper, only selected assumptions of the simplified vehicle operation process model have been presented. The values of indices characterizing the analysed process are determined with the use of computer simulation of Markov decision process, being a mathematical model of technical objects operation process.

2. The research object

The research object are systems of public urban transportation systems providing transport services on the territory of towns and their suburbs. Providing high level of safety, proper level of vehicles operating readiness and operation reliability and continuous pursuit of improvement of the operation efficiency are basic goals of this type of systems.

An example of the research object chosen to illustrate the study, is a real system of public urban bus transportation, operating in the analysed urban agglomeration. This system is one of subsystems of the whole public transportation system.

Safety level of services provided by the system depends on the values of features describing: the system of operation and its environment, technical objects used in it, operators of these objects, etc.

There can be distinguished the following subsets of factors affecting the level of safety:

- operation-related factors – conditioned by the technical object operation,
- external factors – characterizing the impact of the environment on technical objects and their operator and people who are involved in the system and its environment,
- anthropotechnical factors – connected with conscious or unconscious human actions.

In the analysed operation system, it is the system of providing serviceability that is responsible for providing the transport means with the required operational readiness and reliability

Providing transport means with high reliability level depends on the quality of subassemblies used for repairs, proper assessment of the transport means state and high quality of maintenance and repairs.

An assumption, that the identified set of technical objects, used in the analysed system of operation, can be divided into n separate subsets of objects, homogeneous in terms of the research purpose, has been accepted. So classified subsets of technical objects are called: categories of objects [6]. In practice, a given technical object (bus) is assigned to a given category on the basis of the following criteria [8, 10]:

- type of object (make and type of bus),
- operational potential,
- operation time,
- others.

Further, in this paper, one category of objects is discussed.

In result of identification of a real urban bus transportation system and its operation process, three finite sets of states and operation-related events significant for an analysis of the system [6, 8, 10], have been distinguished. In the computing model, selected subsets of these states and events are analysed.

3. Mathematical model of the operation process

A natural model of a vehicle (technical object) operation is a random process with a finite state space S and a set of parameters R+ (subset of natural numbers ≥ 0) [2, 3, 4, 5]. Homogenous stochastic processes, including Markov and semi-Markov processes [3, 9] are commonly used for modelling of operation state changes. This is an oversimplification of real processes. In result of identification of the process of urban bus transportation, it was found to be a nonhomogeneous process. Also, due to the research objective and the need to model a sequence of the object states, whose changes depend not only on the previous states but also on other factors, the operation process was analysed using the theory of Markov decision processes.

Stochastic model $\{X_t, D_t, t \in T\}$, $t \geq 0$ being a specific case of Markov decision process, was used. In practical applications, it must be decided whether there are reasons to reject the assumptions connected with the mathematical apparatus.

In the next section, the main assumptions accepted for the description of stochastic process $\{X_t, D_t, t \in T\}$, $t \geq 0$ being a mathematical model of the analysed operation system, are discussed.

It is assumed that a stochastic process can describe the process of operation state changes:

$$\{X_t, t \in T\}, t \geq 0, \tag{1}$$

with a finite state space:

$$S = \{1, 2, ..., n\}, n \in N, \tag{2}$$

where:
N – set of natural numbers,
T – subset of real numbers.

For the needs of this research, it is accepted that the states of the discussed stochastic process correspond to operation states of the technical object (vehicle).

Alternative k, accepted upon entering state i, is denoted as ($i \in S, k \in N$). Finite set A_i, of alternatives (decisions) corresponds to each state i, $i \in S$.

Sets of alternatives have to be equal for each state in terms of quantity. Also states, for which the set of alternatives is a single-elements set, can occur. States, for which there is no possibility to choose an alternative, are referred to as non-decision states. Some literature sources refer to non-decision states as to such for which the set of alternatives in s an empty set [8]. It appears that acceptance of one-element sets of alternatives is more consistent for a description of the analysed process $\{X_t, D_t, t \in T\}$, $t \geq 0$.

It is accepted that elements of set A_i, $i \in S$ are elements a_{ik}, ($I \in S$, $k \in N$), that is:

$$A_i = \{a_{i,1}, a_{i,2}, ..., a_{i,\bar{\bar{i}}}\}, \tag{3}$$

where $\bar{\bar{i}}$ – denotes capacity of set A_i.

The set of all the subsets of alternatives is denoted as A, that is:

$$A = \bigcup_{i \in S} A_i . \tag{4}$$

Generally, application of a given alternative upon process $\{X_t, t \in T\}$ entering $i \in S$ State, can have an influence on the process successive state $j \in S$ and the state features (time of being in the state – type of distribution and its parameters, costs or profits obtained by the systems in this particular state, etc.).

The alternatives can represent given modes of operation, events, decisions, etc. which can be assigned to the state of the modelled process [1, 5, 8, 11]. In a real system of operation, there can be different ways of maintenance, repair, surveys, operation modes and scopes, e.g. different transportation routes, on which the vehicle is used. Acceptance of a given alternative can affect costs, income, frequency and kinds of failures, times of operation states, sequences of states, etc. [5, 11.]

Stochastic process:

$$\{D_t, t\in T\}, t \geq 0 \tag{5}$$

with a finite set of alternatives A describes the selection method of $a \in A$ alternatives.

Change of process $\{D_t, t \in T\}$ state occurs in times t, of process $\{X_t, t \in T\}$ state changes, In times t_n $n \in N$ of process $\{X_t, t \in T\}$ state changes alternative $a \in A$ is chosen. If in time t_n, state i is the state of process $\{X_t, t \in T\}$ then $a \in A_i$.

Process $\{X_t, D_t\}$, $t \geq 0$ with a finite state set S and finite set of alternatives A, is called a stochastic decision process. In result of this process, a sequence of states and decisions is obtained from initial time t_0 to time t_n:

$$h_{t_n} = \{i_{t_0}, a_{t_0}, i_{t_1}, a_{t_1}, \ldots, i_{t_n}, a_{t_n}), \tag{6}$$

which is called the process history until time t_n.

Further, it is assumed that the analysed stochastic decision process:

$$\{X_{t_n}, D_{t_n}\}_{n=1}^{\infty}, \tag{7}$$

is Markov decision process. A set of possible implementations of Markov decision process is set $W = \{S \times A\}^{\infty}$. It is also assumed that the probability of choosing alternative $a_{t_n} \in A$ depends merely on state $i_{t_{n-1}} \in S$, and does not depend on the process history $h_{t_{n-1}}$.

In this case, the sequence of process $\{X_t, t\in T\}$ states is a nonhomogeneous Markov chain. [3, 4].

In order to define the analysed stochastic decision process $\{X_t, D_t\}$, $t \geq 0$ it is also necessary to define:

- Method for choice of alternatives for process $\{D_t, t\in T\}$,
- Initial distribution of $\{X_t, t\in T\}$,
- Conditional probabilities of process $\{X_t, t\in T\}$ state changes,
- Random variables of process $\{X_t, t\in T\}$ states times.

Simplifying, the rule used for determination of choice of alternative $a \in A$, upon entering state $i \in S$, is referred to as a strategy. The manner of choosing the alternative upon entering the process state can be of random or determined character.

Formula:

$$p = [p_1, p_2, \ldots, p_n], \ \sum_{i\in S} p_i = 1, \ p_i \geq 0, \ i \in S, \tag{8}$$

is used for determination of $\{X_t, t \in T\}$. Giving values of p_i of elements of the initial distribution vector p, determines the probability that, in time t, process $\{X_t, t \in T\}$ will be in state i.

The probability, that upon entering state i, process $\{X_t, t \in T\}$ will, in one step, change its state from state i, $I \in S$ into j, $j \in S$, for accepted alternative $a \in A_i$, has been denoted as p_{ij}^a.

Condition:

$$\sum_{j\in S} p_{ij}^a = 1, \ p_{ij}^a \geq 0, \ i, j \in S, \ a \in A_i, \tag{9}$$

is satisfied.

Stochastic matrix $P^{(i,a)}$, defining conditional probability of transition p_{ij}^a, can be assigned to each state $i \in S$ and alternative $a \in A_i$. The set of matrixes assigned to state $i \in S$ has capacity equal to $\overline{\overline{i}}$ (capacity of set A_i).

Matrix $P^{(i,a)}$ is a matrix made up of stochastic verses, which determine probabilities of transition from a state marked with the verse number to all the remaining states. Element p_{ij}^{a}, situated on an intersection of a verse with number i and a column with number j in matrix $P^{(i,a)}$, is the probability of transition from state $i \in S$ to state $j \in S$ as long as alternative $a \in A_i$ has been used upon entering state $i \in S$.

Random variable, denoting duration time of state $i \in S$ of process $\{X_t, t \in T\}$, when the successive state is $j \in S$, and when upon entering state i decision $a \in A_i$ is made about distribution defined by distribution function $F_{ij}^{a}(t)$, is denoted as T_{ij}^{a}.

For the purpose of simplification, it was assumed that:

$$F_{ij}^{a}(t) = F_{i}^{a}(t) = F_{ia}(t),\ i, j \in S,\ a \in A_i\,. \tag{10}$$

This means that the duration time of state $i \in S$ does not depend on the process successive state. Function $F_{ia}(t)$ is a function of state $i \in S$ duration time distribution on condition that, decision a is made upon entering this state.

Random variable denoting duration time of state $i \in S$, with distribution defined by distribution function $F_{ia}(t)$, is denoted as T_{ia}.

In order to evaluate economic aspects involved in the modelled operation process it is necessary to additionally determine appropriate values of economic categories connected with the manner of the process states entering and staying in them.

4. Computing model

In result of identification of the research object and its processes, it is possible to determine finite phase spaces of the vehicles operation states, which affect such features as efficiency, safety, costs of technical objects operation and operating readiness of technical objects, etc. The kind of operation states and size of the phase space depend on the research goals [5-7, 9, 10].

In order to illustrate the discussion, the following operation states of a bus have been analysed:

- S_1 – state connected with provision of transport services, that is a state in which a bus and its operator perform the transport task,
- S_2 – state of maintenance,
- S_3 – state of waiting for performance of transport tasks, that is standby of vehicles on the territory of a bus depot while not being in operation,
- S_4 – state of post–repair servicing, that is a state in which the object, after being repaired undergoes control of the object condition (the so-called post-repair diagnosing) and quality of performed repairs.

In order to illustrate the discussion, influence of the type of bus subassembly damage on the change of probability of transition between states and duration times of the states as well as costs connected with the vehicle being in a particular state, has been modelled as well. The following major assumptions have been accepted:

- after completion of the vehicle subassemblies repairs it is necessary to control the object state, that is, state S_4 is possible to reach only after a repair of the vehicle identified damage,
- if in result of control of the object state (state S_4), it is still found to be unable to perform its transport tasks it is repaired at the station where control of its state was performed (remains in state S_4),
- the repair cost per time unit (state S_2) is connected with the type of failure,
- the repair duration time (state S_2) depends on the type of the damaged vehicle subassembly,
- duration time of post-repair object (state S_4) depends of the bus state control result, that is whether the vehicle is allowed to perform its transport tasks or needs to undergo another repair.

Buses used in the research objects were decomposed into systems. For the needs of the simplified computing model, the vehicle systems (their elements damage) were divided into three subsets:

- Systems (their elements damage) characterized by low repair labour consumption, the repair time is relatively short (frequently the process of repair is performed outside a bus depot by units of the so called standby service) denoted by code U_1,
- Systems (their elements damage) which need post-repair service denoted by code U_2,
- Systems (their elements damage) which do not need post-repair service characterized by high labour consumption, whose average time of repair is relatively long (multiple of system denoted by U_1 code), denoted by code U_3,

In the computing model, tests results of sixty vehicles obtained in the period of two years, are used, including data on the number of failures of the bus distinguished systems. Tests results cover failures repaired both in the system of providing vehicles with serviceability and in the environment of the operation system including repairs provided by technical service.

Stochastic decision process $\{X_t, D_t\}$, $t \geq 0$ is described by the model of operation state changes of the vehicles used in the research object. In the analysed example states $i \in S$ of process $\{X_t, t \in T\}$ correspond to the identified operation states S_i, $i = 1, 2, 3, 4$. States $i = 1, 3$ of process $\{X_t, t \in T\}$ are non-decision states, that is, subsets of alternatives A_1 i A_3 are one-element subsets. In the same states, alternatives are of only formal character (maintaining the notation consistence) and they have no influence on the analysed process $\{X_t, D_t\}$, $t \geq 0$ course. In state $i = 2$ of alternatives $a \in A_2$ of $\{D_t, t \in T\}$ process correspond to the codes of the bus distinguished damaged systems and represent their failures (elements). Set A_2 contains the following elements $A_2 = \{a_{2,1}, a_{2,2}, a_{2,3}\}$. Interpretation of entrance to state $i = 2$ of process $\{X_t, t \in T\}$, in time t and occurrence of alternative a_{23} of process $\{D_t, t \in T\}$ is as follows; a bus failure occurred and its repair, and the damaged system is the system denoted by code U_3. In state $i = 4$ alternatives $a \in A_4$ of process $\{D_t, t \in T\}$ correspond to the codes of the vehicle state control results. Set A_4 contains the following elements $A_4 = \{a_{4,1}, a_{4,2}\}$. $a_{4,1}$ and $a_{4,2}$ were used to denote respectively the control which finished with admitting the vehicle to perform transport tasks and recommending a post repair.

The rule for choice of alternatives a in state i is determined by distribution of probability of the analysed alternatives occurrence. It was assumed that q_{ik}, $i \in S, k \in N$ means probability of alternative a_{ik} occurrence upon entering state i.

Expression:

$$q_i = [q_{i1}, q_{i2}, ..., q_{ik}],\ q_{ik} \geq 0,\ i \in S,\ k \in N \tag{11}$$

is used for denotation of vector of alternatives occurrence distribution in state i. Elements of vector q_i meet the condition:

$$\sum_{i \in S, k \in N} q_{ik} = 1. \tag{12}$$

In the considered example, elements of this vector for $i = 2$ denote probabilities of a given system failure occurrence.

For the discussed assumptions, simulation of the process of a single technical object operation involves simulating the described stochastic decision process, being a model of the process of operation state changes of vehicles used in the analysed research object.

A program enabling simulation of the stochastic decision process has been developed.

Data indispensable for determination of the described process need to be specified basing on the results of tests performed in the analysed research object.

In order to perform the simulation it is necessary to use data indispensable for determination of the described process $\{X_t, D_t\}$ $t \geq 0$.

For the assumption about independence of the analysed technical object systems (elements) failures, vector q_i, i = 2 of probability distribution can be determined on the basis of frequency of the analysed systems damage occurrence.

Simulation experiments have been performed to illustrate of the discussion.

The assumption that random variables T_{ia} for $i \in S$, $a \in A$ have gamma distributions with different parameters, and their being in the process states is connected with gaining profits (state 1) and bearing costs by the operation system, has been accepted for the needs of the simulations. The values defining conditional probability of stochastic matrix $P^{(i,a)}$ $i, j \in S$, $a \in A$ transition p_{ij}^{a}, have been estimated on the basis of initial experimental tests.

Initial distribution $p = [p_1, p_2, ..., p_n]$, n = 4 of process $\{X_t, t \in T\}$ is not of big importance for the analysed example, in case of simulation of a big number of state changes. Values p_i of the accepted for calculation initial distribution vector elements are only of hypothetical character. In order to eliminate the influence of erroneously estimated values of the initial distribution vector, a fragment of the process has been analysed with omission of its initial phase.

Values of probabilities q_{ik}, i = 2, k = 1, 2, 3 of alternative a_{ik} occurrence have been determined basing on data concerning the bus failures. For the description simplification, alternatives are further denoted by code k.

The remaining values of parameters used for simulation experiments have been estimated on the basis of results of initial tests performed in the research object.

It is necessary to accept that the parameters values of random variables T_{ia} $i \in S$, $a \in A$, used in the model, are of hypothetical character. The parameters values were estimated basing on a small size data set.

Calculations for one category of objects consisting of 46 vehicles were performed, over a period of 700 days. Selected calculation results are presented in Fig. 1 to 4.

5. Conclusions

One the research goals is to present the possibilities of using Markov decision process for modelling the process of technical objects operation and analysis of the operation system behaviour after changing the model initial parameters values. Change in the model parameters initial values can enhance the influence of different factors on the behaviour of the system and characteristics of a vehicle operation process.

The analysed model of the operation process, due to the assumed degree of its description general character, can be used for analysis of operation processes of technical objects operated in transport means operation systems, as well as other categories of systems, providing the basis for their evaluation, safety control, operational readiness and reliability. The values of the mathematical model parameters can represent different features of the system, its operation related processes and objects used in them.

The analysis of results, obtained on the basis of performed simulations, makes it possible to choose a variant optimal in terms of the discusses system efficiency and reliability.

Rational control (management) of a complex system of technical objects operation and maintenance by decision makers allows analysing values of decision indices and prognosis of the system behaviour. This type of indices include those, which refer to the serviceability assurance subsystem, specifically concerning the number of repairs, labour involved in repairs, demand for specials equipment, etc. Prognosis of the analysed indices change in time is also of significance.

It should be noted that for analyses of the diagnostic and subsystems of serviceability assurance, apart from the information concerning the vehicle failures, it is necessary (indispensable) to provide data on the type of damaged system (element) failure. The kind of damaged system (element) affects such characteristics as:

- time of repair (time of the vehicle being in the subsystem providing serviceability),
- labour consumption involved in a repair (measured by the number of work hours),
- demand for specialist diagnostic and repair stations,

– time of the vehicle being out of service,
– necessity of using another vehicle,
– costs connected with restoring the vehicle's running order,
– others.

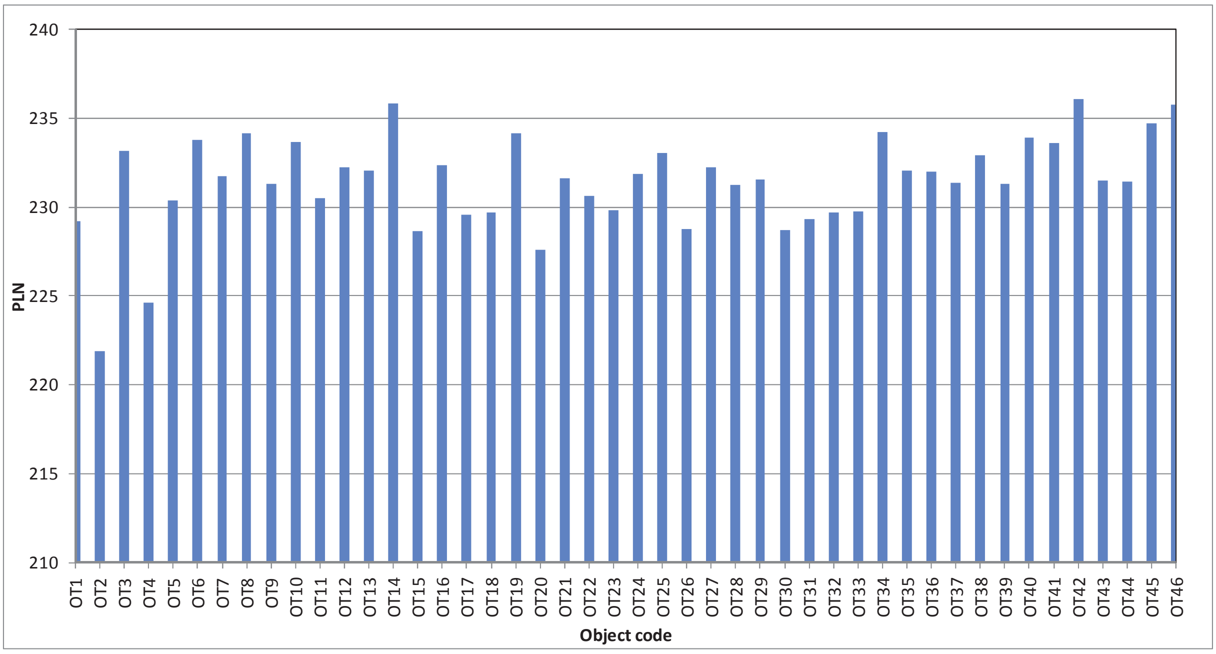

Fig. 1. Average value of the daily income generated by the system due to performance of the transport tasks by the particular objects

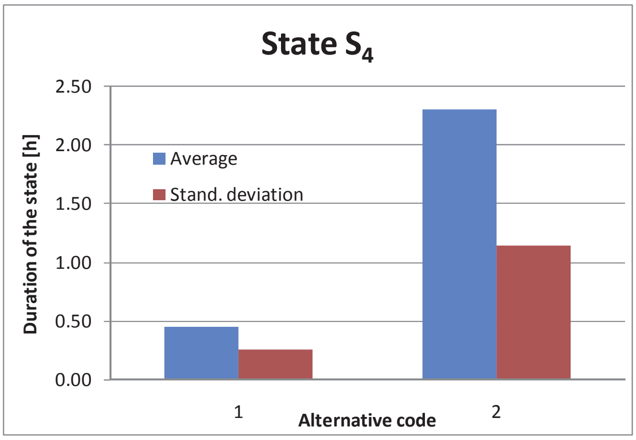

Fig. 2. Durations of the analysed states (state S_4)

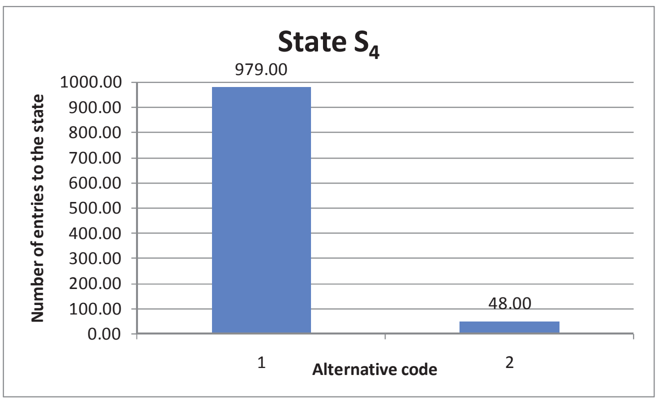

Fig. 3. Number of entries to the state S_4

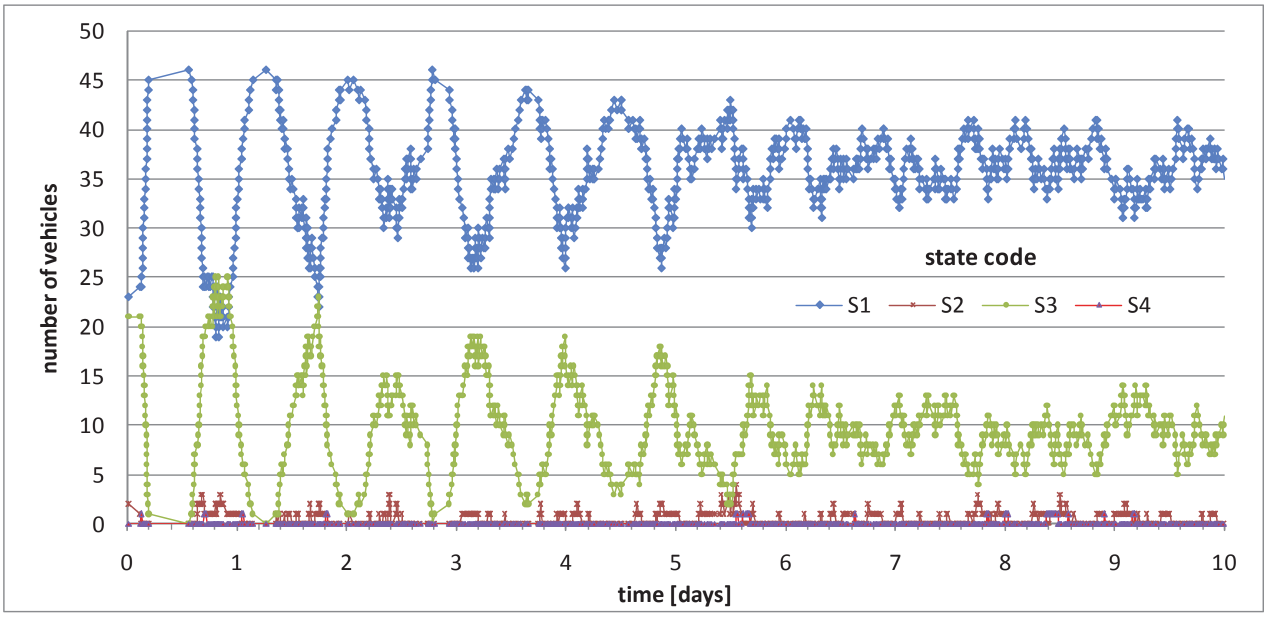

Fig. 4. Relationship between the number of the technical objects (vehicles) in the analysed operational states and the time

It appears that prognosis of the operation system behaviour based on a simulation of changes in technical objects operation sates as well as determination and analysis of values of chosen decision indices can facilitate the control of the discussed system.

In his work, selected assumptions of technical objects operation process model, simplified for the needs of this paper objectives, which can be used for simulations of the operation states changes. Values characterizing the discussed process are determined on the basis of computer simulation of Markov decision process, being a mathematical model of technical objects operation process.

Markov decision process, applied for modelling and analysis of a public urban transportation system operation process, enables development of tools to support decision makers involved in the discussed system operation and maintenance.

Simulation experiments of operation processes of the analysed research object have been conducted by means of the developed computer program. In this way, initial verification of the model was performed. Sample implementations of the model have been developed. There is a possibility to perform analyses concerning both the technical and economic aspects. The set of indices possible to determine includes operational readiness, times of repairs, efficiency of provided transport services, costs and others. Experimental tests for different categories of objects make it possible to evaluate selected technical and economic aspects of replacement of old objects with new ones and assessment of the objects usability for a given system.

Results of the conducted simulation experiments have proved that the model is sensitive to a change in its initial parameters values. Analysis of the simulation experiments results confirms high stability of the results.

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