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ANALYSIS OF DISCRETE-CONTINUOUS MODEL OF VIBROISOLATION SYSTEM OF RAILWAY SUBSTRUCTURE

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Abstract

Based on analysis of the railroads, construction can be concluded that the wide variety of design of the track rail vehicles is applicable in the country. The constructions of railways tracks can be divided into two main groups: separate trackways and built-in trackways. In the paper, a construction of vibroisolated trackway built-in or located on engineering structure such as bridge or viaduct is presented. This proposed construction of trackway consists of an embankment of land on which the bottom slab is located. On the concrete bottom slab, the perforated rubber plates are stacked constituting the elastic vibration isolation system. On the rubber platen, the concrete slab constituting the so-called inertial mass is mounted. Then, on the platen, the sand bed and the two layers of gravel with a variable grain size are placed. The final layer of crushed stone reaches the level of railway ties to which the elastic rail pads are attached. The several mathematical models of such vibration isolation systems of railway tracks were proposed. One of these models allowing for an analysis that allows to formulate both optimum conditions due to the selection and distribution of elastic-damping elements to ensure the translational movement of the platen and the vehicle so the vibration isolation system was "highly tuned", i.e. the natural frequency of the system was higher than the frequency of excitation caused by rail vehicles. It is a new trend in the design and selection of vibration isolation systems of railway tracks. In the paper, the conception of discrete-continuous vibration isolation system in which a continuous part is the beam supported on discretely distributed elastic elements is presented. The load to the system is modelling as solid body simulating the moving rail vehicle. The results of this analysis are the main part of the proper selection of vibration isolation system of railway tracks.

Keywords: *vibroisolation, modelling of railway tracks systems, elastic-damping elements*

1. Introduction

Based on analysis of the railroads construction can be concluded that the wide variety of design of the track rail vehicles is applicable in the country. The constructions of railways tracks can be divided into two main groups: separate trackways and built-in trackways. In the paper a construction of vibroisolated trackway built-in or located on engineering structure such as bridge or viaduct is presented. This proposed construction of trackway consists of an embankment of land on which the bottom slab is located (Fig. 1) [1].

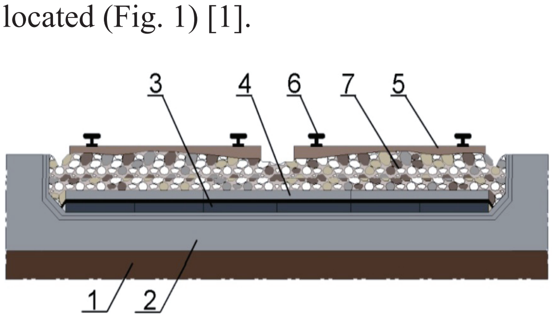

Fig. 1. Concept of vibration isolation system of railway track: 1 – subsoil, 2 – bituminous concrete, 3 – perforated rubber plates, 4 – platen, 5 – railway sleepers, 6 – rails, 7 – Breakstone ballast

On the concrete bottom slab, the rubber-perforated plates are stacked forming elastic vibration isolation layer [2, 3, 5, and 6]. The concrete platen (so-called inertial mass) is mounted on elastic layer. Then, a sand bed and the two layers of gravel with a variable grain size is disposed on the platen. The final layer of crushed stone reaches the level of prestressed concrete railway sleepers to which rails are attached by elastic pads. Attaching the rails to the substrate is achieved by means of springs SB30c. In the case of urban structures on which track runs is necessary to use a double waterproofing, which in this case is a layer of rubber tapes located on a concrete slab and on bottom layer of vibration isolation system (on the perforated rubber plates) mounted by the use of the thermoplastic mass. This type of track ensures an even distribution of the static and dynamic loads. The construction of vibration isolated railways lines, in relation to the railways lines without the vibration energy-absorbing elements, has the following advantages:

– greater durability,
– a significant reduction in noise and vibration,
– no special requirements for specific equipment
– execution time does not exceed the execution time of the traditional track.

The two basic disadvantages of such vibration isolation system of railway track is the necessity of frequent inspection of rails fasteners and systematically, especially during autumn and winter, inspection of the proper functioning of dehydration system.

Both of these defects are also disadvantages of other types of railway tracks. In summary, based on the above presented review, it is clear that the best solution for the construction of railway tracks is vibration isolation system of railways tracks presented in Fig. 1, due to the decreasing of dynamic and sound impacts from transport on environment. The use of any other solutions particularly in large urban areas or railway stations can cause the emission of excessive noise and dynamic loads on the environment during railway vehicle rides [4].

2. Discrete-continuous model of vibration isolation system of railway lines

Based on the analysis presented in chapter one, several basic models were established and dynamic analysis of them was carried out. For example in Fig. 2 the model of the discrete-continuous concept of vibration isolation system of subgrades rail vehicles built in roadways is presented. A pressure plate fixed on massless springs is the continuous part of the model and the vehicle was modelled as a rigid body.

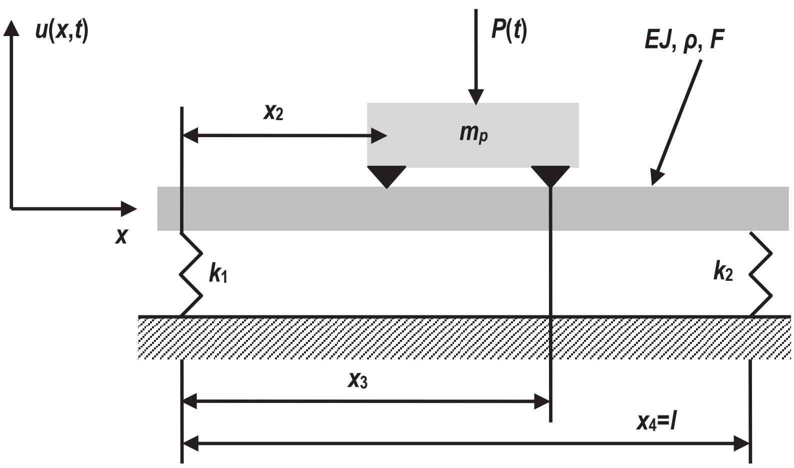

Fig. 2. The considered discrete-continuous model of vibration isolation system of track

Based on this models can be formulated both optimum conditions due to the selection and distribution of elastic-damping elements to ensure the translational movement of the platen and the vehicle so the vibration isolation system was “highly tuned”, i.e. the natural frequency of the system was higher than the frequency of excitation caused by rail vehicles. It is a new trend in the design and selection of vibration isolation systems of railway tracks. The equation of motion describing the so-accepted model has the following form:

$$EJ\frac{\partial^4 u}{\partial x^4}+\left[\rho F+\alpha m_w\delta_3+(1-\alpha)m_w\delta_4\right]\frac{\partial^2 u}{\partial t^2}=-k_1 u\delta_1-k_2 u\delta_2+\beta P(t)\delta_3+(1-\beta)P(t)\delta_4\,, \quad (1)$$

where:
J – moment of inertia of the platen,
ρ – density of the platen,
F – cross-section of the platen,
m_w – vehicle weight,
α – ratio of mass of a rigid body distribution (vehicle), $\alpha \in (0,1)$,
β – ratio of exciting force distribution $P(t)$, $\beta \in (0,1)$,
δ – Dirac delta applied at the points x_i (i = 1, 2, 3, 4),
k_1, k_2 – stiffness coefficients of the springs supporting platen.

For equation (1) we introduce the following boundary conditions:

$$k_1 u(0,t)=-EJ\frac{\partial^3 u(0,t)}{\partial x^3}\,, \quad (2)$$

$$k_{21} u(l,t)=-EJ\frac{\partial^3 u(l,t)}{\partial x^3}\,, \quad (3)$$

$$\frac{\partial^2 u(0,t)}{\partial x^2}=\frac{\partial^2 u(l,t)}{\partial x^2}=0\,. \quad (4)$$

For equation (1) we introduce the following initial conditions:

$$u(x,0)=\varphi(x)\,, \quad (5)$$

$$\frac{\partial u(x,0)}{\partial t}=\psi(x)\,. \quad (6)$$

For the considered system, we assume that the solution will have the form:

$$u(x_i,t)=X(x)T(t)\,. \quad (7)$$

Substituting the expected solution (7), in case that we consider the free vibrations, i.e. $P(t) = 0$, to the equations (1), (2), (3) and (4) we get:

$$\ddot{T}_k+\omega^2 T_k=0\,, \quad (8)$$

$$X^{\mathrm{IV}}-\alpha^4=a_1X(0)\delta_1-a_2X(1)\delta_2+a_3\lambda^2X(x_3)\delta_3+a_4\lambda^2X(x_4)\delta_4\,, \quad (9)$$

$$a_1X(0)=-X'''(0)\,, \quad (10)$$

$$a_2X(1)=-X'''(1)\,, \quad (11)$$

$$X''(1)=X''(0)=0\,, \quad (12)$$

where:

$$\lambda^4=\frac{\rho F}{EJ}\,,\quad a_1=\frac{k_1}{EJ}\,,\quad a_2=\frac{k_2}{EJ}\,,\quad a_3=\frac{\alpha\, m_k}{\rho F}\,,\quad a_4=(1-\alpha)\frac{m_k}{\rho F}\,.$$

The solution of equation (9) in the class of generalized functions has the following form:

$$X(x) = R\sin\lambda s + P\cos\lambda c + Q\sinh\lambda s + S\cosh\lambda c - \frac{1}{\lambda^3}\{a_1 X(0)(\sinh\lambda x - \sin\lambda x)H_1 + ...$$
$$... + a_2 X(1)[(\sinh\lambda(x-1) - \sin\lambda(x-1)]H_2\} + \lambda\{a_3 X(x_3)[\sinh\lambda(x-x_3) - ...$$
$$... - \sin\lambda(x-x_3)]H_3 + a_4 X(x_4)[\sinh\lambda(x-x_4) - \sin\lambda(x-x_4)]H_4\}, \tag{12}$$

where H_i are the Heaviside functions.

On the basis of the solution (12), after determining the distribution of derivative of second and third order, at the boundary conditions (10), (11), (12) and the geometric conditions $x = x_3$, $x = x_4$, $x = 1$, we obtain a homogeneous system of linear equations which allows for the calculation of constants R, P, Q, $X(0)$, $X(1)$, $X(x_3)$, $X(x_4)$:

$$\mathbf{AW} = \mathbf{0}, \tag{13}$$

where:

$\mathbf{W}^{\mathrm{T}} = \{R, Q, X(0), X(1), X(x_3), X(x_4)\}$,

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix},$$

$a_{11} = a_2\lambda_3(\sinh\lambda l + \sin\lambda l) + \lambda^6(\cosh\lambda l - \cos\lambda l)$,

$a_{12} = \frac{1}{2}(\cosh\lambda l + \cos\lambda l) + \frac{1}{2}\lambda^6(\sinh\lambda l + \sin\lambda l) - a_1 a_2(\sinh\lambda l - 4\sin\lambda l) - ... - a_1 a_2(\cosh\lambda l + 4\cos\lambda l)$,

$a_{13} = 0$,

$a_{14} = \lambda^4 a_1 a_3[\sinh\lambda(l-x_3) - \sin\lambda x_1] + \lambda^7 a_3[\cosh\lambda(l-x_3) + \cos(l-x_3)]$,

$a_{15} = \lambda^4 a_2[\sinh\lambda(l-x_3) - \sin(l-x_3)] + \lambda^7 a_3[\cosh(l-x_3) + \cos(l-x_3)]$,

$a_{21} = 2\lambda^5\sinh\lambda l$,

$a_{22} = \lambda^5\cosh\lambda l - 2a_1\lambda^2\sinh\lambda l$,

$a_{23} = -\lambda^5$,

$a_{24} = 2\lambda^6 a_3\sin\lambda(l-x_3)$,

$a_{25} = 2\lambda^6 a_4\sin\lambda(l-x_4)$,

$a_{31} = \lambda^3(\sinh\lambda l + \sin\lambda l)$,

$a_{32} = \frac{1}{2}\lambda^3(\cosh\lambda l + \cos\lambda l) - a_1(\sinh\lambda l - 4\sin\lambda l)$,

$a_{33} = -\lambda^5$,

$a_{34} = \lambda^6 a_3[\sinh(l-x_3) - \sin\lambda(l-x_1)]$,

$a_{35} = \lambda^4 a_4[\sinh\lambda(l-x_3) - \sin(l-\lambda x_1)]$,

$a_{41} = \sinh\lambda x_3 + \sin\lambda x_3$,

$a_{42} = \frac{1}{2}a_2\lambda^3(\cosh\lambda x_3 + \cos\lambda x_3) + a_1(\sinh\lambda x_3 - 4\sin\lambda x_3)$,

$a_{43} = a_2[\sinh\lambda(l-x_3) - \sin\lambda(l-x_3)]$,

$a_{44} = -\lambda^3$,

$a_{45} = \lambda^4 a_4[\sinh\lambda(x_3 - x_4) + \sin\lambda(x_4 - x_3)]$,

$a_{51} = \lambda^3(\sinh\lambda x_4 + \sin\lambda x_4)$,

$a_{52} = \frac{1}{2}a_2\lambda^3(\cosh\lambda x_4 + \cos\lambda x_4) - a_1(\sinh\lambda x_3 - 4\sin\lambda x_3)$,

$a_{53} = a_2[\sinh\lambda(l - x_4) - \sin\lambda(l - x_4)]$,

$a_{54} = a_2[\sinh\lambda(l - x_4) - \sin\lambda(l - x_4)]$,

$a_{55} = -\lambda^3$.

The system of algebraic equations (13) has a nontrivial solution if the following condition is satisfied:

$$\det(A) = 0. \tag{14}$$

Condition (14) allows the determination of their eigen value, which can be set in an increasing sequence $\{\lambda_n\}$ (n = 1, 2, 3, ..., i). If the vibration isolation system is to be "high tuned" the stiffness of the platen must be chosen so as to fulfil the condition on the frequency of vibration platen in the form of:

$$\omega_{0n} = 2\omega_w, \quad n = 1,2,3,...,i, \tag{15}$$

where ω_w is the excitation frequency.

3. Selection of platen parameters

Using the relationship between the eigenvalues and vibration frequencies of the platen, which is expressed by the following equation:

$$\lambda_1^4 = \frac{\rho F}{EJ}\omega_1^2, \tag{16}$$

one can set up a condition, which should meet the weight and stiffness of the platen in the form of:

$$\frac{m}{\rho} = \int \frac{\lambda_1^4}{4\rho\,\omega_w^2} EJ\,\mathrm{d}x, \tag{17}$$

from which, using additional functional $Q(h)$, is possible to determine the height of the platen at the assumed length and width resulting from the construction conditions. Minimization of the functional $Q(h)$, which is a measure of the stiffness of the vibration isolation system platen, allows to the minimization of its weight at assumed dimension. The solution of the functional $Q(h)$ is determined for the worst case occurs for vibration isolation system, i.e. when the vehicle mass is applied in the middle of the platen (Fig. 3).

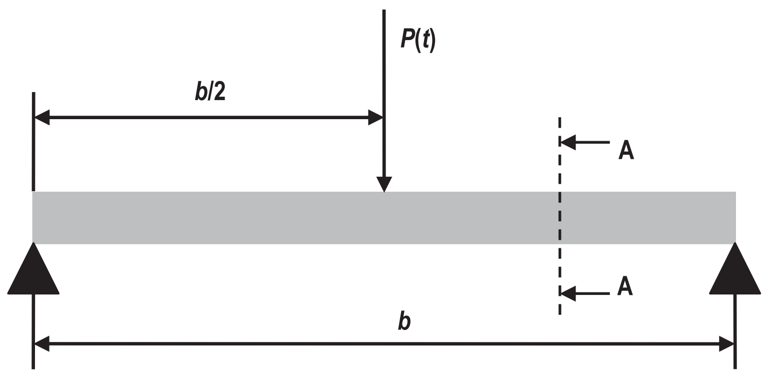

Fig. 3. Schematic model of the vibration isolation system platen

Assuming that the cross section AA is a rectangular platen its moment of inertia is given by the following equation:

$$J = \frac{1}{4} d\, h b^2 , \tag{18}$$

where:
b – length of the platen,
d – width of the platen,
h – height of the platen.

The transverse section of the platen is expressed by the relationship:

$$F = d\, h . \tag{19}$$

For this case, the functional $Q(h)$ according to Lagrangian takes the following form:

$$Q(h) = \int \left(\frac{P x^2}{E\, d\, h b^2} + \mu\, b\, d \right) \mathrm{d}x , \tag{20}$$

where μ is the Lagrange multiplier.

Taking into account the relations (17) and (20) and the condition for the extremum of functional equation ones can get the platen height as a function of its stiffness. This condition takes the following form:

$$h = \frac{\omega_w}{\lambda_i^2} \sqrt{\frac{14 m_p}{3EJ\, b\, d}} . \tag{21}$$

4. Summary

Presented above analysis of the discrete-continuous model of vibration analysis system allows for the analytical determination natural frequencies and the selection of the geometric parameters such as the mass and stiffness of the platen of vibration isolation system. Eigenvalues problem resolved under the assumption that the platten is supported by two springs of different stiffness. Assuming that the vibration isolation system should have only transactional movement that is that the spring stiffness at the both sides are the same greatly simplifies the solution of eigenvalues problem form computational point of view. Then, using for example the first four eigenvalues of the system on can perform optimal selection of the mass and stiffness of the platen. Similarly, we can formulate the conditions on the stiffness of the springs and their distribution, but this requires further research in this area and many of the simulation. This will be the subject of further work.

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