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SELECTED ASPECTS OF MODELING MUFFLERS FOR EXHAUST SYSTEMS OF VEHICLES

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Abstract

The world used several solutions exhaust systems of motor vehicles. Today, in addition to the core function of which is to discharge exhaust gases *from a combustion engine and noise reduction, these systems must comply with the functions associated with aftertreatment and impact on the characteristics of the engine. Requirements increase for noise suppression, at the tendency to increase engine efficiency and reduce fuel consumption. On the other hand, it seeks to reduce production costs. All this forces the search for new solutions and improve existing ones. In the process of the combustion gases discharge it is also important to eliminate frequency bands especially harmful to people and the environment (including other living organisms). Our goal is to develop a new design muffler of exhaust system that will meet these expectations of producers and users of vehicles. For this purpose, mathematical analysis of a classical systems with circular holes in the perforated pipes and proposed by the authors of the longitudinal gaps systems was performed. The paper presents some aspects of mufflers modeling for reducing acoustic noise emitted from the vehicle. Basic criterion for assessing was therefore the damping effect. Built theoretical models have allowed a rough estimate of acoustic performance at specific frequency bands by determining the transfer impedances. Because the noise reduction in automobiles is a complex, for the construction of such systems one of the important elements is to eliminate these frequency bands of acoustic waves in the process of the exhaust gases discharge which are the most dangerous to humans. The comparative analysis of these systems has allowed to determine the direction of further structural changes.*

Keywords*: vehicles, noise reduction , exhaust system, mufflers, efficiency of attenuation*

1. Introduction

The world clearly outline the two tendencies in the development of exhaust systems. The first priority sets itself maximization of vehicle engine performance, the second – as the greatest silence and noise reduction in the process of the combustion gases discharge. Of course, regardless of which path is taken in the construction process, these systems must comply with a legally sanctioned exhaust aftertreatment functions of harmful gases and particulate emissions and noise level standards. An important aspect of today is the economic criterion, often in terms of minimizing costs than maximizing the effects of possible within your budget. As can be seen from the observation of these trends, these systems have many important and sometimes conflicting functions in the vehicle.

By appropriate choice of exhaust pipes diameters, the capacity and shape of the mufflers expansion chambers and in additional the flow-dynamising elements or resonators we can influence both the noise of the vehicle and the engine characteristic curve. The literature contains the study results of the various solutions on the exhaust systems structure as well as muffling materials, designed to fulfill the absorption silencers [5, 6, 10, 11]. Many manufacturers shape also sound of vehicles by the exhaust system, which is the hallmark of the specified model or brand. With a different approach in this area it can be found also in sports cars.

Taking into account variety and complexity of these issues and economic aspects, the process of modeling becomes an indispensable part of the design of exhaust silencers. It also allows you to reduce the costs of developing and implementing new solutions. In reviewing the literature, we can meet with a variety of modeling techniques of exhaust systems and their components[1, 3, 7]. Therefore, the article focuses on selected aspects of modeling these systems, important for the future work of the authors. The purpose of this article is the analysis of the applied systems and proposal of some solutions in this field. This paper mainly focuses on the design of perforated pipe of reactive silencers and their evaluation in terms of predicted effectiveness of sound waves attenuation. Analysis of gas flow and acoustic performance enables as a consequence evaluation of the ability of these systems to reduce noise.

2. Selected aspects of mufflers modeling including new solution

Design of exhaust systems is difficult due to the complicated description of acoustic and thermal phenomena (flow, diffraction, interference). These phenomena are characterized by many variables. Used sound absorbing materials, sizes and shapes of the silencers chambers and pipes, used perforation and other parts such as the flow-dynamising elements have the impact on their course. It can be said that each of the above factors and parameters can greatly affect the flow and distribution of exhaust gas energy both in the exhaust system as well as the engine system [2, 8]. Therefore, the possibility of research in this area are very large. Some studies focused on finding an optimal porosity, which is defined as the ratio of the total area of the holes in the duct to the total area of the analyzed section of pipe [11]. Porosity is relevant for the calculations in terms of the choice of the equivalent transfer impedance of muffler for the specific dominant frequency. As mentioned earlier, the objective of this paper is to analyze the system of perforated elements. Current solutions used in the world are based primarily on systems with circular holes [3, 6, 11]. Most of the research relates to systems of perforated tubes with circular holes, treated as Helmholtz resonators [11].

Perforated tubes in mufflers have been modeled by the transfer impedance approach proposed by Sullivan and Crocker [11]:

$$\xi = \frac{1}{\rho a \sigma}(2.4 + 0.02 f), \tag{1}$$

where:
ξ – the dimensionless transfer impedance,
ρ – the mean density of the fluid,
σ – porosity of perforated tube (ratio between the open area and total area),
f – the frequency of sound,
a – the speed of sound.

More accurate relationship includes a hole diameter and wall thickness of the perforated tube:

$$\xi = \frac{1}{\sigma}[6 \times 10^{-3} + ik(t + 0.75 d_h)], \tag{2}$$

where:
k – wavenumber,
t – the wall thickness of the tube,
d_h – the hole diameter,
i – imaginary part $(\sqrt{-1})$.

Pressure jump across the perforated tube wall is related to the equivalent normal velocity by:

$$p^{-} - p^{+} = \rho c \xi v_n, \tag{3}$$

where:
p^- – the sound pressure on the other side of the perforated tube,
p^+ – the sound pressure in the normal direction,
v_n – the equivalent normal velocity,
c – the speed of sound,
σ – porosity of perforated tube.

The above empirical formulas give high conformity with experimental results and have been used in many calculations by the BEM method [1, 12]. Boundary Element Method (BEM) is now fairly commonly applied in the modeling elements of silencers using computer techniques [4, 12].

Other formulas using the Fock function proposed Kordziński. In his work [8] he analyzes the effectiveness of the whole muffler. Notes, however, that the variation of diameter holes in the perforated duct has a beneficial effect on the frequency range of attenuation, while increasing the porosity of the tube improves the sound reduction. The effectiveness of attenuation was determined by the formula:

$$\Delta L = 10\log\left(1 + \frac{\frac{C_0 V}{4F_r^2}}{\left(z - \frac{1}{z}\right)^2}\right), \tag{4}$$

where:
C_o – the hole conductivity,
V – the chamber volume excluding the pipe capacity,
F_r – cross-section of the tube,
$z = \frac{f}{f_n}$ – trimming ratio considering the natural frequency f_n.

The natural frequency is defined as:

$$f_n = 0.69 \frac{a}{2\pi} \sqrt{\frac{zF_o}{F_W b l_o}}, \tag{5}$$

where:
F_W – cross section of the outlet pipe,
b – the distance between the centers of the holes,
l_o – the hole length,
a – speed of sound.

The hole conductivity is determined from the formula:

$$C_o = \frac{F_o}{l_e}, \tag{6}$$

where:
F_o – the total area of all tube holes,
l_e – the efficient, effective length of one hole.

The effective length of the hole takes into account a certain extent the inertia of the exhaust gas in the holes and can be defined by using the Fock function $\varphi(\zeta)$ [8] :

$$l_e = g + \frac{\pi d_o}{4\varphi(\zeta)}, \tag{7}$$

where:
g – the thickness of the tube wall,
d_o – diameter of the holes in tube,
ζ – Fock function parameter.

Parameter of the Fock function takes into account the distance of the holes from each other:

$$\zeta = \frac{d_o}{b}, \tag{8}$$

where:
b – the distance between the centers of the holes.

Computational method(the substructuring technique) based on the impedance matrix(Z) synthesis approach has been proposed by Lou, Wu and Cheng [9]. In this method, the muffler is divided into a number of substructures and the impedance matrices of all substructures are combined together. The impedance matrix relates the sound pressure(p) at the inlet and outlet of the any substructure to the corresponding particle velocities(v). For example, for the first and second substructure, this relationship is follow:

$$\begin{Bmatrix} p_i \\ p_1 \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{Bmatrix} v_i \\ v_1 \end{Bmatrix}, \tag{9}$$

$$\begin{Bmatrix} p_2 \\ p_o \end{Bmatrix} = \begin{bmatrix} Z_{31} & Z_{32} \\ Z_{41} & Z_{42} \end{bmatrix} \begin{Bmatrix} v_2 \\ v_o \end{Bmatrix}. \tag{10}$$

The sound pressure and particle velocity at the inlet of the first substructure are denoted by p_i and v_i, respectively and the corresponding variables at the outlet of the first substructure are denoted by p_1 and v_1. p_2 and v_2 represent the sound pressure and particle velocity, respectively, at the inlet of the second substructure and p_o and v_o represent the corresponding variables at the outlet of the second substructure. The effectiveness of acoustic attenuation of each of them it can be expressed using a 'four-pole-type'transfer matrix. The acoustic property of the filter element that connects the two substructures are described by the matrix[9]:

$$\begin{Bmatrix} p_1 \\ v_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} p_2 \\ v_2 \end{Bmatrix}. \tag{11}$$

The sub-matrices A, B, C and D of the transfer matrix describe the flow parametres of a straight duct as functions of speed of sound and mean density. After solving the equations describing them we substitute results into a matrix and obtain the final results.

Of course we can model any part of the muffler, for example, the use of several different muffling materials, different diameter and holes arrangement in the perforated tubes, the location of the chambers, pipes, resonators, shapes and size outside of the muffler, and even wall thickness of components. If you multiply these variables by the number of used computational methods and models (in this article are just a few examples) we get a picture of how wide the possibilities of research are in the field of exhaust systems. The authors chose the issue of influence the shape and porosity size of perforated pipes on the effectiveness of attenuation the wave frequency emitted by internal combustion piston engines. As an alternative to the used elements of circular holes, perforation with rectangular gaps of tube has been adopted. The choice of such a shape is not indifferent. In his work[8], Kordziński in the chapter devoted the issue of lossless noise attenuation namely without loss of engine power, therefore such systems, which have a high acoustic resistance and flow resistance as small as possible. He suggests attenuation only constituent of turbulent flow, transient. They are the main source of noise. However, elements of slit shape of holes emit 'slotted' sounds at high frequencies and low intensity, caused by laminar flow through them. Such steady streams should flow by the outlet system without resistance. Therefore, the use of hole-slot is intended to reduce the resistance of the outlet system and the noise by the arrangement of exhaust gas flow.

In the proposed method, the perforated pipe is modeled as a cylindrical tube with the porosity percentage and the thickness of the walls and gap size given as input depending on the formula used. The frequency of 'slotted' sounds is defined as:

$$f_s = \frac{w}{25F_w}, \tag{12}$$

where:
w – the flow rate through the gap,
F_w – the cross of the duct-gap.

To compare the attenuation effectiveness of muffler with circular holes perforation and slotted perforation we should adopt the following assumptions:

- size of the exhaust pipe, muffler chamber and grade porosity of both mufflers must be the same,
- measurement should be conducted under the same conditions.

After transformations we obtain the final dependence on efficient, effective length of one slot(gap):

$$l_{es} = g + \frac{l_s + h_s}{2}, \tag{13}$$

where:
l_s – the gap length,
h_s – the gap width,
F_o – the total area of all tube gaps ,
g – the wall thickness.

This corresponds to 0.75dh in formula Sullivan and Crocker [11], approximately half of the circumference of the circle. Using equation (2) we can determine the transfer impedance of the proposed system. In the end, we obtain the following equation for the gaps:

$$\xi = \frac{1}{\sigma}[6\times10^{-3} + ik(g + \frac{l_s + h_s}{2})], \tag{14}$$

where:
k –wavenumber,
i –imaginary part $(\sqrt{-1})$,
σ –porosity of perforated tube.

3. Preliminary assessment of the analyzed solutions of mufflers

There are several parameters which describe the acoustical performance of a muffler. These include Noise Reduction (*NR*), Insertion Loss (*IL*), Attenuation (*ATT*), and the Transmission Loss (*TL*). Some of them depend not only on the muffler, but also on the source impedance and the radiation impedance [2]. These acoustical quantities will be determined in further studies. Given the limited scope of this article and the complexity of the presented issues, the authors show the effect of changing the shape of the holes in the perforated pipe at the transfer impedance ratio as a primary indicator affecting the effectiveness of the silencer in accordance with the relationships and models described in the previous chapter. Built theoretical models have allowed a rough estimate of both the gas flow and predict acoustic performance of a muffler perforated tube, with various modifications including the shapes of the holes. This models have been used to determine the transfer impedances at the frequency bands, that are most dangerous to humans and simultaneously allow for a calculation using empirical formulas, which will ensure a high level of compliance with the experiments[11]. Therefore, the analysis covers the range from 0 to 3700Hz.

The calculations were conducted for three cases of muffler perforated tube. In the first case the perforated tube is modeled as a cylindrical pipe with the porosity percentage and the thickness of the tube and hole diameter given as input depending on the formula used. In the second case, as proposed by the authors, the perforated tube is modeled as a cylindrical pipe with the porosity percentage and the thickness of the tube and rectangular gaps. The third case, based on a simplified formula (1) represents the model that does not take into account the diameter and shape of the holes and the thickness of the walls. For comparative purposes, in all the cases analyzed in this article it was assumed the following common parameters:

- the diameter of the duct – 0.05 m,
- porosity – 0.264,
- the length of the cylindrical tube 0.120 m,
- the thickness of the walls – 0.001 m.

Figure 1 shows the results of the transfer impedances for three cases of muffler perforated tube described above. The classical solution is represented by a perforated muffler with a circular hole with a diameter of 3mm, and proposed solution by perforated system with rectangular gaps with ls=101 mm and hs=1.54 mm.

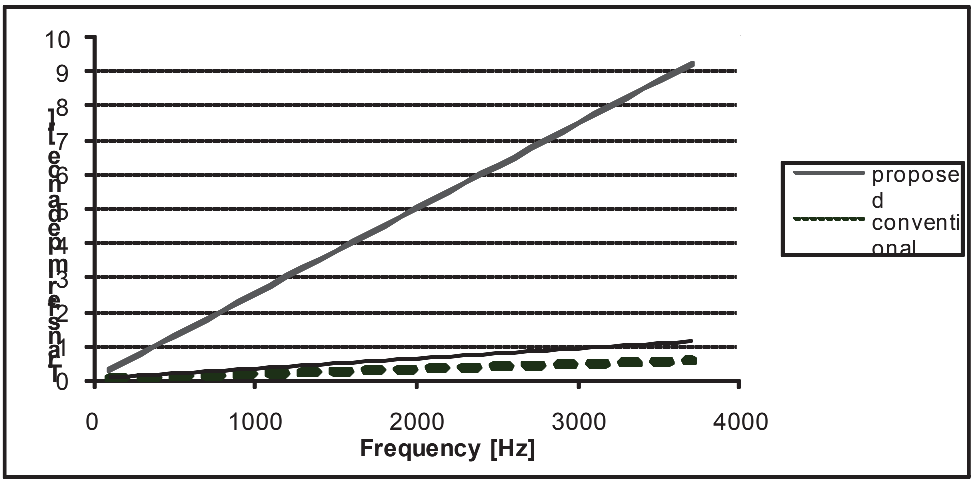

Fig. 1. Comparison of the transfer impedances for three cases of muffler perforated tube

Based on a comparison of the results obtained for three cases of muffler perforated tube it can be concluded that the transfer impedances highly increased in the entire frequency range for the benefit of the proposed system (for some frequencies near 9 fold), which predicts a significant reduction in the noise level using the system with rectangular gaps instead of circular holes.

4. Conclusion

A mufflers is an important devices to control the noise that produced by automobile exhaust. Mechanical performance of the perforated muffler can be controlled by the porosity, distribution and shape of holes. Computational methods is the current trend on automotive field in reducing the cost effect for the analysis of various models and solutions.

This article describes some of the modeling methods of perforated muffler elements, both simplified and based on empirical formulas and details involving computer programs. To describe a variety of phenomena we should use varied, fit models.

The authors proposed the new solution of the perforated muffler with rectangular gaps. A comparison of a new system with used solutions (based on circular holes) was made. The study was carried out in order to evaluate the effectiveness and potential benefits of the proposed solution. For this purpose, the theoretical models of analysed systems were built and the transfer impedances for the harmfull frequency range (from 100 to 3700 Hz) were determined. Preliminary work has shown the validity of the approach and the significant realizable value acoustic performance of the proposed system and, consequently, reduction of noise, which encourages further work and a more detailed analysis of the new solution.

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