# FLIGHT PATH OPTIMIZATION OF AN AIRCRAFT AIDED DURING TAKE-OFF BY MAGLEV SYSTEM

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### Abstract

*Very high air traffic density in the largest airports and in their vicinity involves that the air traffic in the largest airports and their areas of operations approaches the capacity limits. Such high density of the air traffic adversely influences the natural environment in the vicinity of the airports due to the increasing cumulative noise level and the concentration of environmentally hazardous substances. One of the possibilities to improve the situation is to work out innovative solutions aimed at decreasing the aircraft pollution and improving the transport effectiveness. One of the major concepts is using magnetic levitation (MAGLEV) technology to support aircraft take-off and landing. If the aircraft could take-off and start the initial climb phase with the ground power, the installed power may be reduced, resulting in less weight, less drag and less overall fuel consumption which leads to emission reduction. These advantages, the lower fuel consumption and emissions, increase sustainability of the transportation system. Different conditions of the take-off give possibilities to shape the trajectory after the take-off in order to decrease the negative influence on the environment. The aim of the present work was to determine the optimal conditions of the take-off and the optimal climb trajectory of the aircraft aided in the ground phase of the take-off by the system using the MAGLEV technology, minimizing the fuel consumption. The simplified algorithm of optimization of the flight trajectory was used in this work; it uses the method of approximation of the flight path by the third degree polynomial.*

*Keywords: air transport, magnetic levitation, trajectory optimization*

## 1. Introduction

Europe is one of the Earth's most densely populated areas. There are approximately 1270 airports and 1300 airfields in Europe. The total number includes 737 European airports that are equipped for IFR operations. In 2010, approximately 9.5 million IFR flights were performed in Europe and the forecast for 2017 assumes 21 per cent increase in the number of IFR flights, which is an equivalent to 11.5 million take-offs, and the same number of landings [5]. As much as 44 per cent of the total air traffic is concentrated on only 25 largest European airports. That results in a very high air traffic density in the largest European airports and in their vicinity. What it involves, air traffic in the largest airports and their areas of operations (AOA) approaches the capacity limits. Such high density of air traffic adversely influences the natural environment in the vicinity of airports by increasing cumulative noise level and the concentration of environmentally hazardous substances. These factors bring about a considerable decrease of the comfort of living for the inhabitants of areas in the vicinity of large commercial airports. The activities carried out at present with the aim of reducing the harmful influence of air transport on the environment include the implementation of special noise abatement procedures and designing aircraft engines that are quieter and more environmentally friendly (lowered emissions level). The problem of air traffic density increase in hub airports is being solved by reducing the separation between aircraft to the required minimum and optimizing the queuing procedures concerning the aircraft involved in the airport area operations. All these steps seem to be ad hoc rather than system solutions, addressing only immediate needs. One of the remedies to the situation is the implementation of innovative solutions, e.g. a take-off-assisting system utilizing magnetic levitation (MAGLEV) [8]. In case of

using the magnetic levitation technology, the airframe weight can be considerably reduced, since the undercarriage system could be lighter or even ignored. Such a system, owing to its reduced engine power requirement in the take-off phase, will influence the diminishing of the adverse effect of air traffic on the natural environment by reducing not only the emissivity during take-off and landing but also the noise level in the airport and in its vicinity. Application of the innovatory solution of aided take-off is connected with modification of the departure procedures after the take-off in order to minimize the negative effect of the aircraft on the surrounded environment. Optimization of the departure trajectory minimizing fuel consumption and noise emissions can become the basis for working out new procedures for a new kind of take-off modified in relation to the optimal solution which will increase safety of this segment of the flight.

## 2. Departure trajectory optimisation

The climb performance of an aircraft is an important design requirement for establishing trajectories to reach a specified altitude and airspeed after take-off in some optimal manner. Usually for transport aircraft, a climb segment may follow a trajectory designed to achieve an optimal fuel consumption or a minimum time. Trajectory optimization problems to minimize aircraft fuel consumption, noise or time of climb had been studied by various contributors [1, 6, 7, 12, 13, 16].The problem is formulated as the calculus of variations where an objective function such as a flight time or required fuel is minimized while satisfying initial/final conditions and path constrains for a flying vehicle. Although some techniques such as the dynamic programming of Bellman [3] and the maximum principle of Pontryagin [11] have been developed and used. A simplified realization of the Ritz-Galerkin method was used in this work which uses an approximate solution to boundary value problems for determining the optimal flight trajectory described by Taranienko [15]. The method allows determining the optimal trajectory of the flight satisfying the initial/final conditions and control functions and path constrains for an aircraft. The case of optimal ascending and acceleration in the vertical plane with the determined conditions and appropriate constrains was considered. General stating of the task supposes determining the optimal trajectory of movement of a flying vehicle (Fig. 1) described by the system of ordinary differential equations:

$$x_i = f_i(x_i, \dots, x_n, u_1, \dots, u_m), \quad i = 1, 2, \dots, n, \quad m \leq n, \tag{1}$$

fulfilling boundary conditions:

$$x(t_0) = \{x_1(t_0), x_2(t_0), \dots, x_n(t_0)\}, \quad x \in \mathbf{R}^n, \tag{2}$$
$$x(t_f) = \{x_1(t_f), x_2(t_f), \dots, x_n(t_f)\}$$

and constraint for state variables and control variables:

$$x_{i\,min}(t) \leq x_i(t) \leq x_{i\,max}(t), \tag{3}$$
$$u_{j\,min}(t) \leq u_j(t) \leq u_{j\,max}(t), \quad j = 1, 2, \dots, m,$$

where:
$x_i$    – state variables,
$u_j$    – control variables,
$t_0, t_f$ – initial and final times.

The technical characteristics of the flying vehicle are known and can be written as:

$$x_T = (x_{T1}, \dots, x_{Tr}), \quad x_r \in \mathbf{R}^r. \tag{4}$$

The optimal trajectory $x(t)$, $t \in (t_0, t_f)$ has to be found which minimizes the functional:

$$J(x(t)) = \int_{t_0}^{t_f} f_0(X, U) dt \tag{5}$$

and the control corresponding to it is:

$$U(t) = (u_1(t), \dots, u_m(t)), \quad U \in \mathbf{R}^m. \tag{6}$$
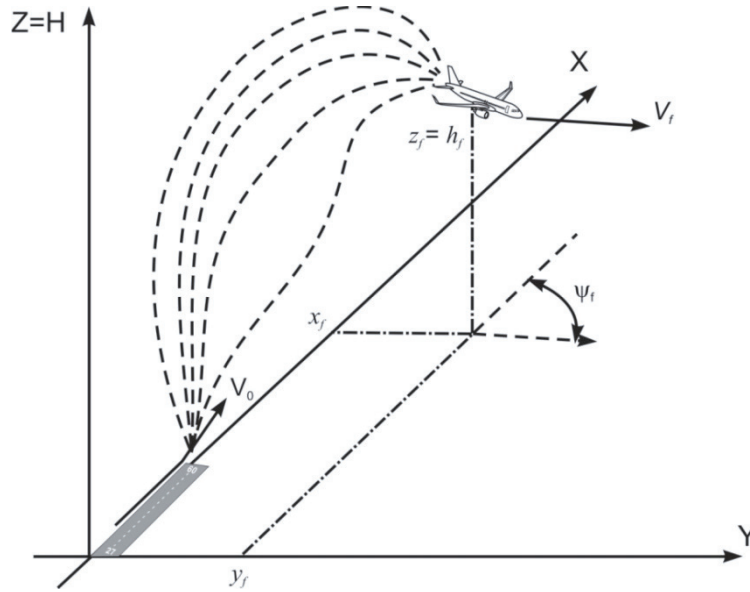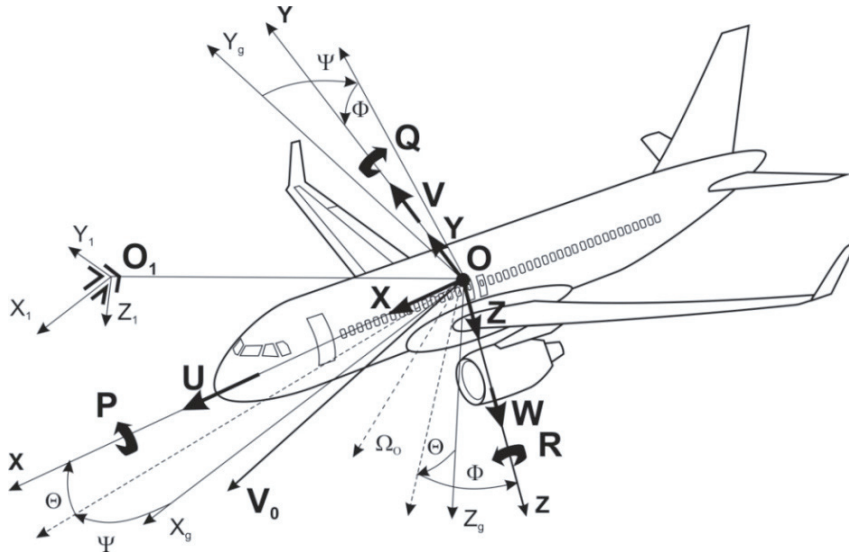
*Fig. 1. Trajectory optimization problem*



*Fig. 2. Forces and aerodynamic moments which influence the aircraft in flight*

In the case of the current task the system of differential equations (1) commonly employed in the aircraft trajectory analysis is the following six-dimension system derived at the centre of mass of the aircraft (Fig. 2) [4]:

$$\dot{V} = g\left(\frac{T\cos\alpha - D}{mg} - \sin\gamma\right),$$

$$\dot{\gamma} = \frac{1}{mV}\left((T\sin\alpha + L)\cos\varphi - mg\cos\gamma\right),$$

$$\dot{\psi} = \frac{(T\sin\alpha + L)\sin\varphi}{mV\cos\gamma}, \tag{7}$$

$$\dot{m} = \frac{dm}{dt} = C_s,$$

$$\dot{x} = V\cos\gamma\cos\psi, \quad \dot{y} = V\cos\gamma\sin\psi, \quad \dot{z} = \dot{h} = V\sin\gamma,$$

where: $V$, $\gamma$, $\psi$, $\alpha$ and $\varphi$ are respectively the speed, the angle of descent, the yaw angle, the angle of attack and the roll angle. $(x, y, z = h)$ is the position of the aircraft, $D, L, m$ (technical characteristics – eq. 4) and $g$ are respectively the engine thrust, the drag force, the lift force, the aircraft mass and the gravitational acceleration.

It was consider a 2 Degrees of Freedom (DOF) dynamic model that describes the point

variable-mass motion of the aircraft over a flat Earth model. A standard atmosphere is defined with ISA (International Standard Atmosphere). Lift coefficient $C_L$ is, in general, a function of the angle of attack $\alpha$ and the Mach number $M$, i.e., $C_L = C_L(\alpha, M)$. The lift coefficient is used as a variable rather than the angle of attack. The aircraft performance model is used from [9] and [10].

Given a commercial flight profile of climb, initial and final conditions, and a set of path constraints, the goal is to find the minimum-fuel consumption trajectory of the aircraft. Optimal total flight time are also to be determined.

The path constraints of the problem are those that conform the aircraft's flight envelope and have been taken from the calculations and literature [2]. $C_{Lmax}$ and $V_{stall}$ vary depending on the flight configuration. There is a relationship between the variables from equations (1) and (7):

$$x_1 = V, \quad x_2 = \gamma, \quad x_3 = \psi, \quad x_4 = m, \quad x_5 = x, \quad x_6 = y, \quad x_7 = h. \tag{8}$$

The boundary conditions are written in the following way:

$$t = 0, x(0) = x_0, y(0) = y_0, h(0) = h_0, V(0) = V_0, \gamma(0) = \gamma_0, \psi(0) = \psi_0, m(0) = m_0 \ ,$$
$$t = t_f, x(t_f) = x_f, y(t_f) = y_f, h(t_f) = h_f, V(t_f) = V_f, \gamma(t_f) = \gamma_f, \psi(t_f) = \psi_f, m(t_f) = m_f. \tag{9}$$

Apart from the conditions written above (9), it is necessary to determine the boundary conditions for the control variables:

$$t = 0, \quad \gamma = \gamma_0, \quad T = T_0, \quad \alpha = \alpha_0 ,$$
$$t = t_f, \quad \gamma = \gamma_f, \quad T = T_f, \quad \alpha = \alpha_f . \tag{10}$$

In the case of optimization of the flight trajectory the final value of the time $t_f$, the speed and final acceleration should be determined. That is why, the speed and acceleration were made independent from the shape of the trajectory, i.e. from the factors $x$, $y$ and $h$, instead of the time a new argument $\tau$ is introduced which is connected with the physical time $t$ with the help of the scale factor:

$$\frac{dt}{d\tau} = \frac{1}{\lambda}, \tag{11}$$

where:
$\lambda$ – unknown scale factor.

The system of equations describing the movement of the aircraft (7) will be described regarding the new variable $\tau$ and will be completed by the equation (11). The new system of equations which connects 12 unknown variables, 7 phase variables (8), time $t$, three control variables (10) and function $\lambda$ (11), is not a closed system. In order to make the system closed it is necessary to add four additional conditions, i.e. supporting functions. The relations from $\tau$ coordinates $x(\tau)$, $y(\tau)$, $h(\tau)$ will fulfil these functions. In the fourth relation it is possible to add the air-speed variation principle between the initial and final points described by the function $V(\tau)$. Then, the function $\lambda$ can be determined on the basis of the relation:

$$\lambda = \frac{V}{\sqrt{\acute{x}^2 + \acute{y}^2 + \acute{h}^2}}, \tag{12}$$

and angle of descent and yaw angle

$$\sin \gamma = \frac{\acute{h}}{\sqrt{\acute{x}^2 + \acute{y}^2 + \acute{h}^2}} = \frac{\acute{h}\lambda}{V}, \tag{13}$$
$$\tan \psi = -\frac{\acute{y}}{\acute{x}}.$$

As the supporting function any continuous function can be accepted which is smoothly varying in the range from $\tau_0$ to $\tau_f$. The experience shows that it is the best to accept the supporting function in the form of three cubic polynomials described in the following way:

$$x_i(\tau) = C_{i0} + C_{i1}\tau + \frac{1}{2}C_{i2}\tau^2 + \frac{1}{6}C_{i3}\tau^3. \tag{14}$$

The derivatives of the supporting function will be:

$$\dot{x}_i(\tau) = C_{i1} + C_{i2}\tau + \frac{1}{2}C_{i3}\tau^2,$$
$$\ddot{x}_i(\tau) = C_{i1} + C_{i2}\tau + \frac{1}{2}C_{i3}\tau^2, \tag{15}$$
$$\dddot{x}_i(\tau) = C_{i3},$$

where coefficients $C_{ij}$ are equal:

$$C_{i0} = x_{i0} - [k_1]\left(\frac{a_{i2}}{6}\right)\tau_{i1}^3 - [k_2]\left(\frac{a_{i3}}{6}\right)\tau_{i2}^3,$$
$$C_{i1} = \dot{x}_{i0} + [k_1]\left(\frac{a_{i2}}{2}\right)\tau_{i1}^2 - [k_2]\left(\frac{a_{i3}}{2}\right)\tau_{i2}^2,$$
$$C_{i2} = \ddot{x}_{i0} - [k_1]a_{i2}\tau_{i1} - [k_2]a_{i3}\tau_{i2}, \tag{16}$$
$$C_{i3} = a_{i1} + [k_1]a_{i2} + [k_2]a_{i3},$$
$$k_1 = 0 \ \ for \ \tau \leq \tau_{i1} \ \ and \ \ k_1 = 1 \ \ for \ \tau > \tau_{i1},$$
$$k_2 = 0 \ \ for \ \tau \leq \tau_{i2} \ \ and \ \ k_2 = 1 \ \ for \ \tau > \tau_{i2}.$$

## 3. Results

In the frames of this work the optimal trajectory of climbing of the aircraft with the characteristics of A320 was analysed, whose take-off is aided by the MAGLEV technology. Three different scenarios to operate the GABRIEL system during take-off were analysed (Tab. 1).

*Tab. 1. Definition of take-off scenarios*

| Aircraft weight (kg) | Scenario | Take-off speed, m/s | Use of aircraft engine power at lift off, % | Flap |
|---|---|---|---|---|
| 71 168,44 (aircraft without landing gear) | I – Accelerated | 75 | $\cong 0$ | CONF 2 |
| | II – Conventional | 75 | 100 | CONF 2 |
| | III – Unconventional | 110 | 100 | CONF 0 |

*where: CONF – see [2] and [10]*

Both optimized and flight profiles are stated as hybrid optimal control problems. To solve the reformulated optimal control problem (1)-(6) a Ritz-Galerkin method [15] has been used. The resulting sparse nonlinear programming problem (NLP), has been solved using own elaborated software. The typical profiles computation has been performed with a tool combining 3-DoF flight dynamics differential equations with procedure-oriented flight control. Tab. 2 show the typical, optimized and free flight climb procedures used herein for the numerical simulation and the boundary conditions of the flight.

*Tab. 2. Boundary conditions for climb path optimization*

| Parameter | Value | Unit |
|---|---|---|
| $x_0$ | 0 | [m] |
| $h_0$ | 0 | [m] |
| $V_0 = V_{LOF}$ (depending on scenario) | Tab. 1. | [m/s] |
| $\gamma_0$ | 0.02 | [rad] |
| $h_f$ | FL300 | [-] |
| $M_f$ | 0.7 | [-] |
| $\gamma_f$ | 0 | [rad] |

Search for optimal trajectories minimizing fuel consumption must be done in a realistic flight domain. Indeed, operational procedures are performed with respect to parameter limits related to the safety of flight and the operational modes of the aircraft. The path constraints are equal for all phases:

- position coordinates: $x \geq 0$, $h \geq 0$,
- lift coefficient: $C_L < C_{Lmax}$, ($\alpha < \alpha_{CR}$),
- load factor: $n < 1.2$,
- thrust: $T(h, V) < T_{max}(h, V)$.

Operations of an aircraft are divided into two parts: the LTO cycle and cruise. The LTO cycle includes all activities near the airport below the altitude of 1,000 m. These activities are taxi in and out, take-off, climb-out, and approach landing. Cruise is defined as all activities at altitudes above 1,000 m. Climb from the end of climb-out in the LTO cycle to cruise altitude, cruise, and descent from cruise altitudes to the start of LTO operations of landing are defined as the cruise part. In the present work the trajectory of climbing to the cruise height of flight (Tab. 2) for different take-off scenarios (Tab. 1) was determined. The taking-off speeds followed from the analysed scripts described in [14]. The calculations were carried out minimizing the fuel consumption during the climbing phase. The values of the optimized parameters were determined by traditional methods of static optimization. Optimum trajectory of the aircraft in the climb phase is shown in Fig. 3. The optimal change of trajectory parameters are presented in Fig. 4-6.
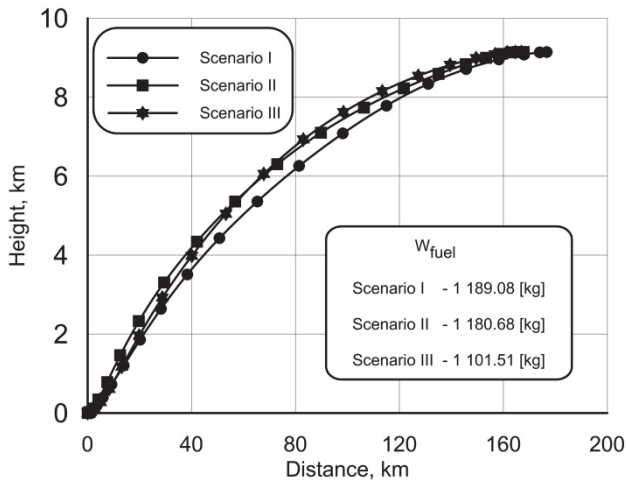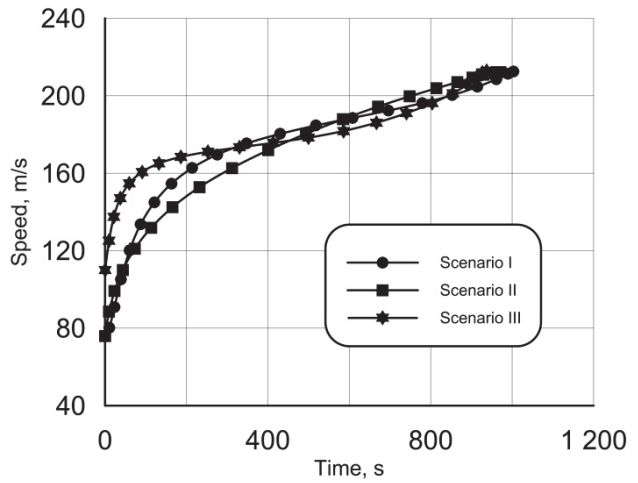


*Fig. 3. The optimal climb trajectory*


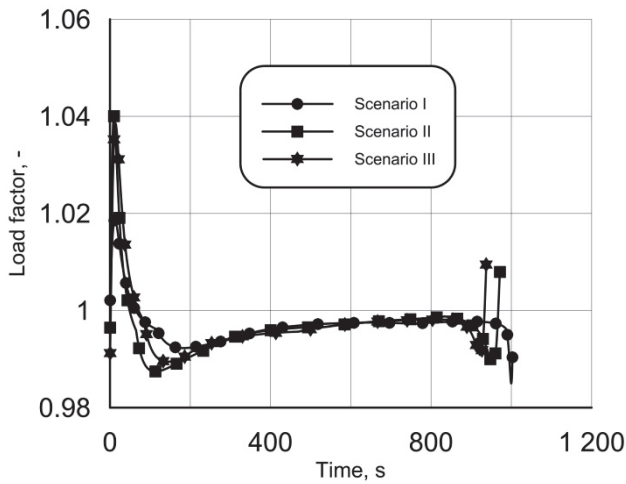
*Fig. 4. The optimal change of speed*



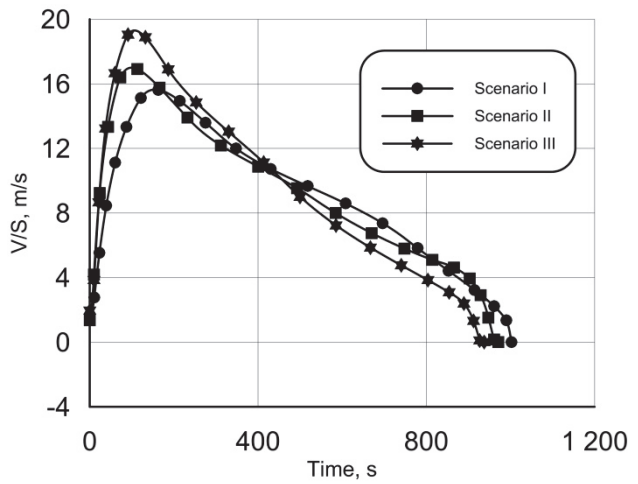*Fig. 5. The optimal change of load factor*



*Fig. 6. The optimal change of vertical speed*

**4. Summary**

The work concerned determining the optimal trajectory of the climb phase of the transport aircraft aided in the phase of acceleration by the system using the magnetic levitation technology. The conditions different from a traditional take-off (higher take-off speed, lack of undercarriage, en route configuration) allows assuming that optimal profile of the aircraft climb should be different from currently performed. Determining the climb trajectory guaranteeing minimization of the fuel consumption would also give a solution characterized by minimal emission of substances harmful for the environment. To determine the optimal trajectory of climb the Ritz-Galerkin method was used which is the method of approximate solution of boundary value problems, which guarantees obtaining the results thorough enough for practical purposes. Further works should concern determining the optimal trajectory regarding minimization of harmful substance and noise emission, taking into account the most densely populated areas avoidance.

**Acknowledgments**

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