

METHOD FOR ASSESSMENT OF RISKS CONNECTED WITH TECHNICAL OBJECT FUNCTIONING ON THE BASIS OF MARKOV MODEL OF OPERATION PROCESS

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Abstract

In the article, a method for assessment of risks connected with a technical object functioning has been studied, in a selected operating system. An urban system of bus transportation, in a selected urban complex, has been chosen as the research object. Functioning of each technical object operating system, including transportation systems, is dependent on its possibility to perform properly the assigned transport task. One of the methods for assessment of the transportation system ability to accomplish the assigned transport task in a proper way is determination of the transport means operation risk. The presented approach involves determination of indexes defining the risks connected with functioning of one technical object (transport means) in an executive subsystem. Indexes of the risk connected with functioning of a single technical object have been determined on the basis of a mathematical model of the operation process, in the studied transportation system. On the basis of the identification of the authentic transport system and the means of transport operation process carried out in it, crucial operational states were designed as well as the possibilities of transferring between the particular states. Based on that, an event-centered model of the use of the means of transport was built, followed by a mathematical model of that process. A Mathematical model of the operation process has been developed with acceptance of an assumption that the process model is to be represented by a homogeneous Markov model $X(t)$. In this paper, the transport means operation risk is determined as a product of probability (or sum of probabilities) of its being in an undesirable state (states) of the Markov model of transport means operation process and the size of the damage in effect of being in the state (states). The size of the suffered damage is measured by a product of time and unit cost whose quantities are connected with being in the operation process undesirable state or states. Assessment of risk connected with technical objects operation and maintenance can be the point of reference to formulate design requirements concerning durability and reliability of the operated objects (transport means) as well as assumptions concerning design or modernization of the technical means necessary for assurance of the vehicle availability in a given transport system.

Keywords: *transport system, operation process, Markov model, risk*

1. Introduction

The main goal of the transport system operation is to meet transport demands through carrying out transports over given routes. Performance of transport tasks is the responsibility of elementary subsystems of the type: driver – transport means. The range of the assigned transport task is defined by frequency of rides and the amount of carried load in a given time interval.

Functioning of every technical object operation system, including the transport ones, should be efficient. Efficiency of a transport system operation depends on how the assigned transport tasks are performed by it. One of the methods for an assessment of the system ability to perform its task properly is to establish the risks connected with the transport means functioning. The presented method involves determination of the risk connected with functioning of a single technical object (transport means) in a transport means executive subsystem. Indexes of a single technical object operation risk have been determined on the basis of parameters of an operation process mathematical model, in a given system of transportation. Due to a random character of the factors determining the course and efficiency of the transport means operation process, in a given complex system, the processes most frequently used for mathematical modelling of the operation process are stochastic ones. Among random processes, the most widely used are Markov and semi Markov processes [1-9]. The operation mathematical model accepted in this work was developed on the basis of a homogenous Markov process $X(t)$.

Basing on the definitions included in the Polish Norms and the subject literature, risk can be defined as [10-14]:

- product of frequency or probability of a given, dangerous event occurrence and economic-social consequences involved,
- product of probability of an undesirable event occurrence and measurements of its consequences.

The probability of an undesirable or dangerous event occurrence is determinable on the basis of data coming from experimental tests carried out in a real system of technical objects operation, e.g. transport means. The effects of an undesirable event occurrence can affect both the technical object (transport means), the human (e.g. operator performing the transport task) and the environment. The effect of an undesirable or dangerous event occurrence can involve financial losses, loss of health or life, most frequently expressed by cost, extent of action and sometimes duration of the inconvenience [10, 13, 14]. Then, the dependence defining the risk, caused by the vehicle being in the i -th, undesirable state of the operation process, is given by formula

$$r_i^{OT} = P_i^{OT} \cdot \bar{K}_i^{OT}, \quad i \in S_{NP}, \quad (1)$$

where:

P_i^{OT} - probability of the transport means being in the i -th state of the operation process,

\bar{K}_i^{OT} - mean cost born by the transport system, caused by the vehicle being in the i -th state of the operation process,

S_{NP} - subset of the operation process states, being undesirable ones.

2. Event model of transport means operation process

An event model of the operation process was developed on the basis of an analysis of the operation states and events of transport means (urban buses) used in the real, studied transportation system [15]. In result of identification of the analyzed system and its performance of a multi-state transport means operation process, its significant operation states and possible transitions between these states have been determined. On this basis, a graph of the operation process state changes was built and presented in Fig. 1.

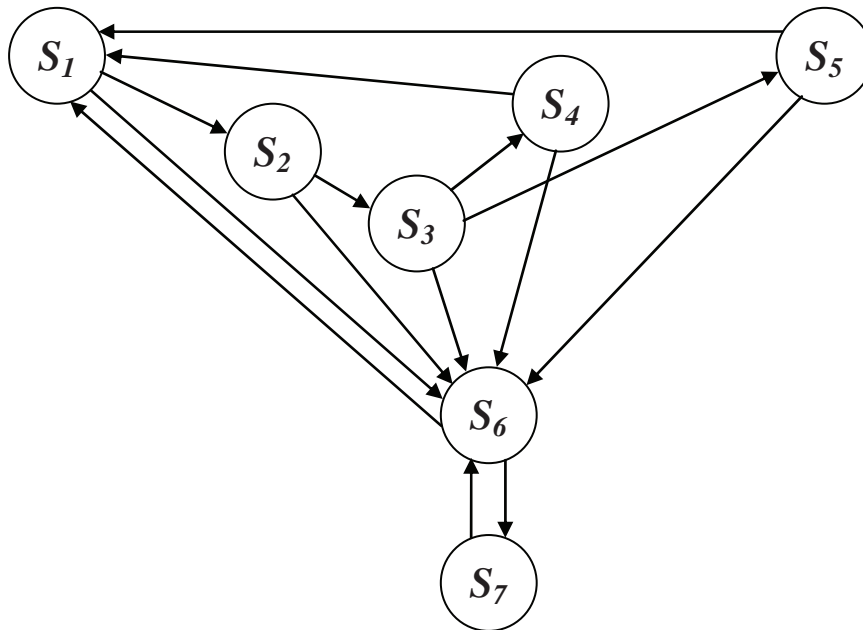


Fig. 1. Directed graph of transport means operation process imaging; S_1 – transport task performance, S_2 – failure during the ride, S_3 – estimation of the damage by an emergency service unit, S_4 – repair by an emergency service unit without losing a ride, S_5 – repair by an emergency service unit with losing a ride, S_6 – standby at the bus depot, S_7 – making the technical object available at proper repair stations

3. Mathematical model of transport means operation process

Basing on the carried out analysis of assumptions and constraints, it was assumed that the mathematical model for the transport means operating process to be used is Markov process $X(t)$ [15]. In order to determine boundary probabilities p_i^* of the vehicle being in the states of $X(t)$ process, S_6 – standby at the bus depot is assumed to be the process initial state. Using Markov processes for the operation process mathematical modelling, the following assumptions have been accepted:

- the modelled process of operation has a finite number of states S_i , $i = 1, 2, \dots, 7$,
- if the technical object is in state S_i , in time t , then, $X(t) = i$, where $i = 1, 2, \dots, 7$,
- random process $X(t)$, being a mathematical model of the operating process, is a homogenous process.

According to the accepted assumptions and on the basis of the graph (Fig. 1), the initial distribution has the form:

$$p_i(0) = \begin{cases} 1 & \text{if } i = 6, \\ 0 & \text{if } i \neq 6, \end{cases} \quad (2)$$

where:

$$p_i(0) = P\{X(0) = i\}, \quad i = 1, 2, \dots, 7. \quad (3)$$

In order to determine the values of boundary probabilities p_i^* of the vehicle being in the Markov model of transport means operation process, on the basis of the graph presented in Fig. 1, matrixes P of the state change probabilities and Λ of the state change intensity of process $X(t)$:

$$P = \begin{bmatrix} 0 & p_{12} & 0 & 0 & 0 & p_{16} & 0 \\ 0 & 0 & p_{23} & 0 & 0 & p_{26} & 0 \\ 0 & 0 & 0 & p_{34} & p_{35} & p_{36} & 0 \\ p_{41} & 0 & 0 & 0 & 0 & p_{46} & 0 \\ p_{51} & 0 & 0 & 0 & 0 & p_{56} & 0 \\ p_{61} & 0 & 0 & 0 & 0 & 0 & p_{67} \\ 0 & 0 & 0 & 0 & 0 & p_{76} & 0 \end{bmatrix}, \quad (4)$$

$$A = \begin{bmatrix} -\lambda_{11} & \lambda_{12} & 0 & 0 & 0 & \lambda_{16} & 0 \\ 0 & -\lambda_{22} & \lambda_{23} & 0 & 0 & \lambda_{26} & 0 \\ 0 & 0 & -\lambda_{33} & \lambda_{34} & \lambda_{35} & \lambda_{36} & 0 \\ \lambda_{41} & 0 & 0 & -\lambda_{44} & 0 & \lambda_{46} & 0 \\ \lambda_{51} & 0 & 0 & 0 & -\lambda_{55} & \lambda_{56} & 0 \\ \lambda_{61} & 0 & 0 & 0 & 0 & -\lambda_{66} & \lambda_{67} \\ 0 & 0 & 0 & 0 & 0 & \lambda_{76} & -\lambda_{77} \end{bmatrix}, \quad (5)$$

where:

p_{ij} - probability of transition from state S_i to state S_j of $X(t)$ process,

λ_{ij} - intensity of transition from state S_i to state S_j of $X(t)$ process.

Next, on the basis of matrixes P and A , a system of linear equations was built

$$\sum_i \lambda_{ij} \cdot p_i^* = 0, \quad j = 1, 2, \dots, 7. \quad (6)$$

In result of solving system of equations (6), formulas describing boundary probabilities p_i^* of being in states of the Markov process $X(t)$:

$$p_1^* = \frac{1}{1 + \frac{\lambda_{12}}{\lambda_{22}} \cdot \left[1 + \frac{\lambda_{23}}{\lambda_{33}} \cdot \left(1 + \frac{\lambda_{34}}{\lambda_{44}} + \frac{\lambda_{35}}{\lambda_{55}} \right) \right] + a \cdot \left(1 + \frac{\lambda_{67}}{\lambda_{77}} \right)}, \quad (7)$$

$$p_2^* = \frac{\lambda_{12}}{\lambda_{22}} \cdot p_1^*, \quad (8)$$

$$p_3^* = \frac{\lambda_{12} \cdot \lambda_{23}}{\lambda_{22} \cdot \lambda_{33}} \cdot p_1^*, \quad (9)$$

$$p_4^* = \frac{\lambda_{12} \cdot \lambda_{23} \cdot \lambda_{34}}{\lambda_{22} \cdot \lambda_{33} \cdot \lambda_{44}} \cdot p_1^*, \quad (10)$$

$$p_5^* = \frac{\lambda_{12} \cdot \lambda_{23} \cdot \lambda_{35}}{\lambda_{22} \cdot \lambda_{33} \cdot \lambda_{55}} \cdot p_1^*, \quad (11)$$

$$p_6^* = a \cdot p_1^*, \quad (12)$$

$$p_7^* = a \cdot \frac{\lambda_{67}}{\lambda_{77}} \cdot p_1^*, \quad (13)$$

where:

$$a = \frac{\lambda_{11} - \frac{\lambda_{12} \cdot \lambda_{23}}{\lambda_{22} \cdot \lambda_{33}} \cdot \left(\frac{\lambda_{34} \cdot \lambda_{41}}{\lambda_{44}} + \frac{\lambda_{35} \cdot \lambda_{51}}{\lambda_{55}} \right)}{\lambda_{61}}. \quad (14)$$

4. Assessment of transport means functioning risk

In this work the risk connected with transport means functioning is defined as a product of probability or a sum of probabilities of the vehicle being in an undesirable state or states of the Markov model for the transport means operation process, and the extent of loss resulting from being in this state (states). The extent of loss is measured by a product of time and a unit cost which quantities are associated with the vehicle being in an undesirable state or states of the operation process. Then, taking into consideration formula (1), the dependence describing the risk caused by the vehicle being in the i -th undesirable state of the Markov transport means operation process model is given by:

$$r_i^{OT} = p_i^* \cdot T_i^{OT} \cdot k_i^{OT}, \quad i \in S_{NP}, \quad (15)$$

where:

p_i^* - boundary probability of the transport means being in the i -th state of the Markov transport means operation process model,

T_i^{OT} - mean time of the transport means being in the i -th state of the Markov transport means operation process model, defined by formulas from (7) to (13),

k_i^{OT} - unit cost born by the transport system, caused by the vehicle being in the i -th state of the Markov transport means operation process model.

Below, there have been presented measurements of the risk of undesirable events occurrence, connected with transport means failures while performance of transport tasks, and the repairs involved:

1. The risk connected with the need to perform a repair of the transport means by a unit of the technical service without losing a ride (during the break between successive rides), presented by dependence:

$$r_N^{(1)} = \sum_i (P_i^{OT} \cdot \bar{K}_i^{OT}) = \sum_i (P_i^{OT} \cdot T_i^{OT} \cdot k_i^{OT}), \quad i \in S_N^{(1)}, \quad (16)$$

where:

$S_N^{(1)}$ - subset of the operation model process states in which occur costs connected with the transport means repairs by a unit of an emergency service without losing a ride.

For the operation process model presented in Fig. 1, formula (16) has the form:

$$r_N^{(1)} = P_4^{OT} \cdot \bar{K}_4^{OT} = p_4^* \cdot T_4^{OT} \cdot k_4^{OT}, \quad (17)$$

that is:

$$r_N^{(1)} = \frac{\lambda_{12} \cdot \lambda_{23} \cdot \lambda_{34} \cdot T_4^{OT} \cdot k_4^{OT}}{\lambda_{22} \cdot \lambda_{33} \cdot \lambda_{44}} \cdot \left[\frac{1}{\lambda_{61}} \cdot \left(1 + \frac{\lambda_{67}}{\lambda_{77}} \right) \right] \cdot \left[\lambda_{11} - \frac{\lambda_{12} \cdot \lambda_{23}}{\lambda_{22} \cdot \lambda_{33}} \cdot \left(\frac{\lambda_{34} \cdot \lambda_{41}}{\lambda_{44}} + \frac{\lambda_{35} \cdot \lambda_{31}}{\lambda_{55}} \right) \right] \cdot \left[1 + \frac{\lambda_{12}}{\lambda_{22}} \cdot \left[1 + \frac{\lambda_{23}}{\lambda_{33}} \cdot \left(1 + \frac{\lambda_{34}}{\lambda_{44}} + \frac{\lambda_{35}}{\lambda_{55}} \right) \right] \right]$$

2. The risk connected with the vehicle repairs by an emergency service unit with losing a ride (necessity of replacing the damaged object with a standby one), has been given by dependence

$$r_N^{(2)} = \sum_i (P_i^{OT} \cdot \bar{K}_i^{OT}) = \sum_i (P_i^{OT} \cdot T_i^{OT} \cdot k_i^{OT}), \quad i \in S_N^{(2)}, \quad (18)$$

where:

$S_N^{(2)}$ - subset of the operation process states in which the costs born are connected with the transport means repairs performed by an emergency service unit with losing a ride.

For the operation process model presented in Fig. 1, formula (18) has the form:

$$r_N^{(2)} = P_5^{OT} \cdot \bar{K}_5^{OT} = p_5^* \cdot T_5^{OT} \cdot k_5^{OT}, \quad (19)$$

that is:

$$r_N^{(2)} = \frac{\lambda_{12} \cdot \lambda_{23} \cdot \lambda_{35} \cdot T_5^{OT} \cdot k_5^{OT}}{\lambda_{22} \cdot \lambda_{33} \cdot \lambda_{55}} \cdot \left[\frac{1}{\lambda_{61}} \cdot \left(1 + \frac{\lambda_{67}}{\lambda_{77}} \right) \right] \cdot \left[\lambda_{11} - \frac{\lambda_{12} \cdot \lambda_{23}}{\lambda_{22} \cdot \lambda_{33}} \cdot \left(\frac{\lambda_{34} \cdot \lambda_{41}}{\lambda_{44}} + \frac{\lambda_{35} \cdot \lambda_{51}}{\lambda_{55}} \right) \right] \cdot \left[1 + \frac{\lambda_{12}}{\lambda_{22}} \cdot \left[1 + \frac{\lambda_{23}}{\lambda_{33}} \cdot \left(1 + \frac{\lambda_{34}}{\lambda_{44}} + \frac{\lambda_{35}}{\lambda_{55}} \right) \right] \right]$$

3. The risk connected with the transport means repair by an emergency service unit (both without and with losing a ride), is expressed by dependence

$$r_N^{(3)} = \sum_i (P_i^{OT} \cdot \bar{K}_i^{OT}) = \sum_i (P_i^{OT} \cdot T_i^{OT} \cdot k_i^{OT}), \quad i \in S_N^{(3)}, \quad (20)$$

where:

$S_N^{(3)}$ - subset of the operation process states in which there are born costs connected with the transport means repairs by an emergency service unit.

For the operation process model presented in Fig. 1, formula (20) has the form:

$$r_N^{(3)} = P_4^{OT} \cdot \bar{K}_4^{OT} + P_5^{OT} \cdot \bar{K}_5^{OT} = p_4^* \cdot T_4^{OT} \cdot k_4^{OT} + p_5^* \cdot T_5^{OT} \cdot k_5^{OT}, \quad (21)$$

that is:

$$r_N^{(3)} = \frac{\lambda_{12} \cdot \lambda_{23} \cdot \left(\frac{\lambda_{34}}{\lambda_{44}} \cdot T_4^{OT} \cdot k_4^{OT} + \frac{\lambda_{35}}{\lambda_{55}} \cdot T_5^{OT} \cdot k_5^{OT} \right)}{\lambda_{22} \cdot \lambda_{33}} \cdot \left[\frac{1}{\lambda_{61}} \cdot \left(1 + \frac{\lambda_{67}}{\lambda_{77}} \right) \right] \cdot \left[\lambda_{11} - \frac{\lambda_{12} \cdot \lambda_{23}}{\lambda_{22} \cdot \lambda_{33}} \cdot \left(\frac{\lambda_{34} \cdot \lambda_{41}}{\lambda_{44}} + \frac{\lambda_{35} \cdot \lambda_{51}}{\lambda_{55}} \right) \right] \cdot \left[1 + \frac{\lambda_{12}}{\lambda_{22}} \cdot \left[1 + \frac{\lambda_{23}}{\lambda_{33}} \cdot \left(1 + \frac{\lambda_{34}}{\lambda_{44}} + \frac{\lambda_{35}}{\lambda_{55}} \right) \right] \right]$$

4. The risk connected with a repair of the transport means damaged during performance of a transport task (both by an emergency service and at the subsystem stations for providing the vehicle with availability), expressed by dependence

$$r_N^{(4)} = \sum_i (P_i^{OT} \cdot \bar{K}_i^{OT}) = \sum_i (P_i^{OT} \cdot T_i^{OT} \cdot k_i^{OT}), \quad i \in S_N^{(4)}, \quad (22)$$

where:

$S_N^{(4)}$ - subsystem of the operation process model states in which there are born costs of the transport means repair connected with a failure which occurred during performance of the transport task.

For the operation process model presented in Fig. 1, formula (22) after appropriate transformations has the form:

$$r_N^{(4)} = P_4^{OT} \cdot \bar{K}_4^{OT} + P_5^{OT} \cdot \bar{K}_5^{OT} + P_{7(N)}^{OT} \cdot \bar{K}_7^{OT} = p_4^* \cdot T_4^{OT} \cdot k_4^{OT} + p_5^* \cdot T_5^{OT} \cdot k_5^{OT} + \left\{ \frac{P_{12} \cdot [p_{26} + p_{23} \cdot (p_{36} + p_{34} \cdot p_{46} + p_{35} \cdot p_{56})] \cdot P_{67}}{p_{16} + p_{12} \cdot [p_{26} + p_{23} \cdot (p_{36} + p_{34} \cdot p_{46} + p_{35} \cdot p_{56})]} \right\} \cdot p_{67} \cdot p_7^* \cdot T_7^{OT} \cdot k_7^{OT}, \quad (23)$$

that is:

$$r_N^{(4)} = \frac{\lambda_{12} \cdot \lambda_{23} \cdot \left(\frac{\lambda_{34}}{\lambda_{44}} \cdot T_4^{OT} \cdot k_4^{OT} + \frac{\lambda_{35}}{\lambda_{55}} \cdot T_5^{OT} \cdot k_5^{OT} \right) + \left\{ \frac{P_{12} \cdot [p_{26} + p_{23} \cdot (p_{36} + p_{34} \cdot p_{46} + p_{35} \cdot p_{56})] \cdot P_{67}}{p_{16} + p_{12} \cdot [p_{26} + p_{23} \cdot (p_{36} + p_{34} \cdot p_{46} + p_{35} \cdot p_{56})]} \right\} \cdot \left[\lambda_{11} - \frac{\lambda_{12} \cdot \lambda_{23}}{\lambda_{22} \cdot \lambda_{33}} \cdot \left(\frac{\lambda_{34} \cdot \lambda_{41}}{\lambda_{44}} + \frac{\lambda_{35} \cdot \lambda_{51}}{\lambda_{55}} \right) \right] \cdot \frac{\lambda_{67} \cdot T_7^{OT} \cdot k_7^{OT}}{\lambda_{61} \cdot \lambda_{77}}}{1 + \frac{\lambda_{12}}{\lambda_{22}} \cdot \left[1 + \frac{\lambda_{23}}{\lambda_{33}} \cdot \left(1 + \frac{\lambda_{34}}{\lambda_{44}} + \frac{\lambda_{35}}{\lambda_{55}} \right) \right] + \left[\lambda_{11} - \frac{\lambda_{12} \cdot \lambda_{23}}{\lambda_{22} \cdot \lambda_{33}} \cdot \left(\frac{\lambda_{34} \cdot \lambda_{41}}{\lambda_{44}} + \frac{\lambda_{35} \cdot \lambda_{51}}{\lambda_{55}} \right) \right] \cdot \left[\frac{1}{\lambda_{61}} \cdot \left(1 + \frac{\lambda_{67}}{\lambda_{77}} \right) \right]}$$

where:

$P_{7(N)}^{OT}$ - probability of the transport means being in state S_7 when the damage occurred during performance of the transport task.

5. Conclusion

In further stages of the experiment, in order to determine values of indexes of the transport means functioning risk presented in this work, tests will be performed in a selected real transport

system a and there will be worked out a method for determination of criteria-based values of the risk assessment. Assessment of the risks connected with technical objects operation can be a reference point for formulation of the design requirements concerning strength and reliability of the utilized technical objects (transport means) as well as assumptions connected with a design and modernization of the servicing infrastructure providing vehicles of a given operation system with availability.

The assessment of the risk connected with technical objects functioning is an issue of complex character and of great importance. Obtainment of reliable information on the subject of risk assessment involves determination of the risk numerical value and values of criteria which can make it possible to make a decision as to whether so determined risk level can be accepted or not. The process of the risk acceptance is prone to error of making a wrong decision, measured by a pre accepted probability of such an error to be made. Dependencies defining the risk can be treated as a random variable. Then, the risk assessment and a decision on its acceptance are carried out on the basis of probability distribution for so defined random variable [10].

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