DEFORMABLE GROUND INFLUENCE ON THE FRICTION DRIVE TRANSMISSION BETWEEN DRIVE PULLEY AND ELASTOMETRIC TRACK

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Abstract

Industrial vehicles with undercarriage with elastomeric tracks are frequently used in agriculture and forestry, which is connected with moving on deformable ground. A friction method of transmitting the drive between a drive pulley and a track, which is often applied, is sensitive to adverse ambient conditions, whereas the influence of the ground on friction feedback conditions in the area of the ground working on the drive system is significant. Deformable ground may be a source of contamination, which considerably reduces a friction coefficient on wrap of a drive pulley. The article includes the analysis of deformable ground influence on the volume of driving moment transmitted between a drive pulley and elastomeric track. It has been demonstrated that the influence of deformable ground, as a result of additional pressures working on wrap of a drive pulley may, in some circumstances, considerably increase the moment transmitted by means of friction feedback of a drive pulley with elastomeric track. Increase of driving moment may be higher than 50%. The analysis includes limited applicability of Euler and Entelwein's law in the description of transmitting the moment between a drive pulley with elastomeric lining and elastomeric track.

Keywords: *elastomeric track, track undercarriage, operation of tracked vehicles, power transmissions*

1. Introduction

A universal character of undercarriage with elastomeric tracks allows for using industrial vehicles equipped with them in varied work conditions and on varied grounds. At the stage of designing the drive transmission system, it requires including applicability area of track wheel and steering system. Assuming the use of a machine on deformable ground, the influence of the ground on the undercarriage is presented (Figure 1).

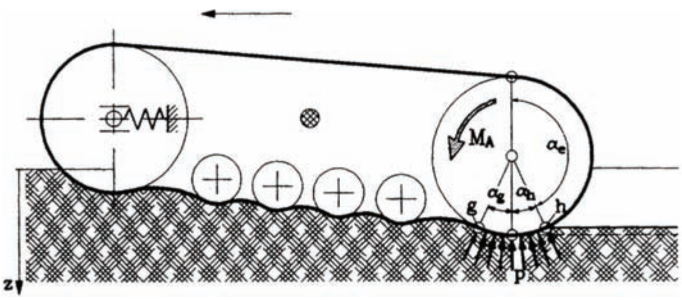

Fig. 1. Track undercarriage on the deformable ground [3]

The analysis of power transmission process in a typical track undercarriage of a working machine requires separate consideration of two sub-areas of a drive pulley interaction with the track (Figure 1): wrap section free from ground impact (α_e) and wrap section, in which ground impact must be considered (α_g and α_h).

The analysis of drive transmission process was conducted on the track undercarriage, whose first roller at the same time is a tension pulley, and the last roller serves as a track sprocket. The selection of such a structure is justified with more advantageous force distribution in the track – maximum track tension is observable only within a short section of the track [3].

2. Drive transmission process analysis between drive pulley and elastomeric track

Analysing the research results presented in the literature [4, 5], it has been noted that within the drive transmission system under consideration, pressure is the most significant parameter influencing a friction coefficient value and, therefore, the following dependance between a friction coefficient and pressure has been derived [2]:

$$\mu = a + b \cdot \ln(p), \tag{1}$$

where:

a, *b* – approximation coefficients of experimental research results,
p – pressure.

2.1. The coupling of the drive pulley with the track on wrap beyond the influence of the soil

The forces acting on a separate element of track are presented in Figure 2. From equilibrium equations and considering that sin(dα/2)= dα/2 and dS·sin(dα/2)≈0 we obtain:

$$\frac{dp}{d\alpha} - p \cdot \mu(p) = 0 \tag{2}$$

and considering (1) we obtain:

$$\frac{dp}{d\alpha} - p \cdot [a + b \cdot \ln(p)] = 0 . \tag{3}$$

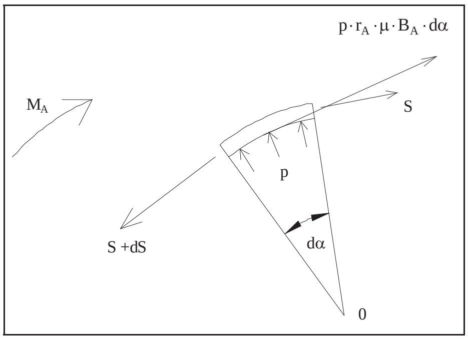

Fig. 2. The forces acting on a separate element of track [2]

Equation (3) a nonlinear first order differential equation, whose result is as follows:

$$p = e^{\frac{1}{b}\left(e^{b\cdot(\alpha + C_1)} - a\right)}. \tag{4}$$

Considering boundary condition ($p(\alpha=0)=p_1$), on substitution to equation (4) we obtain:

$$p(\alpha) = \exp\left[\frac{1}{b}\left(e^{b\cdot\left(\alpha + \frac{1}{b}\cdot\ln[a + b\cdot\ln(p_1)]\right)} - a\right)\right]. \tag{5}$$

The above equation enables calculation of unit pressures value in each point of a belting curve. Applying dependance: $p=2S/(D_A B_A)$, it is possible to define force in the track in any point of wrap:

$$S(\alpha) = p \cdot z_1 = \frac{B_A \cdot D_A}{2} \cdot p = \frac{B_A \cdot D_A}{2} \cdot \exp\left[\frac{1}{b}\left(e^{b\cdot\left(\alpha + \frac{1}{b}\cdot\ln\left[a + b\cdot\ln\left(\frac{S_V}{B_A \cdot D_A}\right)\right]\right)} - a\right)\right]. \qquad (6)$$

A driving moment transmitted by friction feedback without ground impact (eg. vehicle moving on hard ground) may be calculated (for vehicle moving forwards) within equation:

$$M_A = \frac{D_A}{2}\left\{\frac{B_A \cdot D_A}{2} \cdot \exp\left[\frac{1}{b}\left(e^{b\cdot\left(\alpha_0 + \frac{1}{b}\cdot\ln\left[a + b\cdot\ln\left(\frac{S_V}{B_A \cdot D_A}\right)\right]\right)} - a\right)\right] - \frac{S_V}{2}\right\}, \qquad (7)$$

where:

M_A – drive moment,
D_A – diameter of drive pulley,
B_A – width of drive pulley,
S_V – force in the tensioning device,
α_0 – angle of wrap,
a, b – approximation coefficients, (eq. 1).

The formula (7) is not continuous for force in a tension device equalling zero ($S_V=0$). It is caused by an accepted form of aproximation function (1). Calculations applying equation (7) must be conducted for unit pressures in the area of approximate experimental results.

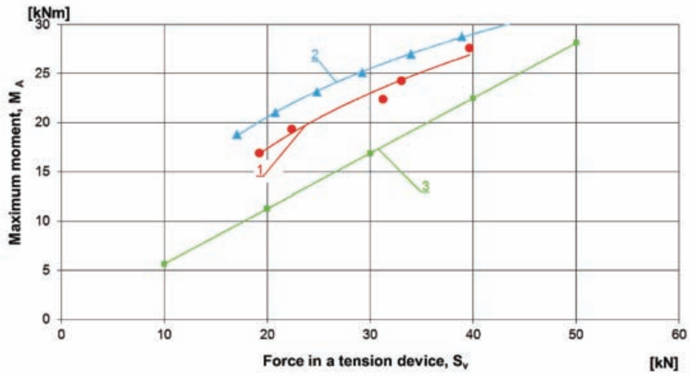

Fig. 3. Comparison of the results of measurements (1) with calculations based on: (2)-equation (7), (3)-Euler's law (μ=0.8)[2]

Figure 3 presents a drive moment transmitted to friction feedback depending on initial tension of elastic band. In calculations, friction coefficient values were used as well as function coefficents characteristic for identical working conditions.

As it can be seen (figure 3), a course of moment calculated from Euler formula is considerably different from the one obatined experimentally. Totally different character of both graphs is particularly noticeable. It is different with calculations conducted according to the elaborated model. However, there are differences as to the value although the character of both graphs is similar, which appears to confirm the selection of the method of considered feedback coefficient variation.

2.2.1. Drive pulley feedback with the track in the impact area of deformable ground

Ground deformation may be resilient or plastic in its nature. In the latter case, deformation may be caused by means of either density or floating of the ground. Ground deformation is mainly dependent on ground type (granulation) and ground condition (porosity, humidity). The simplest dependence between pressure height working on the soil and soil squeeze is [7]:

$$p = k \cdot z^n, \qquad (8)$$

where: p – unit pressure on the ground, z – depth of soil sinkage under load, k – soil deformability module, n – dimensionless index defining soil structure and type.

The studies [1] demonstrated that parameter n does not depend on load area size, but on soil type and its homogeneity. Parameter k is dependent not only on ground type and condition, but also on geometrical types of stanchion. Therefore, Bekker [1] divided coefficient k into two modules of soil vertical deformation: dependent on its cohesion k_c and dependent on its internal friction k_φ:

$$p = \left(\frac{k_c}{b} + k_\varphi \right) \cdot z^n , \tag{9}$$

where:
b – width of stanchion.

Many authors (eg. Reece [6]) attempted to develop formulas (8) and (9). Accepting a similar model and accepting Terzaghi's equation [8] as a basis:

$$p_o = c \cdot N_C + \frac{1}{2} \gamma \cdot b \cdot N_\gamma + \gamma \cdot z \cdot N_g , \tag{10}$$

where: c – cohesion, γ - specific gravity of soil, N_c, N_γ, N_g – coefficients,

Reece [6] suggests:

$$p = \left(c \cdot k'_c + \gamma \cdot b \cdot k'_\varphi \right) \cdot \left(\frac{z}{b} \right)^n . \tag{11}$$

Dependences presented above: pressure-sinkage, are adequate only in the case of original ground load. When considering a track vehicle moving on deformable ground, it is necessary to be aware of ground behaviours in case of unloading and reloading, which is observable when subsequent rollers of the track undercarriage move. The course of an exemplary curve loading-unloading-reloading is presented in Figure 4. Wong [9] suggests, in first approximation (for sections AB and CD, figure 4), the following linear dependence:

$$p = p_{Ei} - k_{Ei} \cdot \left(z_{Ei} - z \right), \tag{12}$$

where: p_{Ei} – pressure at the beginning of unloading, z_{Ei} – sinkage at the beginning of unloading, k_{Ei} – parameter during i-th unloading-loading cycle.

This assumption is possible, since participation of hysteresis in total deformation is negligible, and with sandy ground, it disappears completely. Is has been determined that the value of coefficient k_{Ei} during unloading and reloading changes along with changing sinkage z_{Ei}:

$$k_{Ei} = k_o + \alpha_E \cdot z_{Ei} , \tag{13}$$

where: k_o and α_E are soil parameters obtained through experiments.

2.2.2. Analytical assessment of deformed ground impact on the volume of driving moment transmitted

Movement of vehicle with track undercarriage on deformable ground, on account of drive moment transmitted, has two significant aspects (Figure 5):

- additional pressures in the tangential area of pulley on the ground – fragments α_g and α_h,
- increase of wrap – area α_g.

Dimension of wrap under consideration α_g and α_h within the ground impact area depend on the range of significant parameters. The most significant ones are as follows:

- type and condition of deformable ground,
- drive pulley width and diameter,
- moving vehicle velocity,

– ground deformation velocity,
– loading history (multi-pass-effect),
– type and properties of undercarriage elements,
– track initial tension,
– physical and mechanical properties of elastomeric track and tread form,
– drive pulley vertical loading.

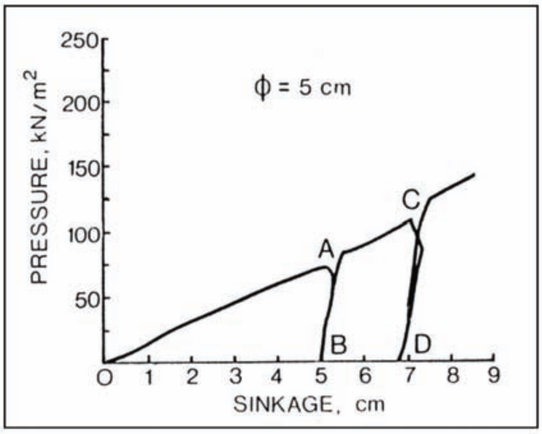

Fig. 4. Response of mineral soil for cyclic loading [9]

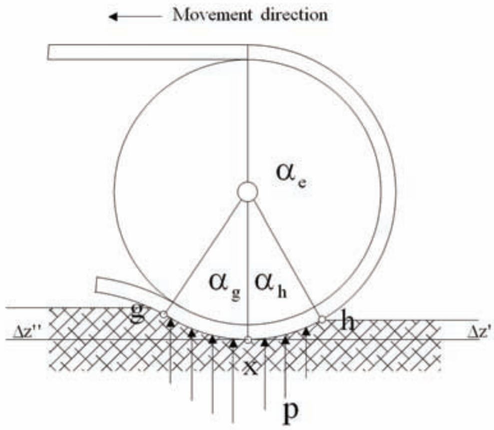

Fig. 5. Interaction of a drive pulley and elastomeric track with pressures on deformable ground (α_e –wrap range without ground impact, α_g - additional wrap resulting from ground deformation through a drive pulley, α_h – wrap range with relaxation of the soil)[2]

Determining values α_g and α_h on an analytical basis is highly complex and requires elaborating track undercarriage model including interaction with the ground. On account of this issue complexity, impact interaction assessment of a drive pulley with deformable ground on the moment transmitted volume is limited to:
– increase of wrap angle with additional angle α_g,
– influence of unit pressures from the ground in wrap angle area α_h, whereby volume α_h (at the same time $\Delta z'$) is a function of elasticity properties of the soil including loading history (multi-pass-effect).

Dependence between elastic reflection $\Delta z'$ and α_h is as follows $\Delta z'=0,5\cdot D_A(1-\cos(\alpha_h))$, hence, angle $\alpha_h=\arccos(1-2\cdot\Delta z'/D_A)$. Analogously $\Delta z''=0,5\cdot D_A(1-\cos(\alpha_g))$ and $\alpha_g=\arccos(1-2\cdot\Delta z''/D_A)$.

Figure 6 presents the increase of driving moment transmitted, caused by wrap angle increase with additional angle α_g. Calculations were conducted using equation (8), the influence of additional unit pressures from the ground on the track were not included (data for calculations: D_A=0,8 m, B_A=0,276 m, S_V=80 kN, $\alpha_0=\pi + \alpha_g$ rad, a=2.80229, b=-0.172314 [1/ln(Pa)]). In order to include the influence of pressures from the ground, the section of the track h-x was subject to thorough analysis (Figure 6).
Pressures distribution working on the separated track element in this area is presented in (Figure 7).

From equilibrium equations in the radial and tangential directions, considering that: $\sin(d\alpha/2)\approx d\alpha/2$, $\cos(d\alpha/2)\approx 1$, $dS\cdot\sin(d\alpha/2)\approx 0$ and applying the following dependences: $dR=\mu\cdot dN$, $\tau=\tau(p)$ and $p=p(z)$ upon transformations, the following first order differential equation was obtained:

$$\frac{dS}{d\alpha}-\mu\cdot S-B_A\cdot r_A[\mu\cdot p(z)-\tau(p)]=0\,. \qquad (14)$$

Applying the following dependence:

$$z=r_A\cdot\cos(\alpha_h-\alpha), \qquad (15)$$

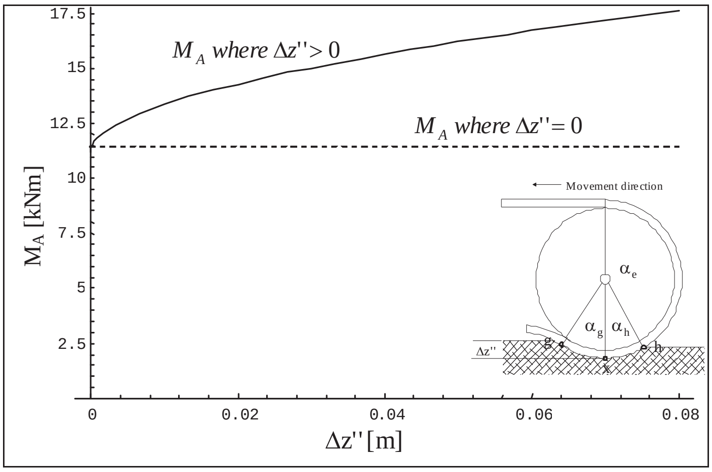

Fig. 6. Increase of driving moment M_A caused by wrap angle increase with additional angle $\alpha_g = f(\Delta z'')$ [2]

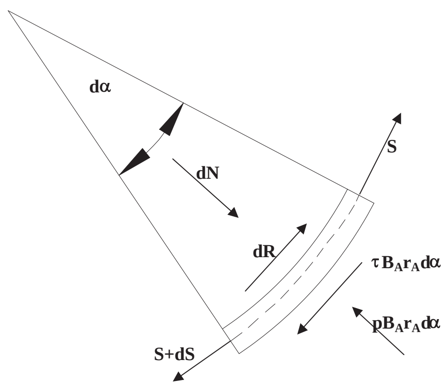

Fig. 7. Distribution of pressures acting on the track element from the tangential area of a drive pulley with deformed ground [2]

where:
α – angle coordinate beginning from h,
functions p=p(z) through p=p(α) and τ(p)=τ (α) were verified:

$$\frac{dS}{d\alpha} - \mu \cdot S = B_A \cdot r_A \left[\mu \cdot p(\alpha) - \tau(\alpha)\right] . \tag{16}$$

On account of relatively low values of angles α_g and α_h and minor value changes of feedback coefficient μ=f(p), the above equation (16) is solved for μ=const. The result of homogeneous equation:

$$\frac{dS}{d\alpha} - \mu \cdot S = 0 \tag{17}$$

is:

$$S_a = C_1 \cdot e^{\mu \cdot \alpha} . \tag{18}$$

A peculiar result was obtained when introducing anticipated equation result (14):

$$S_b = C(\alpha) \cdot e^{\mu \cdot \alpha} \tag{19}$$

on differentiation: $S'_b = C'(\alpha) \cdot e^{\mu \cdot \alpha} + C(\alpha) \cdot \mu \cdot e^{\mu \cdot \alpha}$ and substituting to equation (14) the following was obtained:

$$C'(\alpha) \cdot e^{\mu \cdot \alpha} + C(\alpha) \cdot e^{\mu \cdot \alpha} - \mu \cdot C(\alpha) \cdot e^{\mu \cdot \alpha} = B_A \cdot r_A \cdot \left[\mu \cdot p(\alpha) - \tau(\alpha)\right]. \tag{20}$$

On transformations, the following was obtained:

$$S_b = e^{\mu \cdot \alpha} \int B_A \cdot r_A \cdot \left[\mu \cdot p(\alpha) - \tau(\alpha)\right] \cdot e^{-\mu \cdot \alpha} d\alpha . \tag{21}$$

Since $S(\alpha) = S_a(\alpha) + S_b(\alpha)$ the following final result was obtained:

$$S = e^{\mu \alpha} \left\{ C_1 + B_A \cdot r_A \int \left[\mu \cdot p(\alpha) - \tau(\alpha)\right] \cdot e^{-\mu \alpha} d\alpha \right\} \cdot \tag{22}$$

Recent considerations include neither thickness nor structure of the track (reinforcement lines). Nevertheless, with more precise analysis of tangential tensions influence from the ground on the generated feedback force, it may be noticed that tangential force on the external track surface and tread $\tau \cdot B_A \cdot r_A \cdot d\alpha$ (Figure 7) causes elastomeric layers deformation, but only to lines surface separating the track section, whose lengthening, on account of low values of tangential force and high rigidity of lines, is negligible. Therefore, the influence of tangential forces from the ground does not reach the tangential sphere of the track and drive pulley and does not influence feedback process significantly. Thus, equation (22) was solved without tangential tensions impact ($\tau(\alpha)=0$). The above equation was solved as first within the range g-x. Introducing pressure-sinkage function into equation (22):

$$p(z)=k\cdot\Delta z', \qquad (23)$$

the following was obtained:

$$S=e^{\mu\alpha}\left\{C_1+\mu\cdot B_A\cdot r_A\cdot k\int\left[r_A\cdot\cos(\alpha_h-\alpha)-\Delta z'\right]\cdot e^{-\mu\alpha}d\alpha\right\}. \qquad (24)$$

Linear equation (8) (n=1) was accepted on account of inconsiderable ground deformations (practically within resilience range). Dividing the integral of the above equation into the sum of integrals:

$$A=A_1+A_2=\int r_A\cdot\cos(\alpha_h-\alpha)\cdot e^{-\mu\alpha}d\alpha-\int\Delta z'\cdot e^{-\mu\alpha}d\alpha\ . \qquad (25)$$

Introducing the solved integrals (A_1 and A_2) into equation (22) the following was obtained:

$$S=e^{\mu\alpha}\left\{C_1+B_A\cdot r_A\cdot k\cdot\left[e^{-\mu\alpha}\left\{\frac{-r_A\cdot\mu}{1+\mu^2}\left[\mu\cdot\cos(\alpha_h-\alpha)+\sin(\alpha_h-\alpha)\right]+\Delta z'\right\}+C_2+C_3\right]\right\}, \qquad (26)$$

taking: $C=C_1+\mu B_A r_A(C_2+C_3)$ the following was obtained:

$$S=C\cdot e^{\mu\alpha}+B_A\cdot r_A\cdot k\cdot\left[\Delta z'-\frac{r_A\cdot\mu}{1+\mu^2}\left[\mu\cdot\cos(\alpha_h-\alpha)+\sin(\alpha_h-\alpha)\right]\right]. \qquad (27)$$

Constant C was determined based on the boundary condition $S(\alpha=0)=S_h$ and the following was obtained:

$$S=S_h e^{\mu\alpha}+B_A r_A k\left[\frac{r_A\mu}{1+\mu^2}\left\{e^{\mu\alpha}\left[\mu\cos(\alpha_h)+\sin(\alpha_h)\right]-\mu\cos(\alpha_h-\alpha)-\sin(\alpha_h-\alpha)\right\}-\Delta z'\left(e^{\mu\alpha}-1\right)\right]. \qquad (28)$$

In order to calculate a moment transmitted through feedback, it is necessary to calculate force in the belt in point h (taking $\alpha=\alpha_h$):

$$S_2=S_h e^{\mu\alpha_h}+B_A r_A k\left[\frac{r_A\mu}{1+\mu^2}\left\{e^{\mu\alpha_h}\left[\mu\cos(\alpha_h)+\sin(\alpha_h)\right]-\mu\right\}-\Delta z'\left(e^{\mu\alpha_h}-1\right)\right], \qquad (29)$$

force in point h (S_h) was solved taking $\alpha_0=\pi-\alpha_h$. Total driving moment (including the deformed ground impact may be calculated from the below formula:

$$M_A=\frac{D_A}{2}\left\{\left(\left(\frac{B_A\cdot D_A}{2}\cdot\exp\left[\frac{1}{b}\left(e^{b\cdot\left(\alpha_e+\frac{1}{b}\cdot\ln\left[a+b\cdot\ln\left(\frac{S_V}{B_A\cdot D_A}\right)\right]\right)}-a\right)\right]\cdot e^{\mu\alpha_h}+\right.\right.\right.$$

$$\left.\left.+B_A r_A k\left[\frac{r_A\mu}{1+\mu^2}\left\{e^{\mu\alpha_h}\left[\mu\cos(\alpha_h)+\sin(\alpha_h)\right]-\mu\right\}-\Delta z'\left(e^{\mu\alpha_h}-1\right)\right]\right)-\frac{S_V}{2}\right\}\ . \qquad (30)$$

Taking data from previous example (Fig. 6) for calculations and applying equation (30), the drive moment transmitted was calculated including vertical ground reactions (Figure 8).

3. Conclusions

The above analyses indicate that vertical ground reactions may exert significant influence on the driving moment transmitted through drive system increasing it even by 50%. This increase may be higher if beyond sphere α_h, the second area of cooperation of a drive pulley with deformable ground α_g will is considered. Comparable pressure values in curve sphere αg cause at least the same (or higher) increment of the moment transmitted in comparison to curve sphere α_g.

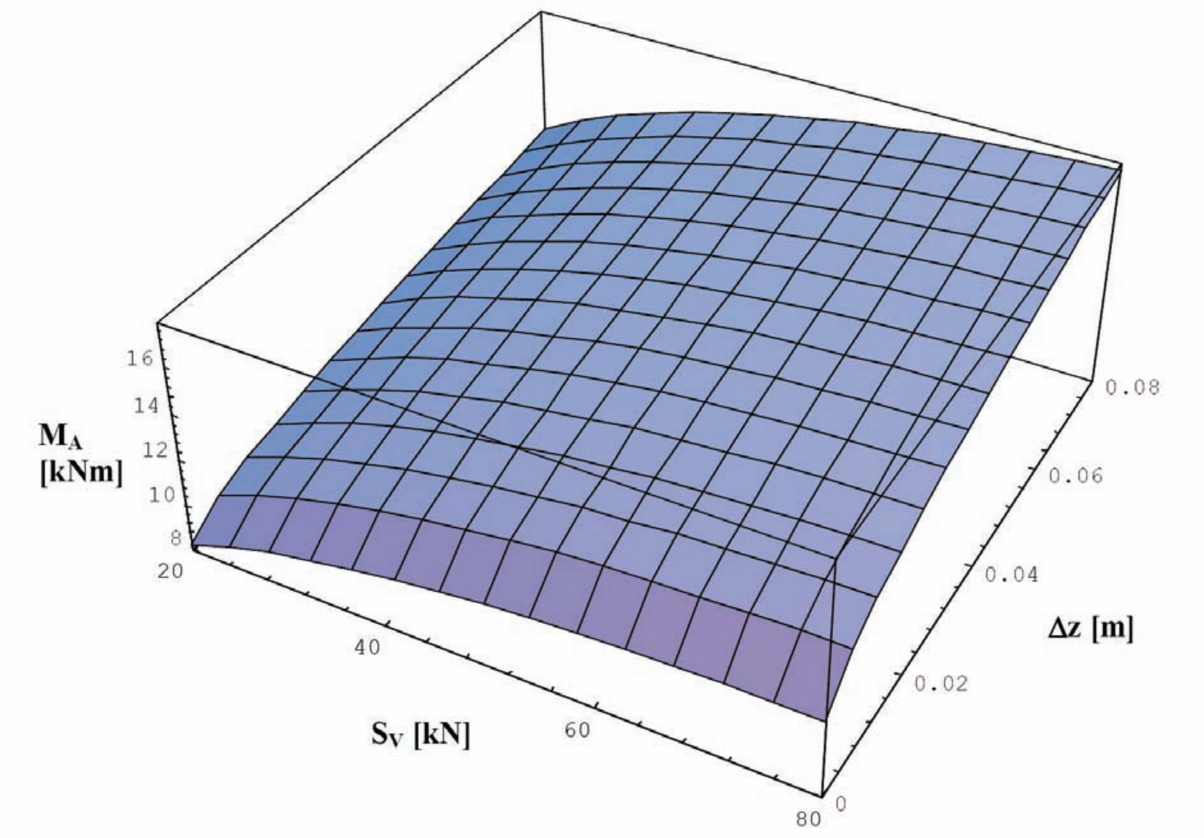

Fig. 8. Driving moment as function of drive pulley sinkage and force in a tension device [2]

Therefore, total increment of the transferred drive moment caused by ground impact may be even twice as much higher than increment obtained in the above calculations, thus ground vertical reactions may cause the transmitted drive moment increase even by 100%.

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