

THE MODEL OF DEFINING THE EFFICIENCY OF LOGISTICS SUBSYSTEM POSTS IN THE TRANSPORT SYSTEM

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Abstract

The tasks of transport systems include the realization of the assigned transport tasks by the utilization subsystem comprised of elementary subsystems of the man – technological object (driver – vehicle) type. In order to assure the appropriate realization of the transport task it is necessary to maintain the required number of vehicles at the state of task worthiness. It is achieved as a result of the realization of service and repair processes at the logistics subsystem posts. In case of the transport system, the task assigned to the logistics subsystem is determined by the number of technological objects (vehicles) which should be serviced as well as the length of the time interval during which they are to be serviced. The task determined in this way is realized by teams of specialists at each logistics subsystem post, equipped with appropriate devices and tools. One of the methods of the evaluation of the level of the realization of worthiness is defining the efficiency of the logistics subsystem posts. The article presents the model of defining the efficiency of logistics subsystem posts in the transport system. The presented model measures the efficiency of the logistics subsystem through the probability of servicing of the required number of vehicles over an assigned time interval. The obtained characteristics have been presented in general terms as well as for exponential and Erlang distributions. Then, for the utilization data obtained from the tests conducted in an existing transport system, the values of analyzed characteristics were defined. The presented results are the effect of research conducted as part of a larger research project pertaining to the creation of a decision-making model of controlling the availability of a transport system.

Keywords: transport system, logistics subsystem, efficiency of posts

1. Introduction

For appropriate realization of the assigned transport tasks in transport systems, a required amount of task-efficient technological objects (means of transport) is necessary. During the realization of the transport task the means of transport used become damaged under the influence of a number of factors. From the point of view of the effectiveness of the operation of the transport system, the damaged technological objects should be repaired in the shortest possible time. Because of this, most transport systems have their own technological infrastructure (logistics subsystem) equipped with an appropriate amount of service and repair posts. The task of the logistics subsystem is the repair of a required number of damaged means of transport in a time interval assigned for this purpose, defined by the timetable of the realized transport tasks. One of the methods of evaluating the degree of realization of the processes of ensuring worthiness carried out at the logistics subsystem posts is the defining of the efficiency of this subsystem posts.

The paper presents the method of defining the efficiency of logistics subsystem posts measured by the value of probability of the repair of the number k of technological objects in the time interval τ assigned for this goal. The evaluation of the efficiency of the logistics subsystem posts may form the basis for decisions on the change of the number and type of the posts used in the tested transport system. The suggested method may be used for a single post, a group of posts of a given type, e.g. diagnostic posts, as well as for the logistics subsystem as a whole. The model

presented in the paper has been built taking into consideration the issues pertaining to the theory of the regeneration of technological objects.

2. The efficiency of the logistics subsystem posts

Let the random variable S mean the period of the technological object remaining at the logistics subsystem with the CDF

$$A(t) = P(S < t). \quad (1)$$

If $N(\tau)$ means the number of technological objects repaired at the logistics subsystem in the time interval τ , then the distribution of the number of technological objects repaired at the logistics subsystem in the time interval τ is defined by the relation

$$P(N(\tau) = n) = A^{(n)}(\tau) - A^{(n+1)}(\tau), \quad n = 0, 1, 2, \dots, \quad (2)$$

where:

$$A^{(0)}(\tau) = 1, \quad (3)$$

$$A^{(1)}(\tau) = A(\tau), \quad (4)$$

$$A^{(n)}(\tau) = \int_0^{\tau} A^{(n-1)}(\tau - x) dA(x), \quad n = 2, 3, \dots, \quad (5)$$

or

$$A^{(n)}(\tau) = \int_0^{\tau} A^{(n-1)}(\tau - x) a(x) dx, \quad n = 2, 3, \dots, \quad (6)$$

where

$$a(t) = \frac{dA(t)}{dt}, \quad (7)$$

whereas the probability of the number of repaired technological objects in the logistics subsystem in the time interval τ being lower than n is defined by the relation

$$V(n) = P(N(\tau) < n) = 1 - A^{(n)}(\tau), \quad n = 1, 2, \dots \quad (8)$$

Let the random variable T mean the period of the technological object remaining outside of the logistics subsystem (in the utilization subsystem) with the CDF

$$B(t) = P(T < t). \quad (9)$$

If $L(\tau)$ means the number of technological objects driven in the time interval τ for the logistics subsystem, then the distribution of the number of technological objects directed to the logistics subsystem in the time period τ is defined by the relation

$$P(L(\tau) = n) = B^{(n)}(\tau) - B^{(n+1)}(\tau), \quad n = 0, 1, 2, \dots, \quad (10)$$

where:

$$B^{(0)}(\tau) = 1, \quad (11)$$

$$B^{(1)}(\tau) = B(\tau), \quad (12)$$

$$B^{(n)}(\tau) = \int_0^{\tau} B^{(n-1)}(\tau - x) dB(x), \quad n = 2, 3, \dots, \quad (13)$$

or

$$B^{(n)}(\tau) = \int_0^{\tau} B^{(n-1)}(\tau - x)b(x) dx, \quad n = 2,3,\dots, \quad (14)$$

where

$$b(t) = \frac{dB(t)}{dt}, \quad (15)$$

whereas the probability of the number of technological objects directed to the logistics subsystem in the time interval τ being lower than n is defined by the relation

$$U(n) = P(L(\tau) < n) = 1 - B^{(n)}(\tau). \quad (16)$$

Taking the above into consideration, the number $M(\tau)$ of technological objects unrepaired in the logistics subsystem in the time interval τ is defined as follows

$$M(\tau) = L(\tau) - N(\tau), \quad (17)$$

then its CDF reflecting the probability of technological objects unrepaired in the time interval τ being lower than $n = N - k$, i.e. the number of the technological objects repaired in the time interval τ equals at least k , where:

N - the number of technological objects used in the transport system,

k - the number of technological objects repaired in the time interval τ at the logistics subsystem posts, is the characteristic feature describing the efficiency of the logistics subsystem posts and is defined as follows

$$\begin{aligned} W(N - k) = W(n) &= P(M(\tau) < n) = P(L(\tau) - N(\tau) < n) = \quad , \quad n = 1,2,\dots, \\ &= \sum_{b=0}^{\infty} P(N(\tau) = b, L(\tau) < n + b) = \\ &= \sum_{b=0}^{\infty} P(N(\tau) = b) \cdot P(L(\tau) < n + b) = \quad (18) \\ &= \sum_{b=0}^{\infty} [A^{(b)}(\tau) - A^{(b+1)}(\tau)] \cdot U(n + b) = \\ &= \sum_{b=0}^{\infty} [A^{(b)}(\tau) - A^{(b+1)}(\tau)] \cdot [1 - B^{(n+b)}(\tau)]. \end{aligned}$$

Then the average number of technological objects unrepaired at the logistics subsystem in the time interval τ is defined by the formula:

$$\bar{M}(\tau) = \bar{L}(\tau) - \bar{N}(\tau), \quad (19)$$

where

$$\bar{L}(\tau) = E[L(\tau)] = \sum_{n=0}^{\infty} n P(L(\tau) = n), \quad (20)$$

and

$$\bar{N}(\tau) = E[N(\tau)] = \sum_{n=0}^{\infty} n P(N(\tau) = n), \quad (21)$$

are the respective expected values of the number $L(\tau)$ of technological objects directed to the logistics subsystem and the number $N(\tau)$ of technological objects repaired at the logistics subsystem posts in the time interval τ .

Random variables S and T meaning the periods of time of the technological object remaining at the logistics subsystem as well as in the utilization subsystem may be defined by distributions of various types. The formulas for determining the values of the index of the efficiency of the logistics subsystem for Erlang and exponential distributions are given below.

Erlang distribution

If random variable S meaning the period of the technological object remaining at the logistics subsystem post has the Erlang distribution of the r type with the CDF:

$$A(t) = P(S < t) = \int_0^{c \cdot t} \frac{u^{r-1}}{(r-1)!} e^{-u} du, \quad x \geq 0, \quad c > 0, \quad (22)$$

then the number $N(\tau)$ meaning the number of technological objects repaired in the logistics subsystem in the time interval τ has the distribution:

$$P(N(\tau) = n) = \sum_{k=n-r}^{n+r-1} \frac{(c \cdot \tau)^k}{k!} e^{-c \cdot \tau}, \quad n = 0, 1, 2, \dots \quad (23)$$

If random variable T means the time of the technological object remaining outside of the logistics subsystem (in the utilization subsystem) has the Erlang distribution of the s type with the CDF:

$$B(t) = P(T < t) = \int_0^{d \cdot t} \frac{u^{s-1}}{(s-1)!} e^{-u} du, \quad x \geq 0, \quad d > 0, \quad (24)$$

then the number $L(\tau)$ meaning the number of the technological objects directed to the logistics subsystem in the time interval τ has the distribution:

$$P(L(\tau) = n) = \sum_{k=n-s}^{n+s-1} \frac{(d \cdot \tau)^k}{k!} e^{-d \cdot \tau}, \quad n = 0, 1, 2, \dots \quad (25)$$

Then the relation (18) looks as follows:

$$\begin{aligned} W(N - k) &= W(n) = P(M(\tau) < n) = P(L(\tau) - N(\tau) < n) = \\ &= \sum_{b=0}^{\infty} \left[\sum_{k=b-r}^{b+r-1} \frac{(c \cdot \tau)^k}{k!} e^{-c \cdot \tau} \cdot \sum_{l=(n+b) \cdot s}^{(n+b) \cdot s + s - 1} \frac{(d \cdot \tau)^l}{l!} e^{-d \cdot \tau} \right] = \\ &= \sum_{b=0}^{\infty} \left[\sum_{k=b-r}^{b+r-1} \frac{(c \cdot \tau)^k}{k!} e^{-c \cdot \tau} \cdot \sum_{l=(N-k+b) \cdot s}^{(N-k+b) \cdot s + s - 1} \frac{(d \cdot \tau)^l}{l!} e^{-d \cdot \tau} \right]. \end{aligned} \quad (26)$$

Exponential distribution

If random variable S meaning the time of the technological object remaining at the logistics subsystem has the exponential distribution with the CDF:

$$A(t) = P(S < t) = 1 - e^{-\beta t}, \quad t \geq 0, \quad (27)$$

then the number $N(\tau)$ meaning the number of technological objects repaired at the logistics subsystem in the time interval τ has the Poisson distribution with the $\beta \cdot \tau$ parameter, i.e.:

$$P(N(\tau) = n) = \frac{(\beta \cdot \tau)^n}{n!} e^{-\beta \cdot \tau}, \quad n = 0, 1, 2, \dots, \quad (28)$$

then we have:

$$A^{(n)}(\tau) = 1 - \sum_{b=1}^{n-1} \frac{(\beta \cdot \tau)^b}{b!} e^{-\beta \cdot \tau}, \quad n = 1, 2, \dots \quad (29)$$

If random variable T meaning the time of the technological object remaining outside of the logistics subsystem (in the utilization subsystem) has the exponential distribution with the CDF:

$$B(t) = P(T < t) = 1 - e^{-\gamma t}, \quad t \geq 0, \quad (30)$$

then the number $L(\tau)$ meaning the number of technological objects directed to the logistics subsystem in the time interval τ has the Poisson distribution with the $\gamma \cdot \tau$ parameter, i.e.:

$$P(L(\tau) = n) = \frac{(\gamma \cdot \tau)^n}{n!} e^{-\gamma \tau}, \quad n = 0, 1, 2, \dots \quad (31)$$

then we get:

$$A^{(n)}(\tau) = 1 - \sum_{b=1}^{n-1} \frac{(\gamma \cdot \tau)^b}{b!} e^{-\gamma \tau}, \quad n = 1, 2, \dots \quad (32)$$

Then the relation (18) looks as follows:

$$\begin{aligned} W(N - k) &= W(n) = P(M(\tau) < n) = P(L(\tau) - N(\tau) < n) = \\ &= \sum_{b=0}^{\infty} \left[\frac{(\beta \cdot \tau)^b}{b!} e^{-\beta \tau} \cdot \sum_{c=0}^{n+b-1} \frac{(\gamma \cdot \tau)^c}{c!} e^{-\gamma \tau} \right] = \\ &= \sum_{b=0}^{\infty} \left[\frac{(\beta \cdot \tau)^b}{b!} e^{-\beta \tau} \cdot \sum_{c=0}^{N-k+b-1} \frac{(\gamma \cdot \tau)^c}{c!} e^{-\gamma \tau} \right]. \end{aligned} \quad (33)$$

3. Partial findings

In order to illustrate our considerations, partial results of calculations of the index of the efficiency of the logistics subsystem posts. Tab. 1 includes the values of chosen parameters defining the process of the use of the means of transport in the tested transport system: the average 24-hour numbers and time periods of the technological objects remaining at the logistics subsystem posts and at the utilization subsystem. In the tested transport system, the logistics subsystem includes two fuel retake posts, two service posts on the day of operation, one periodical technical service post, two diagnostic posts, and fifteen repair posts.

Tab. 1. The values of chosen parameters determining the process of the use of the means of transport

Subsystem/Post	Average 24-hour number of technological objects	Average period of time of the stay of the technological object
Logistics subsystem		
Fuel retake post	99	5.57 mins
Service on the day of operation post	101.1	7.57 mins
Periodical technical services post	2.37	5.06 hrs
Diagnostic post	23.77	39.53 mins
Repair post	49.9	4.63 hrs
Utilization subsystem	119	19.89 hrs

Based on the data presented in Tab. 1, the values of the index of the efficiency of the logistics subsystem posts were determined $W(N - k)$ which reflects the probability of the repair of at least k number of technological objects in the time interval t . Partial results were presented in Fig. 1 (determined for the exponential distribution).

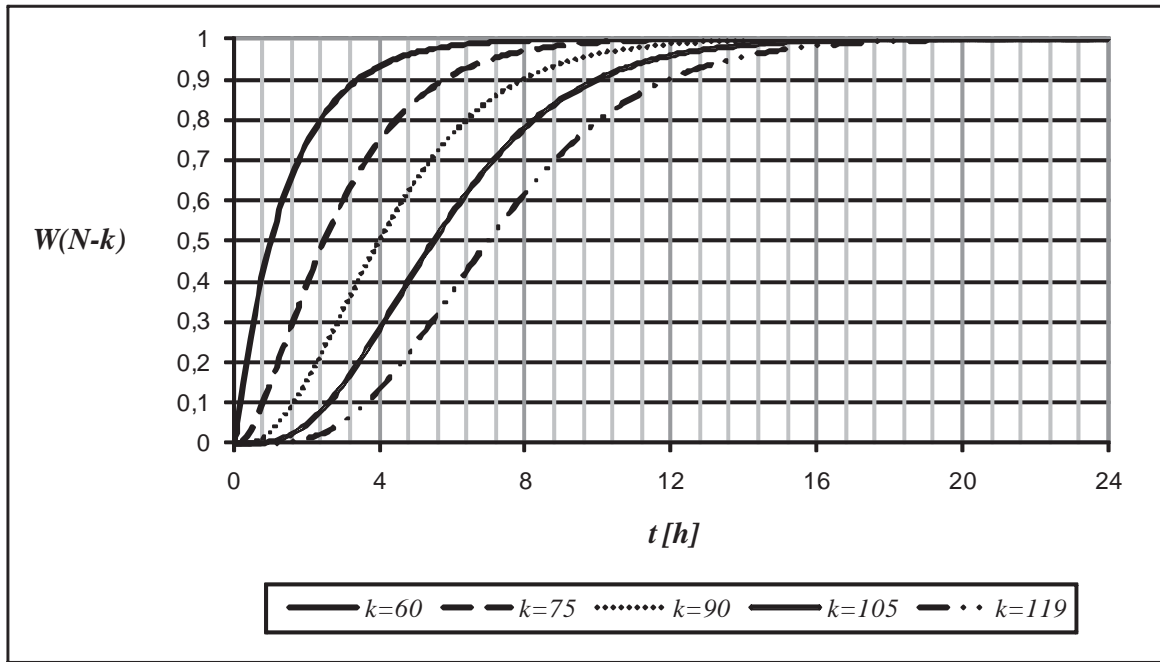


Fig. 1. Value of the index $W(N - k)$ depending on the time of repair t and the value of the number k of the repaired objects

On the basis of the presented graph one may note that while the repair time t of the given number of technological objects k rises, so does the value of the index of the efficiency of the logistics subsystem posts $W(N - k)$, e.g.:

- for $k = 90$ and $t = 4$ hrs, $W(N - k) = 0.5$,
- for $k = 90$ and $t = 8$ hrs, $W(N - k) = 0.9$,
- for $k = 90$ and $t = 14$ hrs, $W(N - k) = 1$.

On the other hand, the minimum repair time t_{min} of the given number of technological objects k rises, along with the number k , e.g.:

- for $k = 60$, $t_{min} \approx 8$ hrs,
- for $k = 75$, $t_{min} \approx 12$ hrs,
- for $k = 90$, $t_{min} \approx 14$ hrs.

4. Summary

On the basis of the models presented in the paper, the determined values of the index of the efficiency of the service and repair posts may be used for the evaluation of both the single posts and the whole logistics system for the realization of the assigned task. The evaluation of the logistics subsystem posts obtained in this way constitutes a piece of information the use of which in the process of operation control shall facilitate rational decision-making pertaining to the conforming of the logistics subsystem to the current needs of the transport system, e.g. as a result of:

- modernization of the logistics subsystem,
- exchange of the posts to more efficient ones,
- change of the number of individual post types,
- using of the universal posts where, if necessity arises, servicing, diagnostics and repair of the means of transport can be carried out,
- change in the organization and conditions of carrying out the processes at logistics subsystem posts,
- change of the number of specialists at the logistics subsystem posts, etc.

Using the developed formulas for the exponential distribution is limited to the case in which random variables defining the periods of time of the means of transport remaining at the logistics

subsystem S as well as in the utilization subsystem T has exponential distributions with β and γ parameters respectively. On the other hand, using the formulas for the Erlang distribution is an option only when the analyzed random variables are the sum of independent random variables with exponential distribution. Then, for example, the random variable T with the Erlang distribution may be presented as follows [1]:

$$T = T_1 + T_2 + \dots + T_m, \quad (34)$$

where T_i – independent random variables with exponential distribution with λ parameter.

Such case takes place when, for example, separate periods of time of repairs of the individual systems of the means of transport are analyzed.

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