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MOTION OF THE LONG-HAUL ARTICULATED TRUCK ASSEMBLY WITH ALL WHEELS STEERED ON THE ROAD CURVE

Jerzy Frankowski

Kielce University of Technology
Tysiąclecia Państwa Polskiego Av. 7,25-314 Kielce, Poland
tel.: +48 41 3424448, fax:+48 41 3424340
e- mail: jfrank@tu.kielce.pl

Abstract

Currently, the majority of cargo transported by multi-axle long and heavy-haul motor vehicles, most of which have possibility of front axle wheels are steering only.

In the case of performing the obstacle avoidance maneuver, the overtaking maneuver or the parking maneuver, they are a significant difficulty for the driver because of the width of the occupied road lane when the vehicle moves on the road curve, lane which is wider than greater is the front wheels steering angle and greater overall vehicle length.

The occurrence of the of excessive lateral tire slip phenomenon when driving on curved road track is the cause of accelerated wear of tire tread, which together with a large number of wheels would lead to higher operational costs of such vehicles.

This work using a spatial dynamic model of the vehicle consisting of a two-axle tractor and three-axle trailer with many degrees of freedom, with the possibility of all the wheels steering, moving on a road curve, is a qualitative analysis of the wheels steering problem aimed at the avoidance of slip in the pneumatic tire - road surface coactions area.

The model of coactions of pneumatic tire with the road surface, side drift included, together with accounting the phenomenon of asymmetrical loading of the vehicle chassis and drive train was adopted.

Control for all the wheels steering should lead to an overlay or the maximum approximation of the front axle wheel tracks with the rear axles wheel tracks for the of articulated long-haul truck assembly moving on the road curve.

The solution of model equations of motion for the vehicle considered will allow for the definition of the relationship between the front axle and rear axle wheels steering angles and velocity of movement of the mass centre of articulated haul truck assembly for the given set of design and operating parameters.

The designation of the vehicle track will be possible.

***Keywords**: long-haul articulated vehicle dynamic model, vehicle curvilinear motion*

1. Introduction

The multi-axis and articulated vehicles and long-haul special cargo vehicles [1, 3, 8, 11, 17], when performing the avoidance or overtaking maneuver, are often exposed to the occurrence of slip phenomenon in the tire with road surface coactions area due to the structure of their chassis. Parking maneuvers of such vehicles are significantly more difficult.

Therefore, the design solution of a steering wheels or steering axles is coming back, both in vehicles as well as in multi-axle trailers.

This work using a spatial dynamic model of the vehicle - the articulated truck unit consisting of a two-axle tractor and three-axle trailer with many degrees of freedom, is a qualitative analysis of the problem of wheels steering aimed at the avoidance of slip in the pneumatic tire - road surface coactions area.

In the literature of this issue [4-7], the flat vehicle models taking into account the hypothesis of lateral drift have been considered in dealing with issues of vehicle stability and controllability.

2. Description of the vehicle model

It was assumed that the vehicle - the articulated truck unit with steerable wheels, the model of which is shown in the figure below is a set of seventeen conventional solids: sprung masses (solids 1s and 1n), the suspension non-sprung masses (five solids), and ten pneumatic wheels (solids Ki, where i = 1-10).

It is assumed, that the tractor and the trailer centres of masses are in a longitudinal vertical symmetry planes of the solids 1s and 1n.

The non sprung suspension masses (solids 2s and 3s and solids 2n-4n) include the non-sprung masses of all elements associated with the wheels.

It is assumed that centres of masses lie on the longitudinal vertical symmetry plane of these solids at the height equal to the dynamic radius of the vehicle wheels.

The tires have elastic properties in the lateral and radial directions, dependent on tire pressure and radial load on the wheel.

Distance from the road surface to the lateral roll axis of all suspension axles is constant.

One neglects the variation of the distance of the solids 1s and 1n masses centres from the roll axis s.

These simplifying assumptions impose constraints on the possible mutual displacements of these solids and driving wheels, limiting the number of model degrees of freedom to the twenty-eight.

They are:

- two displacements of a selected point on the tractor in the parallel to the road horizontal plane - X, Y,
- two displacements of a selected point on the trailer in the parallel to the road horizontal plane - X, Y,
- the rotation of the entire tractor system relatively to the axis perpendicular to the road plane - angle θ_c
- rotation of the entire trailer system relatively to the axis perpendicular to the road plane – angle θ_n,
- rotation of solid 1s relative solids 2s and 3s in a vertical transverse plane –angle ϕ_{1s},
- rotation of solid 1n relative solids 2n, 3n and 4n in a vertical transverse plane –angle ϕ_{1n},
- rotation of solids 2s and 3s in a vertical transverse plane –angles ϕ_{ic} $i = 2 - 3$,
- rotation of solids 2n, 3n and 4n in a vertical transverse plane –angles ϕ_{in} $i = 4 - 6$,
- rotation of the wheels relative their own horizontal rotation axes - the angles ε_i $i = 1 - 10$,
- rotation of the wheels around their vertical steering axes passing through the wheel centre - the angles φ_i $i = 1 - 5$.

The rectangular clockwise Cartesian coordinate systems shown in Fig. 1 were defined:

1. OXYZ – is the absolute stationary coordinate system placed in a fixed reference system for which the Earth was assumed.
2. XY plane is the road surface plane on which the vehicle is moving and the point O is the starting point of the vehicle motion path.
3. The direction of X axis is tangential to the desired vehicle trajectory at the starting point O, and its direction designates desirable vehicle movement direction.
4. The Z axis is directed vertically upwards.

Ssxsyszs and Snxnynzn - auxiliary coordinate systems with axes always parallel to the respective axes of the OXYZ coordinate system.

As the beginning of the relevant system the points Ss or Sn were assumed, lying on the Ss or Sn transverse roll axis.

The points of Ss and Sn are the vertical projections of mass centres Cs of the tractor or Cn of the trailer (at rest) to these axes.

Location of points Ss and Sn is constant respectively of solids 1s, 2s and 3s and respectively of solids 1n, 2n, 3n and 4n.

They are rigidly connected with these solids.

Coordinate systems Ssxsyszs and Snxnynzn are in progressive motion relative stationary OXYZ coordinate system.

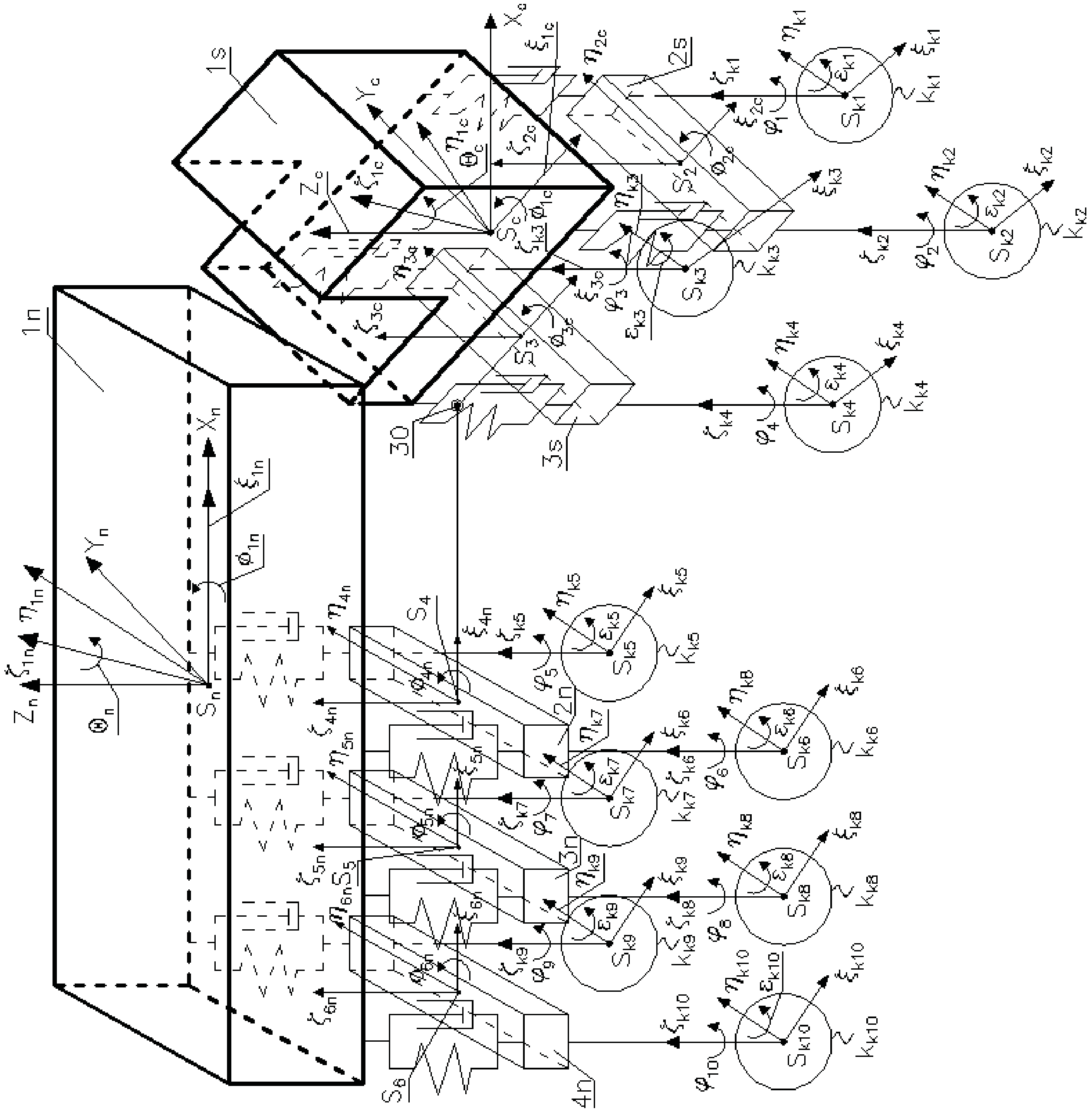

Fig. 1. The five axle vehicle model (a physical model structure)

Ss$\xi_{1c}\eta_{1c}\varsigma_{1c}$ and Sn$\xi_{1n}\eta_{1n}\varsigma_{1n}$ - moving coordinate systems rigidly connected to the solids 1s and 1n, are suspended at the points S_c and S_n, respectively.

Axes ξ_{1c}, ς_{1c} and ξ_{1n}, ς_{1n} lay in the vertical symmetry plane of the solids 1s and 1_n, and axes η_{1c} and η_{1n} are perpendicular to this plane.

The axis ξ_{1c} is the longitudinal axis of the tractor and axis ξ_{1n} is longitudinal axis of the trailer.

Systems Ss$\xi_{1c}\eta_{1c}\varsigma_{1c}$ and Sn$\xi_{1n}\eta_{1n}\varsigma_{1n}$ move in flat motion, in relation to stationary system OXYZ and in rolling motion in relation to moving coordinate systems Ssxsyszs and Snxnynzn respectively to the axes ς_{1c}, ς_{1n} and specified by the angle θ_c or θ_n and rotation about an axis ξ_{1c} or ξ_{1n} defined by an angle ϕ_{1c} or ϕ_{1n}.

$S_i\xi_{ic}\eta_{ic}\varsigma_{ic}$ i = 2, 3 and $S_i\xi_{in}\eta_{in}\varsigma_{in}$ i = 4, 5, 6 - moving coordinate systems rigidly linked to the solids 2s, 3s or solids 4n, 5n and 6n , suspended at the points Si respectively.

Axes ξ_{ic} and ξ_{in} lie in the solids vertical symmetry plane and axes η_{ic} and η_{in} are perpendicular to this plane.

Axes ξ_{ic} and ξ_{in} are the tractor and the trailer longitudinal axes.

Systems $S_i\xi_{ic}\eta_{ic}\varsigma_{ic}$ and $S_i\xi_{in}\eta_{in}\varsigma_{in}$ move in flat motion, in relation to stationary system OXYZ and in rolling motion in relation to moving coordinate systems $S_sx_sy_sz_s$ and $S_nx_ny_nz_n$ respectively to the axes ς_{ic} , ς_{in} and specified by the angle θ_s or θ_n and rotation about an axis ξ_{ic} , ξ_{in} defined by an angle ϕ_{ic} or ϕ_{in}.

$S_{Ki}\xi_{Ki}\eta_{Ki}\varsigma_{Ki}$ i = 1 - 10 - a moving coordinate system associated with the wheel solid Ki suspended at the wheel axis at a S_{Ki} point.

Axes ξ_{Ki} and ς_{Ki} lie in the vertical wheel symmetry plane, and axis η_k is perpendicular to this plane.

3. Construction of mathematical model

In the issue literature [4, 8, 11, 13, 16] one can meet various assumptions for modeling the dynamics of the automobile vehicle, depending on the complexity of the model.

Below are presented the basic assumptions for modeling the transverse motion dynamics of the articulated truck unit with all wheels steered.

The vehicle moves along a dry and smooth road surface, and there are no clearances in connections between the conventional solids.

During ride on the curved track all conventional solids perform appropriate rotations by the angles ϕ_{1c} ,ϕ_{2c} i ϕ_{3c} around the tractor roll axis s_c, and by the angles $\phi_{1n},\phi_{4n},\phi_{5n}$ and ϕ_{6n} around the tractor roll axis Sn.

This assumption is particularly essential in relation to trucks.

In addition, each solid of the vehicle performs a rotation around the vertical axis perpendicular to the road surface by the angle θ_s and θ_n and moves toward the tractor and trailer longitudinal axes as if they were rigidly connected with each other.

It is assumed that in the case of linear motion the tractor and trailer roll axes are collinear.

The aerodynamic forces and moments acting on the vehicle sprung mass are not taken into account.

The elastic and damping properties of the vehicle suspension are taken into account.

The vehicle model takes into account the rotation of the wheel around central main axes of inertia (the wheels angles of rotation ε_i and wheels steering angles φ_i).

Whichever model of coactions between the tire and the road surface can be used [2, 8, 11, 12], with assumption that the constant contact of pneumatic wheels with the road surface is kept, so there are no wheels slipping.

Forcing the direction of vehicle trajectory is caused by changing the steering wheel angle of rotation, and all the forces acting on the vehicle are concentrated.

All vehicle wheels are the steered wheels.

The wheels of the second axle are the driving wheels.

Reactions of the road acting on the wheels: longitudinal, lateral and vertical are concentrated in one point of intersection of the vertical axis of wheel steering with the road plane.

Schematics of articulated haul truck with indications of geometrical dimensions and kinematic values are shown of the Fig. 2.

The change of kinematics of the articulated vehicle occurs depending on the assumed velocity Vc of the tractor centre of mass.

The vehicle is laden with lateral forces Pi between the road surface and tires, and lateral drift angles δ_i assume the small values.

The Fig. 2 shows a setting of vehicle wheels in the curvilinear motion.

The wheels of the first and the second axle and the wheels of the third, fourth and fifth axles are steered in opposite directions.

Thanks to this, the extensions of the wheels axes of rotation can intersect at one point, known as the vehicle instantaneous centre of rotation.

Thanks to this the vehicle wheels can roll with the small lateral slip on the road curve, therefore, the tire wear will be small, a very important parameter for a large number of wheels, and tire durability will be great [13, 14].

Designations posted on the drawings and equations respectively define:

a_1,a_2,a_3,a_4 ,b_1,b_2 ,c_1,c_2, e, - geometrical dimensions of the vehicle model,

h_1 - h_5 - the heights of the respective solids masses centres relative of rolling axis s,

$S_c, S_n S_2$ - S_6 - respective vertical projections of the tractor and trailer solids centres of masses and projections of the respective non-sprung solids centres of masses on the roll axes S_n and S_c,

θ_s and θ_n - angles between the tractor and the trailer longitudinal axis and axes x_c and x_n of coordinate systems $x_s y_s z_s$ and $x_n y_n z_n$,

λ - the angle between the tractor axis and the trailer axis,

$\delta_c, \delta_n, \delta_{20}$ - δ_{60}, δ_i - the average drift angle of the tractor and trailer axles, points $(20 - 60)$ lying on the tractor and trailer axles and on the centres of respective vehicle wheels,

$\vec{R}_i, \vec{V}_i$ - vectors of points positions and velocity,

φ_i - wheels steering angles $i = 1 - 5$,

ε_{Ki} - wheels rotation angles,

V_c, V_n - velocities of the tractor and trailer centres of masses,

$V s_i$ - velocities of vehicle wheels centres,

P_i - the road surface lateral forces effect on the vehicle,

W_1, W_2, and W_5 - W_{10}, - the vehicle movement resistance forces, W_3, and W_4, - driving forces,

Z_1 - Z_{10} - respective wheel vertical load,

J_{si} - the moment of inertia of the i_{th} solid respective rolling axis s,

J_{bi} - the moment of inertia of the i_{th} solid respective axis η_i,

J_i - the moment of inertia of the i_{th} solid respective axis z,

J_{d1} - deviation moment of inertia of solid,

m_i - mass of the i_{th} solid,

m_{Ki} - mass of the i_{th} wheel,

J_K - moment of inertia of the wheel with regard to its rotation axis,

J_{Ka} - moment of inertia of the wheel with regard to ζ axis,

$\dot{\varphi}$ - angular velocity of the wheel steering movement.

In this model, it is assumed that centres of masses of solids 2s, 3s and solids 2n, 3n and 4n lie in the planes $\eta_{ic}\varsigma_{ic}$ or $\eta_{in}\varsigma_{in}$.

It is understood that the angles ϕ_{ic}, ϕ_{in} are small, this also applies to the first derivatives of these angles.

Disorders in the form of wind blowing in a direction perpendicular to the longitudinal axis of the vehicle did not occur.

Therefore, disrupting forces and moments are equal to zero.

We also assume small values of the h_i - the centres of masses heights relative to the rolling axis Sc, Sn (see Fig. 3).

Mass of the transaxle, the tractor and trailer axles (m2c, m3c, m2n ,m3n and m4n) and masses of the wheels are much smaller than the whole vehicle mass.

At the same time they are included in the entire vehicle mass.

In the issue literature, most authors treat deviation moments of inertia of the solids $2-5$ as the higher order small values [16].

It is also assumed, that the road surface lateral forces effect on the vehicle individual wheels comply with the equality: P1=P2 , P3=P4 , P5=P6 , P7=P8 , P9=P10.

The same value of transverse rigidity of front and rear pneumatic wheels and the same side drift angles for the left and right wheels was assumed.

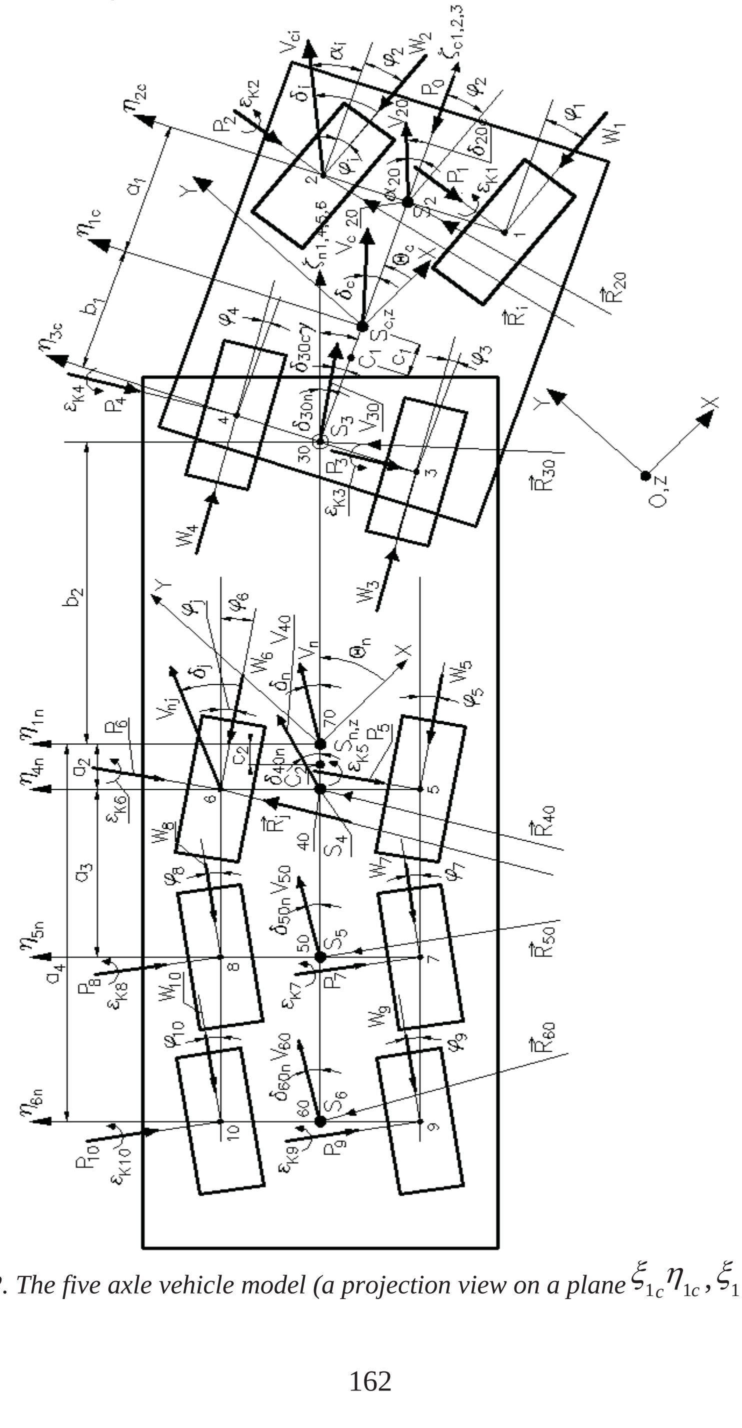

Fig. 2. The five axle vehicle model (a projection view on a plane $\xi_{1c}\eta_{1c}, \xi_{1n}\eta_{1n}$)

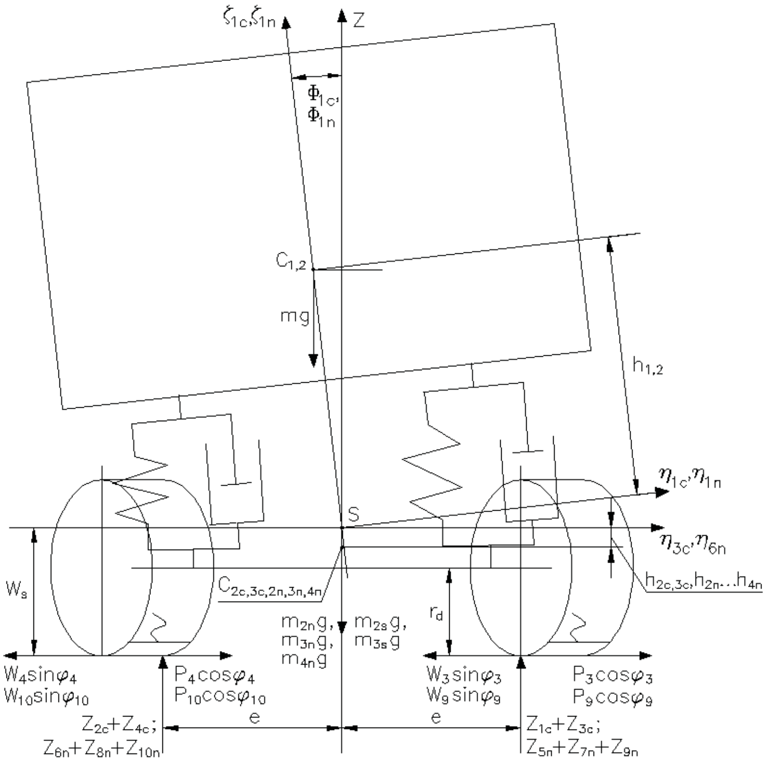

Fig. 3. The five axle vehicle model (a projection view on a plane $\eta_{3c}\varsigma_{3c}$ *or* $\eta_{6n}\varsigma_{6n}$ *)*

The vehicle equations of motion derived using Lagrange equation of the second kind have the following form:

For the tractor:

$$m_c \cdot \dot{V}_{cx} - m_{1c} \cdot c_1 \cdot (\ddot{\theta}_c \cdot \sin\theta_c - \dot{\theta}_c^{\,2} \cdot \cos\theta_c) = 2 \cdot (-P_1 \cdot \varphi_1 + P_3 \cdot \varphi_2 + \\ -W_1 + W_3) \cdot \cos\theta_c + 2 \cdot (-P_1 - P_3 + W_1 \cdot \varphi_1 + W_3 \cdot \varphi_2) \cdot \sin\theta_c, \tag{1}$$

$$m_c \cdot \dot{V}_{cy} + m_{1c} \cdot [c_1 \cdot (\ddot{\theta}_c \cdot \cos\theta_c - \dot{\theta}_c^{\,2} \cdot \sin\theta_c - \ddot{\phi}_{1c} \cdot \phi_{1c} \cdot \sin\theta_c - \dot{\phi}_{1c}^{\,2} \cdot \sin\theta_c + \\ -\dot{\phi}_{1c} \cdot \dot{\theta}_c \cdot \phi_{1c} \cdot \cos\theta_c) + h_1 \cdot (-\ddot{\phi}_{1c} + \dot{\phi}_{1c}^{\,2} \cdot \phi_{1c})] + m_{1c} \cdot h_1 \cdot (-\ddot{\phi}_{1c} + \dot{\phi}_{1c}^{\,2} \cdot \phi_{1c}) = \\ 2 \cdot (-P_1 \cdot \varphi_1 + P_3 \cdot \varphi_2 - W_1 + W_3) \cdot \sin\theta_c + 2 \cdot (-P_1 - P_3 + W_1 \cdot \varphi_1 + W_3 \cdot \varphi_2) \cdot \cos\theta_c, \tag{2}$$

$$m_{1c} \cdot [(-\dot{V}_{cy} \cdot \phi_{1c} \cdot \sin\theta_c - V_{cy} \cdot \dot{\phi}_{1c} \cdot \sin\theta_c - V_{cy} \cdot \dot{\theta}_c \cdot \phi_{1c} \cdot \cos\theta_c) \cdot c_1 + \\ + (-\dot{V}_{cy} + V_{cy} \cdot \dot{\phi}_{1c} \cdot \phi_{1c}) \cdot h_1] + J_{s1c} \cdot (\ddot{\phi}_{1c} \cdot \cos^2\theta_c - \dot{\phi}_{1c} \cdot \dot{\theta}_c \cdot \sin 2 \cdot \theta_c) + \\ + J_{b1c} \cdot (\ddot{\phi}_{1c} \cdot \sin^2\theta_c + \dot{\phi}_{1c} \cdot \dot{\theta}_c \cdot \sin 2 \cdot \theta_c) - m_{1c} \cdot V_{cy} \cdot [-(\dot{\theta}_c \cdot \phi_{1c} \cdot \cos\theta_c + \dot{\phi}_{1c} \cdot \sin\theta_c) \cdot c_1 + \\ + \dot{\phi}_{1c} \cdot \phi_{1c} \cdot h_1] + 2 \cdot [c_{z2c} \cdot (\dot{\phi}_{1c} - \dot{\phi}_{2c}) + c_{z3c} \cdot (\dot{\phi}_{1c} - \dot{\phi}_{3c})] \cdot e^2 + \\ + 2 \cdot [k_{z2c} \cdot (\phi_{1c} - \phi_{2c}) + k_{z3c} \cdot (\phi_{1c} - \phi_{3c})] \cdot e^2 = m_{1c} \cdot h_{1c} \cdot (\phi_1 \cdot g_c + \dot{\theta}_c^{\,2} \cdot r_c'), \tag{3}$$

$$m_{1c} \cdot c_1 \cdot (-\dot{V}_{cx} \cdot \sin\theta_c + \dot{V}_{cy} \cdot \cos\theta_c) + \sum_{i=1}^{5} J_i \cdot \ddot{\theta}_c - 0.5 \cdot \sum_{i=1}^{5} J_{si} \cdot \dot{\phi}_{ic}^{\,2} \cdot \sin 2\theta_c + \\ + 0.5 \cdot \sum_{i=1}^{5} J_{bi} \cdot \dot{\phi}_{ic}^{\,2} \cdot \sin 2\theta_c + J_{d1} \cdot \dot{\phi}_{1c} \cdot \dot{\theta}_c \cdot \theta_c = 2 \cdot [(-P_1 + W_1 \cdot \varphi_1) \cdot a_1 + \\ - P_1 \cdot \varphi_1 \cdot e + (P_3 - W_3 \cdot \varphi_2) \cdot b_1] \,. \tag{4}$$

For the trailer:

$$m_n \cdot \dot{V}_{nx} - m_{1n} \cdot c_{1n} \cdot (\ddot{\theta}_n \cdot \sin\theta_n - \dot{\theta}_n^{\,2} \cdot \cos\theta_n) = 2 \cdot (-P_5 \cdot \varphi_3 + P_7 \cdot \varphi_4 + P_9 \cdot \varphi_5 - W_5 - W_7 - W_9) \cdot \cos\theta_n + 2 \cdot (-P_5 + -P_7 - P_9 + W_5 \cdot \varphi_3 - W_7 \cdot \varphi_4 - W_9 \cdot \varphi_5) \cdot \sin\theta_n , \tag{5}$$

$$m_n \cdot \dot{V}_{ny} + m_{1n} \cdot [c_2 \cdot (\ddot{\theta}_n \cdot \cos\theta_n - \dot{\theta}_n^{\,2} \cdot \sin\theta_n - \ddot{\phi}_{1n} \cdot \phi_{1n} \cdot \sin\theta_n - \dot{\phi}_{1n}^{\,2} \cdot \sin\theta_n - \dot{\phi}_{1n} \cdot \dot{\theta}_n \cdot \phi_{1n} \cdot \cos\theta_n) + h_{1n} \cdot (-\ddot{\phi}_{1n} + \dot{\phi}_{1n}^{\,2} \cdot \phi_{1n})] + m_{1n} \cdot h_{1n} \cdot (-\ddot{\phi}_{1n} + \dot{\phi}_{1n}^{\,2} \cdot \phi_{1n}) = 2 \cdot (-P_5 \cdot \varphi_3 + P_7 \cdot \varphi_4 + P_9 \cdot \varphi_5 - W_5 - W_7 - W_9) \cdot \sin\theta_n + 2 \cdot (-P_5 - P_7 - P_9 + W_5 \cdot \varphi_3 - W_7 \cdot \varphi_4 - W_9 \cdot \varphi_5) \cdot \cos\theta_n , \tag{6}$$

$$m_{1n} \cdot [(-V_{ny} \cdot \dot{\phi}_{1n} \cdot \sin\theta_n - V_{ny} \cdot \dot{\phi}_{1n} \cdot \sin\theta_n - V_{ny} \cdot \dot{\theta}_n \cdot \phi_{1n} \cdot \cos\theta_n) \cdot c_2 + (-\dot{V}_{ny} + V_{ny} \cdot \dot{\phi}_{1n} \cdot \phi_{1n}) \cdot h_{1n}] + J_{s1n} \cdot (\ddot{\phi}_{1n} \cdot \cos^2\theta_n - \dot{\phi}_{1n} \cdot \dot{\theta}_n \cdot \sin 2 \cdot \theta_n) + J_{b1n} \cdot (\ddot{\phi}_{1n} \cdot \sin^2\theta_n + \dot{\phi}_{1n} \cdot \dot{\theta}_n \cdot \sin 2 \cdot \theta_n) - m_{1n} \cdot V_{ny} \cdot [-(\dot{\theta}_n \cdot \phi_{1n} \cdot \cos\theta_n + \dot{\phi}_{1n} \cdot \sin\theta_n) \cdot c_2 + + \dot{\phi}_{1n} \cdot \phi_{1n} \cdot h_{1n}] + 2 \cdot [c_{z4n} \cdot (\dot{\phi}_{1n} - \dot{\phi}_{4n}) + c_{z5n} \cdot (\dot{\phi}_{1n} - \dot{\phi}_{5n}) + c_{z6n} \cdot (\dot{\phi}_{1n} - \dot{\phi}_{6n})] \cdot e^2 + 2 \cdot [k_{z4n} \cdot (\phi_{1n} - \phi_{4n}) + k_{z5n} \cdot (\phi_{1n} - \phi_{5n}) + k_{z6n} \cdot (\phi_{1n} - \phi_{6n})] \cdot e^2 = m_{1n} \cdot h_{1n} \cdot (\phi_{1n} \cdot g_n + \dot{\theta}_n^{\,2} \cdot r_n'), \tag{7}$$

$$m_{1n} \cdot c_2 \cdot (-\dot{V}_{nx} \cdot \sin\theta_n + \dot{V}_{ny} \cdot \cos\theta_n) + \sum_{i=1}^{5} J_{in} \cdot \ddot{\theta}_n - 0.5 \cdot \sum_{i=1}^{5} J_{\sin} \cdot \dot{\phi}_{in}^{\,2} \cdot \sin 2\theta_n + 0.5 \cdot \sum_{i=1}^{5} J_{bin} \cdot \dot{\phi}_{in}^{\,2} \cdot \sin 2\theta_n + J_{d1n} \cdot \dot{\phi}_{1n} \cdot \dot{\theta}_n \cdot \theta_n = 2 \cdot [-P_5 \cdot \varphi_3 \cdot a_2 + (P_5 - W_5 \cdot \varphi_3) \cdot (a_2 + a_3) + (P_7 - W_7 \cdot \varphi_4) \cdot a_4]. \tag{8}$$

Two equations for the solids 2_s and 3_s:

$$J_{si} \cdot (\ddot{\phi}_{ic} \cdot \cos^2\theta_c - \dot{\phi}_{ic} \cdot \dot{\theta}_c \cdot \sin 2 \cdot \theta_c) + J_{bi} \cdot (\ddot{\phi}_{ic} \cdot \sin^2\theta_{sc} + \dot{\phi}_{ic} \cdot \dot{\theta}_c \cdot \sin 2 \cdot \theta_c) = = 2 \cdot c_{zi} \cdot (\dot{\phi}_{1c} - \dot{\phi}_{1c}) \cdot e^2 + 2 \cdot k_{zi} \cdot (\phi_{ic} - \phi_{1c}) \cdot e^2 + 2 \cdot k_{ki} \cdot \phi_{ic} , \tag{9}$$

where i = 2. 3.

Three equations for the solids 2_n ,3_n i 4_n :

$$J_{si} \cdot (\ddot{\phi}_{in} \cdot \cos^2\theta_n - \dot{\phi}_{in} \cdot \dot{\theta}_n \cdot \sin 2 \cdot \theta_n) + J_{bi} \cdot (\ddot{\phi}_{in} \cdot \sin^2\theta_n + \dot{\phi}_{in} \cdot \dot{\theta}_n \cdot \sin 2 \cdot \theta_n) = = 2 \cdot c_{zi} \cdot (\dot{\phi}_{in} - \dot{\phi}_{1n}) \cdot e^2 + 2 \cdot k_{zi} \cdot (\phi_{in} - \phi_{1n}) \cdot e^2 + 2 \cdot k_{ki} \cdot \phi_{in} , \tag{10}$$

where i = 4, 5, 6.

Using the condition of an articulated tractor and trailer connection in section 30 (X_{30}, Y_{30}), one can write an equation for the velocity of the trailer mass centre V_n and lateral drift angle of the trailer centre of mass δ_n, which take the form:

$$V_n = \left[V_c \cdot \cos\gamma + \left(V_c \cdot \delta_c + b \cdot \frac{d\theta_c}{dt} \right) \cdot \sin\gamma \right] / \cos\delta_n ,$$
$$\delta_n = \left[-V_c \cdot \sin\gamma + \left(V_c \cdot \delta_c + b \cdot \frac{d\theta_c}{dt} \right) \cdot \cos\gamma + a_1 \cdot \frac{d\theta_c}{dt} \right] / V_n . \tag{11}$$

The angle γ between the tractor longitudinal axis and the trailer axis can be calculated from the expression:

$$\gamma = \theta_c - \theta_n . \tag{12}$$

Using a simplified wheel theory [2, 5, 8, 13] the forces and moments acting on the wheel can be determined.

The equations of motion for the wheels of the first, third, fourth and fifth axles (these wheels are non-powered) have the form:

$$J_K \cdot \ddot{\varepsilon}_i = r_d \cdot (F_i - Z_i \cdot f - F_{bK} - F_{skr}) \text{ where: } i = 1,2 \text{ or } i = 5 \text{ - } 10 . \tag{13}$$

For the wheels of the second axle, or the driven external and internal wheels the equations of motion take the form:

$$J_K \cdot \ddot{\varepsilon}_i = 0.5 \cdot M_n \pm 0.5 \cdot m_{tr} \cdot \operatorname{sgn} \dot{\theta} - r_d \cdot (F_i - Z_i \cdot f - F_{skr}) \tag{14}$$

for i = 3 (- sign), and for i = 4 (+sign).

The equations of motion for the steering wheel on the assumption, that the steering angles of the respective axles are the same, are as follows:

$$J_K \cdot \ddot{\varphi}_i = \sum_{i=1}^{5} M_{iskr} , \tag{15}$$

where designated:

r_d - the dynamic wheel radius,

F_i - a horizontal force forcing the movement of ongoing wheel, or countering the movement of the driven wheel,

Z_i - the instantaneous vertical wheel load,

f - coefficient of wheel rolling resistance,

F_{skr}- resistance force of the wheel steering movement,

F_{bK}-force of inertia of the wheels in progressive motion,

m_{tr} - torque of friction forces,

M_{skr} - steering wheel torque.

Assuming small values of the lateral drift angles [5] and the same values of steering angles φ_1 for the front axle wheels, and φ_2, φ_3, φ_4 and φ_5 for the steering angles of vehicle wheels of following axles respectively, one gets dependences for angles of the lateral drift for respective wheel axles:

$$\begin{aligned}
&\delta_1 = \varphi_1 + \delta_c - \frac{a_1}{\upsilon_c} \cdot \frac{d\theta_c}{dt} , \qquad \delta_2 = -\varphi_2 + \delta_c + \frac{b_1}{\upsilon_c} \cdot \frac{d\theta_c}{dt} , \\
&\delta_3 = -\varphi_3 + \delta_n + \frac{a_2}{\upsilon_n} \cdot \frac{d\theta_n}{dt} , \quad \delta_4 = -\varphi_4 + \delta_n + \frac{(a_2 + a_3)}{\upsilon_n} \cdot \frac{d\theta_n}{dt} , \\
&\delta_5 = -\varphi_5 + \delta_n + \frac{a_4}{\upsilon_n} \cdot \frac{d\theta_n}{dt} .
\end{aligned} \tag{16}$$

The solution of systems of equations of motion allows to designate the functions $\theta_c(t), \delta_c(\mathrm{t}), \theta_n(t), \delta_n(\mathrm{t})$ necessary to determine the coordinates of points 20 (X_{20} ,Y_{20}), 30 (X_{30} ,Y_{30}), 40 (X_{40} ,Y_{40}) and 50 (X_{50} ,Y_{50}) and then to designate the coordinates of each wheel centre in the XY coordinate system (see Fig. 1).

The components of velocity vectors $\vec{V}_{20}(V_{20X}, V_{20Y}), \vec{V}_{30}(V_{30X}, V_{30Y}), \vec{V}_{40}(V_{40X}, V_{40Y}), \vec{V}_{50}(V_{50X}, V_{50Y})$ hooked in centres of wheel axles designated as 20 (X_{20} ,Y_{20}), 30 (X_{30} ,Y_{30}) , 40 (X_{40} ,Y_{40}),50 (X_{50} ,Y_{50}) (as Fig. 2) can be written in the form of a matrix:

$$\begin{Bmatrix} V_{20X} \\ V_{20Y} \end{Bmatrix}_C = \begin{Bmatrix} V_c \\ \left(V_c \cdot \delta_c - a_1 \cdot \dfrac{d\theta_c}{dt}\right) \end{Bmatrix} \cdot A, \quad \begin{Bmatrix} V_{30X} \\ V_{30Y} \end{Bmatrix}_C = \begin{Bmatrix} V_c \\ \left(V_c \cdot \delta_c + b_1 \cdot \dfrac{d\theta_c}{dt}\right) \end{Bmatrix} \cdot A,$$

$$\begin{Bmatrix} V_{30X} \\ V_{30Y} \end{Bmatrix}_N = \begin{Bmatrix} V_n \\ \left(V_n \cdot \delta_n - b_2 \cdot \dfrac{d\theta_n}{dt}\right) \end{Bmatrix} \cdot B, \quad \begin{Bmatrix} V_{40X} \\ V_{40Y} \end{Bmatrix}_N = \begin{Bmatrix} V_n \\ \left(V_n \cdot \delta_n + a_2 \cdot \dfrac{d\theta_n}{dt}\right) \end{Bmatrix} \cdot B, \tag{17}$$

$$\begin{Bmatrix} V_{50X} \\ V_{50Y} \end{Bmatrix}_N = \begin{Bmatrix} V_n \\ \left(V_n \cdot \delta_n + (a_2 + a_3) \cdot \dfrac{d\theta_n}{dt}\right) \end{Bmatrix} \cdot B, \quad \begin{Bmatrix} V_{60X} \\ V_{60Y} \end{Bmatrix}_N = \begin{Bmatrix} V_n \\ \left(V_n \cdot \delta_n + a_4 \cdot \dfrac{d\theta_n}{dt}\right) \end{Bmatrix} \cdot B.$$

Indices C and N in the matrices denote the tractor and trailer, respectively.
Where A and B are matrices of the form:

$$A = \begin{Bmatrix} \sin\theta_c & (-\cos\theta_c) \\ \cos\theta_c & \sin\theta_c \end{Bmatrix}, \qquad B = \begin{Bmatrix} \sin\theta_n & (-\cos\theta_n) \\ \cos\theta_n & \sin\theta_n \end{Bmatrix}. \tag{18}$$

In further part it is assumed, that the wheel steering angles of the first φ_1 and fourth φ_4 axles are equal.

For this assumption from the condition of perpendicularity of vectors $\vec{V}_{20}$ and $\vec{R}_{20}$ and $\vec{V}_{50}$ and $\vec{R}_{50}$, one can specify the location of the vehicle instantaneous centre of rotation designated as O (X_0, Y_0) from the scalar product of equations:

$$\vec{V}_{20} \cdot \vec{R}_{20} = 0, \qquad \vec{V}_{50} \cdot \vec{R}_{50} = 0. \tag{19}$$

The coordinates of the vehicle instantaneous centre of rotation are determined by the expression:

$$X_0 = \frac{-X_{20} \cdot A1 + X_{50} \cdot B1 + Y_{50} - Y_{20}}{B1 - A1}, \qquad Y_0 = X_{20} \cdot A1 - X_0 \cdot A1 + Y_{20}, \tag{20}$$

where $A1$ and $B1$ determine dependences:

$$\frac{V_{20X}}{V_{20Y}} = A1, \qquad \frac{V_{50X}}{V_{50Y}} = B1. \tag{21}$$

For the third and fourth axle one can write the equations:

$$\vec{V}_{30} \cdot \vec{R}_{30} = 0, \qquad \vec{V}_{40} \cdot \vec{R}_{40} = 0. \tag{22}$$

Therefore, we obtain the expressions:

$$V_{30x} = -\frac{(Y_0 - Y_{30})}{(X_0 - X_{30})} \cdot V_{30y}, \qquad V_{40x} = -\frac{(Y_0 - Y_{40})}{(X_0 - X_{40})} \cdot V_{40y}. \tag{23}$$

Coordinates of points: 30 (X_{30}, Y_{30}) -centre of the third axle and 40 (X_{40}, Y_{40}) -centre of the fourth axle can be determined from the geometrical dimensions of the vehicle (see Fig. 2).
From conditions of equal projections of velocity vectors on the tractor and trailer longitudinal axis of symmetry the equations follows:

$$\begin{aligned} |\vec{V}_{20}| \cdot \cos(\delta_{20} - \varphi_1) &= |\vec{V}_{30}| \cdot \cos(\delta_{30} - \varphi_2), \\ |\vec{V}_{40}| \cdot \cos(\delta_{40} + \varphi_3) &= |\vec{V}_{50}| \cdot \cos(\delta_{50} + \varphi_4). \end{aligned} \tag{24}$$

Taking into account the above equations the coordinates for centres of the fourth and fifth axle - 40 (X_{40}, Y_{40}), 50 (X_{50}, Y_{50}) are determined, then one can specify the components of velocity vectors: $\vec{V}_{40}\left(V_{40X}, V_{40Y}\right), \vec{V}_{50}\left(V_{50X}, V_{50Y}\right)$

After the transformations the steering angles of the fourth and fifth axles φ_4 and φ_5 are designated for the assumed condition of wheel path tracking of the first and fifth axles of articulated vehicle.

The length of the vehicle is greater, them the differences between successive wheel steering angles are bigger.

The distance between the wheel paths of individual axles may be determined from the equation:

$$\begin{aligned} W1 &= \left|\vec{R}_{20}\right| - \left|\vec{R}_{30}\right|, \quad W2 = \left|\vec{R}_{20}\right| - \left|\vec{R}_{40}\right|, \\ W3 &= \left|\vec{R}_{20}\right| - \left|\vec{R}_{50}\right|, \quad W4 = \left|\vec{R}_{20}\right| - \left|\vec{R}_{60}\right|. \end{aligned} \tag{25}$$

The coordinates of the wheels axis centres in the XY coordinate system determine the following expressions:
for the tractor:

$$\begin{aligned} X_i &= \int_0^t [V_c \cdot \sin\theta_c - (V_c \cdot \delta_c - D \cdot \frac{d\theta_c}{dt}) \cdot \cos\theta_c]dt, \\ Y_i &= \int_0^t [V_c \cdot \cos\theta_c + (V_c \cdot \delta_c - D \cdot \frac{d\theta_c}{dt}) \cdot \sin\theta_c]dt, \end{aligned} \tag{26}$$

where index *i* assumes 20 or 30 respectively and designation D value equal to a_1 or ($-b_1$).

For the trailer:

$$\begin{aligned} X_j &= \int_0^t [V_n \cdot \sin\theta_n - (V_n \cdot \delta_n + F \cdot \frac{d\theta_n}{dt}) \cdot \cos\theta_n]dt, \\ Y_j &= \int_0^t [V_n \cdot \cos\theta_n + (V_n \cdot \delta_n + F \cdot \frac{d\theta_n}{dt}) \cdot \sin\theta_n]dt, \end{aligned} \tag{27}$$

where the index *j* assumes 40, 50 or 60 respectively and designation F value equal to b_2, (a_2+a_3) or a_4.
The coordinates of each wheel centre in XY coordinate system determine the terms:

For the tractor:

$$X_k = X_j \mp e \cdot \cos\theta_c \qquad Y_k = Y_j \pm e \cdot \sin\theta_c, \tag{28}$$

where *k*=1, 2, 3, 4, a j=20, 30.

For the trailer:

$$X_k = X_j \pm e \cdot \cos\theta_n \qquad Y_k = Y_j \mp e \cdot \sin\theta_n, \tag{29}$$

where *k*=5-10, a j=40, 50, 60.

4. Summary

The presented model of vehicle with the steered wheels allows for assessment of the impact of selected design parameters, operating parameters, and the wheels steering angles on the vehicle track path.

It is possible to determine the steering angle of rear axle wheels for the assumed value of wheel control angle steered by driver and the driving speed.

The rear axle wheel steering reduces radius of curvature of the vehicle track.

Subsequently, the models of multi axle and multi-articulated vehicles with a greater number of degrees of freedom [3, 10, 13, 14] will be considered next, models with the steered wheels using pneumatic tyre coactions to road surface hypothesis taking into account non-symmetrical vehicle loading by vertical forces during movement on the road curve.

The system of non-slip wheel steering using offset control will be proposed.

In the multi-axle and long-haul trailers the axles with steered wheels are increasingly used.

One can distinguish between self-aligning steering mechanisms (self steering) and the steered wheels controlled from the tractor fifth wheel or tractor drawbar.

Wheel steering angle in the subsequent running axles is set via the system of rods or hydraulic system.

Lateral tire slippage is the cause of accelerated tires wear for non-steered wheels on semi-trailers and long-haul trailers [14].

The wheels steering in the long vehicles have a significant impact on the width of the occupied road lane during a turn, the size of the between walls or between the curbs turning radius as well as the inner turning radius [3, 13, 14].

Subsequently, computer calculations taking into account the impact of design and operating parameters of articulated lorry/truck unit will be carried out.

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