# THE METHOD OF EVALUATION OF THE AVAILABILITY OF THE TRANSPORT SYSTEM FOR THE REALIZATION OF THE ASSIGNED TRANSPORT TASK

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***Abstract***

*The article presents the method of evaluation of the availability of the utilization subsystem in a transport system. The method includes the defining and comparison of the values of the availability of the utilization subsystem to the value of the required availability. The article also deals with the way of defining the minimum availability of an individual technological object (means of transport) as well as the minimum number of technological objects used in the system, in order for the obtained value of the availability of the utilization subsystem to exceed the value of the required availability determined by the parameters of transport task. The values of the availability of the utilization subsystem are defined according to the availability of an individual technological object, the number of objects used in the system as well as the structure by which the individual technological objects are linked. The availability of the individual technological object was defined on the basis of a constructed mathematical model of the utilization process realized in the tested transport system. Then, for the utilization data obtained during tests conducted in an existing transport system, the values of the analyzed characteristics were defined. The presented results are the effect of research conducted as part of a larger research project pertaining to the creation of a decision-making model of controlling the availability of a transport system.*

***Keywords:*** *transport system, technological availability, threshold structure, semi-Markovian model*

## 1. Introduction

The basic objective of the operation of transport systems is fulfilling the transportation needs as a result of the carrying out of transport on particular routes. The direct carrying out of the transport task is performed by elementary subsystems such as operator – means of transport, constituting the utilization subsystem. One of the factors strongly influencing the possibility of correct carrying out of the transport task is the availability of the utilization subsystem to carry out the task.

The technological object (element or system) availability is the object's feature which is characteristic from the point of view of the possibility of timely obtaining or maintaining the state of efficiency (facilitating the realization of goals). The main factors influencing the availability of the utilization subsystem are:
- availability of the technological objects (means of transport) used in the system,
- the number of technological objects used in the system,
- the structure linking the technological objects,
- technological objects' reliability and susceptibility to service and repair,
- efficiency and availability of the worthiness assurance subsystem.

This paper presents the way of evaluating the availability of utilization subsystem as well as the method of defining the required availability of technological objects and the required number of objects used in a transport system in order to assure the appropriate realization of the assigned transport task in the case of technological objects being linked by a threshold structure.

## 2. Availability of the utilization subsystem in a transport system

In complex systems of the use of means of transport, which include municipal bus transport systems, apart from basic technological objects designed for the carrying out of transport tasks there are also additional technological objects, the so called substitute objects. The task of a substitute object is replacing the basic object (continuing the task assigned to the basic object) in case of the carrying out of the task by the basic object becoming impossible, i.e. as a result of damage. When substitute objects are used, the structure which links the technological objects is a threshold structure of the $k$ of $N$ type. A feature of threshold structure systems is the possibility of appropriate realization of an assigned task even when a given number of technological objects used in the system are not available for the carrying out of the task (nonoperational and/or unsupplied objects). The acceptable number of unavailable technological objects in the system may not be higher than $n = N - k_z$, where $N$ is the number of all technological objects used I the system and $k_z$ is the required minimum number of available objects necessary for appropriate realization of the assigned transport task $z = 1, 2, \ldots$ .

The availability of the utilization subsystem with threshold structure including $N$ technological objects, for which the required minimum number of objects available for the realization of the assigned transport task is equal to $k_z$, is described by the following relationship

$$G^{PW} = \sum_{i=k_z}^{N} \binom{N}{i} \cdot \left(G^{OT}\right)^{i} \cdot \left(1 - G^{OT}\right)^{N-i}, \tag{1}$$

where:
$G^{OT}$ - availability of a single technological object (means of transport).

Availability of means of transport $G^{OT}$ may be determined based on the mathematical process of use carried out in the tested transport system (e.g. the Markov model or semi-Markovian model). Then the availability of a single technological object is determined as a sum of border probabilities $p_i^*$ of remaining at operational states $S_i$, $i = 1, 2, \ldots$ belonging to the set of availability states $S_G$

$$G^{OT} = \sum_{i} p_i^*, \quad for \quad S_i \in S_G, \quad i = 1,2,\ldots . \tag{2}$$

Thus, in the case of threshold structure system it is possible to shape its availability by choosing the appropriate number of technological objects used in the system as well as their availability.

## 3. The criteria of evaluating the availability of utilization subsystem

In the presented method the evaluation of the utilization subsystem for the realization of the assigned transport task $z = 1, 2, \ldots$, is done on the basis of the comparison of its value and the value of the required availability which the utilization subsystem should have for the assigned task to be carried out correctly. The required availability of the utilization subsystem for the realization of the assigned transport task is determined depending on the required minimum number of available technological objects $k_z$, defined in the description of the assigned transport task as well as the number of all technological objects used in the transport system $N$, according to the following formula

$$G_z^{PW} = \frac{k_z}{N}. \tag{3}$$

In order to assure the correct realization of the assigned transport task it is necessary for the value of the availability of the utilization subsystem $G^{PW}$ to be at least equal to the value of required availability $G_z^{PW}$ .

Taking into consideration the fact that the required minimum number of objects available for the particular transport task is constant ($k_z = const.$), the availability of the utilization subsystem depends on the number of technological objects used in the system as well as the availability of a single technological object. Then for the evaluation of the availability of the utilization system for the realization of the assigned task the following criteria were adopted:

1. facilitating the definition of a required value of availability of a single technological object $G_z^{OT}$ for a given number of technological objects used in the transport system $N$

$$G^{OT} = G_z^{OT} \Leftrightarrow \min G^{PW}\left(G^{OT}\right) \geq G_z^{PW}\left(G^{OT}\right). \tag{4}$$

2. facilitating the definition of the required number of technological objects used in the transport system $N_z$ for a given value of the availability of a single technological object $G^{OT}$

$$N = N_z \Leftrightarrow min\ G^{PW}\left(N\right) \geq G_z^{PW}\left(N\right). \tag{5}$$

## 4. Model of transport means operation process

The model of operation process was created on the basis of the analysis of state space as well as operational events pertaining to technological objects (municipal buses) used in the analyzed authentic transport system. Due to the identification of the analyzed transport system and the multi-state process of technological object operation utilized in it, crucial operation states of the process as well as possible transfers between the defined states were designated. Based on this, a graph was created, depicting the changes of operation process states, shown in Fig. 1.

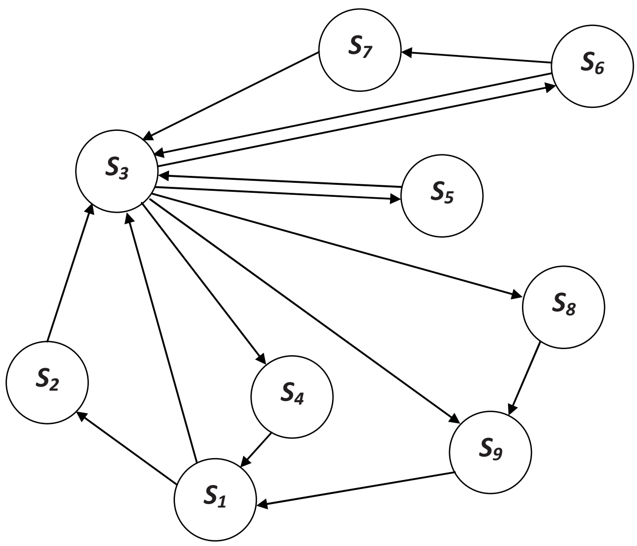


*Fig. 1. Directed graph representing the transport means operation process. $S_1$ – stopover at bus depot parking space, $S_2$ – repair at bus depot parking space, $S_3$ – carrying out of the transport goal, $S_4$ – fuel intake between transport peak hours, $S_5$ – repair by technical support unit without losing a ride, $S_6$ – repair by technical support unit with losing a ride, $S_7$ – awaiting the start of task realization after technical support repair, $S_8$ – emergency exit, $S_9$ – technical object repair at the efficiency implementation subsystem posts*

Using the semi-Markov processes in mathematical modelling of the operation process, the following assumptions were put forward:

- the modelled operation process has a finite number of states $S_i$, $i$ = 1,2,…,9,
- the random process $X(t)$ being the mathematical model of the operation process is a homogenous process,
- at moment $t$ = 0 , the process finds is in state $S_3$ (the initial state is state $S_3$).

The homogenous semi-Markov process is unequivocally defined when initial distribution and its kernel are given. Form our assumptions and based on the directed graph shown in Fig. 1, the initial distribution $p_i(0)=P\{X(0)=i\},\ i=1,2,...,9$ takes up the following form:

$$p_i(0)=\begin{cases}1 \text{ for} & i=3,\\ 0 \text{ for} & i\neq 3,\end{cases} \tag{6}$$

whereas the kernel of process *Q(t)*:

$$Q(t)=\begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & Q_{23}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & Q_{34}(t) & Q_{35}(t) & Q_{36}(t) & 0 & Q_{38}(t) & Q_{39}(t)\\ Q_{41}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & Q_{53}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & Q_{63}(t) & 0 & 0 & 0 & Q_{67}(t) & 0 & 0\\ 0 & 0 & Q_{73}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{89}(t)\\ Q_{91}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{7}$$

where

$$Q_{ij}(t)=P\{X(\tau_{n+1})=j,\tau_{n+1}-\tau_n\le t|X(\tau_n)=i\},\quad i,j=1,2,...,9\,, \tag{8}$$

means that the state of semi-Markovian process and the period of its duration depends solely on the previous state, and does not depend on earlier states and periods of their duration, where $\tau_1, \tau_2, \ldots, \tau_n, \ldots$ are arbitrary moments in time, so that $\tau_1<\tau_2<\ldots<\tau_n<\ldots$,
as well as

$$Q_{ij}(t)=p_{ij}\cdot F_{ij}(t), \tag{9}$$

where:

$$p_{ij}=\lim_{t\to\infty}p_{ij}(t), \tag{10}$$

$p_{ij}$ - means that the conditional probability of transfer from state $S_i$ to state $S_j$,

$$p_{ij}(t)=P\{X(t)=j|X(0)=i\}, \tag{11}$$

as well as

$$F_{ij}(t)=P\{\tau_{n+1}-\tau_n\le t|X(\tau_n)=i,X(\tau_{n+1})=j\},\quad i,j=1,2,...,9\,, \tag{12}$$

is a distribution function of random variable $\Theta_{ij}$ signifying period of duration of state $S_i$, under the condition that the next state will be state $S_j$.

## 5. Availability of transport means

In order to determine the availability of transport system objects (transport means) based on the semi-Markov operation process model, the operational states of the technological object should be divided into availability states $S_G$ and non- availability state $S_{NG}$ of the object to realize the assignment. Technological object availability states are states during which the object, including the operator remains in the operation system, is efficient and supplied or will be repaired and/or supplied in a period of time shorter than the time reserve which is to serve the purpose. Non- availability states are states in which the object or the operator remain outside the operation system (efficient or inefficient), as well as when an inefficient and/or unsupplied object remains in the operation system.

In the presented model, the following technological object availability states were enummerated:
- state $S_1$ – stopover at the bus depot parking space,
- state $S_2$ – repair at the bus depot parking space,
- state $S_3$ – carrying out of the transport task,
- state $S_4$ – fuel retake between traffic rush hours,
- state $S_5$ – repair by technical support unit without losing a ride,
- state $S_7$ – awaiting the start of task realization after technical support repair.

In the tested model the states $S_2$, $S_4$, and $S_5$ were qualified as availability based on the adopted assumption that the supplying of the technological object at state $S_4$ as well as repair at states $S_2$ and $S_5$ during the time reserve break assigned for this purpose do not interfere with the carrying out of the task, at the same time do not bring about the necessity to replace the supplied or repaired object with a different one (substitute object).

In order to assign the values of limit probabilities $p_i^*$ of staying in the states of semi-Markovian model of transport means operation, based on the directed graph shown in Fig. 1, the following were created: matrix $P$ of the states change probabilities and matrix $\Theta$ of conditional periods of duration of the states in process $X(t)$:

$$P = \begin{bmatrix} 0 & p_{12} & p_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{34} & p_{35} & p_{36} & 0 & p_{38} & p_{39} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{63} & 0 & 0 & 0 & p_{67} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{13}$$

$$\Theta = \begin{bmatrix} 0 & \overline{\Theta}_{12} & \overline{\Theta}_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\Theta}_{34} & \overline{\Theta}_{35} & \overline{\Theta}_{36} & 0 & \overline{\Theta}_{38} & \overline{\Theta}_{39} \\ \overline{\Theta}_{41} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{53} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{63} & 0 & 0 & 0 & \overline{\Theta}_{67} & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{73} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{\Theta}_{89} \\ \overline{\Theta}_{91} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{14}$$

Limit probability $p_i^*$ of staying in states of semi-Markov process were assigned on the basis of limit theorem for semi-Markovian processes:

*If hidden Markov chain in semi-Markovian process with finite state S set and continuous type kernel contains one class of positive retu rning states such that for ea ch state* $i \in S, f_{ij} = 1$ *and positive expected values* $E(\Theta_i), i \in S$ *are finite, limit probabilities*

$$p_i^* = \lim_{t \to \infty} p_i(t) = \frac{\pi_i \cdot E(\Theta_i)}{\sum_{i \in S} \pi_i \cdot E(\Theta_i)}, \tag{15}$$

*exist where probabilities* $\pi_i, i \in S$ *constitute a stationary distribution of a hidden Markov chain, which fulfils the simultaneous linear equations*

$$\sum_{i \in S} \pi_i \cdot p_{ij} = \pi_j, \; j \in S, \; \sum_{i \in S} \pi_i = 1, \tag{16}$$

Then, with the use of the MATHEMATICA software, the limit probability $p_i^*$ of staying in states of semi-Markov process and the availability of technological objects of the transport system were determined, defined by the following dependencies:

$$G^{OT} = \sum_i p_i^* = p_1^* + p_2^* + p_3^* + p_4^* + p_5^* + p_7^*, \; dla \; S_i \in S_G, \; i = 1,2,...,9, \tag{17}$$

$$G^{OT} = \frac{(p_{34}+p_{38}+p_{39}) \cdot (\overline{\Theta}_1 + p_{12} \cdot \overline{\Theta}_2) + \overline{\Theta}_3 + p_{34} \cdot \overline{\Theta}_4 + p_{35} \cdot \overline{\Theta}_5 + p_{36} \cdot p_{67} \cdot \overline{\Theta}_7}{\left[(p_{34}+p_{38}+p_{39}) \cdot (\overline{\Theta}_1 + p_{12} \cdot \overline{\Theta}_2)\right] + \overline{\Theta}_3 + p_{34} \cdot \overline{\Theta}_4 + p_{35} \cdot \overline{\Theta}_5 + \left[p_{36} \cdot (\overline{\Theta}_6 + p_{67} \cdot \overline{\Theta}_7)\right] + p_{38} \cdot \overline{\Theta}_8 + (p_{38}+p_{39}) \cdot \overline{\Theta}_9} \tag{18}$$

## 6. Findings

Based on the source data obtained from operational research carried out in an existing municipal bus transport system, the values of the elements of matrix $P$ (13) and matrix $\Theta$ (14), the absolute values of the duration of process states, the values of stationary distribution embedded in Markov chain process were estimated. Then the values of the border semi-Markovian process distribution was established (the values of border probabilities for remaining of technological objects at process states). Results are given in Tab. 1.

The data of use pertained to 182 technological objects (municipal buses) used in an existing municipal bus transport system for the period from April 2009 to December 2009. Operational research conducted with the use of passive experiment in natural operational conditions for the tested technological objects.

*Tab. 1. Values of probability* $p_i^*$ *and border semi-Markov process distribution*

| | | |
|---|---|---|
| $p_1^*$ = 0.30296 | $p_2^*$ = 0.00042 | $p_3^*$ = 0.54054 |
| $p_4^*$ = 0.00652 | $p_5^*$ = 0.00049 | $p_6^*$ = 0.00215 |
| $p_7^*$ = 0.00085 | $p_8^*$ = 0.00272 | $p_9^*$ = 0.14334 |

Based on the values of border probabilities $p_i^*$ of remaining at semi-Markovian process states presented in table 1 the value of the availability of a single technological object used in the tested transport system were determined

$$G^{OT} = p_1^* + p_2^* + p_3^* + p_4^* + p_5^* + p_7^* = 0.8518.$$

The availability of the utilization subsystem for the realization of the assigned transport task was determined for two cases (assigned transport task):

1. when the required number of technological objects amounts to $k_z$ = 98 ($N$ = 182, $G^{OT}$ = 0.8518) – pertaining to the realization of transport task outside of traffic rush hours, then

$$G^{PW}(k_z = 98) = 1 > G_z^{PW}(k_z = 98) = 0.5385.$$

2. when the required number of technological objects amounts to $k_z$ = 159 ($N$ = 182, $G^{OT}$ = 0.8518) – pertaining to the realization of transport task during traffic rush hours, then

$$G^{PW}(k_z = 159) = 0.2380 < G_z^{PW}(k_z = 159) = 0.8736.$$

The above shows that for the given number of technological objects used in the tested transport system $N$ = 182 and their availability $G^{OT}$ = 0.8518, the appropriate realization of the transport task by the utilization subsystem is possible only outside of traffic rush hour. On the other hand, during traffic rush hours the condition for the appropriate realization of the task is fulfilled ($G^{PW} < G_z^{PW}$). In order to determine the required number of objects used in the system $N_z$ (for the given availability of a single technological object $G^{OT}$) the graphs presented in Fig. 2 were prepared.

a) b)

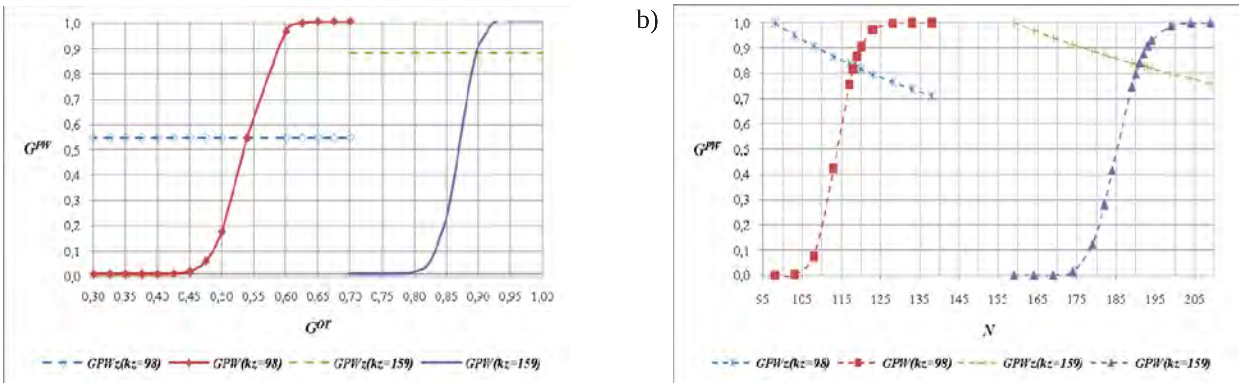


*Fig. 2. The availability of the utilization subsystem with threshold structure $G^{PW}$: a) in the function of the availability of a single technological object $G^{OT}$ for a given number of objects required for the realization of task $k_z$ as well as the number of objects used in the system N = 182, b) in the function of the number of objects used in the system N, for a given number of objects required for the realization of task $k_z$ as well as the availability of technological object $G^{OT}$ = 0.8518*

Based on the graph presented in Fig. 2a, the required availability of technological objects for the correct realization of the assigned transport task amounts to:

- outside of traffic rush hours $G_z^{OT}(k_z = 98) = 0.5393$, then

$$G^{PW}(k_z = 98) = 0.5394 > G_z^{PW}(k_z = 98) = 0.5385 ,$$

- during traffic rush hours $G_z^{OT}(k_z = 159) = 0.8969$, then

$$G^{PW}(k_z = 159) = 0.8742 > G_z^{PW}(k_z = 159) = 0.8736 ,$$

then, based on the chart presented in Fig. 2b, one may conclude that the minimum required number of technological objects for the correct realization of the assigned transport task amounts to:

- outside of traffic rush hours $N_z(k_z = 98) = 119$, then

$$G^{PW}(k_z = 98) = 0.8665 > G_z^{PW}(k_z = 98) = 0.8235 ,$$

- during traffic rush hours $N_z(k_z = 159) = 191$, then

$$G^{PW}(k_z = 159) = 0.8416 > G_z^{PW}(k_z = 159) = 0.8325 .$$

## 7. Summary

The presented method facilitates the evaluation of the correct realization of the assigned transport task as a result of comparing the value of the availability of the utilization subsystem to the value of the required availability. Based on the obtained evaluation of the availability of the utilization subsystem as well as the values of the parameters characteristic of the magnitude of the transport task it is possible to choose the values of the availability of technological objects as well as the number of objects used in the transport system. The minimum required values of the availability of technological objects as well as the number of the objects used in the transport system are determined in such a way, so that the value of the availability of the utilization subsystem was equal to or higher than the value of the required availability which the utilization subsystem should have in order to correctly realize the assigned transport task in given conditions.

At the following stage of the conducted research, a model of the evaluation of the availability and efficiency of the logistics subsystem posts will be prepared. The effects of the conducted work will facilitate the construction of a decision-making model of controlling the availability of the transport system.

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