# THE IDENTIFICATION OF MODEL PARAMETERS FOR A SEMI-EMPIRICAL MODEL OF WORKING PROCESS IN THE COMPRESSION-IGNITION ENGINE

#### Krzysztof Brzozowski, Jacek Nowakowski

University of Bielsko-Biała Willowa Street 2, 43-309 Bielsko-Biała tel.: +48 33 8279289, fax: +48 33 8279289 e-mail: kbrzozowski@ath.eu

#### Abstract

The paper concerns the identification process applied for a semi-empirical model of working process in the CI engine. The identification is based on pressure courses in the cylinder recorded during the experimental measurements on the test stand and is performed for estimation of values of the model parameters. Appropriate estimated values of the model parameters ensure minimization of the difference between measured and modelled pressure courses in the cylinder. The identification process can be divided into two stages. The first stage concerns identification of discrete values of the model parameters for a set of discrete engine operating conditions. The task of discrete identification is formulated as a dynamic optimisation task which is solved using a genetic algorithm. The accuracy of the identification process is evaluated by comparison of measured and calculated values of main parameters which characterize the working cycle such as: the mean indicated pressure, thermal efficiency, the mean indicated pressure in working part of the cycle, maximal pressure of the cycle, the mass of the medium in the cylinder and the crankshaft angle for which the maximal pressure occurred. The second stage concerns generalization of the results for any technically possible engine operating conditions which is solved by means of approximation. Feedforward multilayer artificial neural networks are used for the approximation. The accuracy of the identification and some examples of verification of the model predictions are presented as well.

Keywords: modelling, working cycle, artificial intelligence

### **1. Introduction**

Many different mathematical models can be used in research of engine technology. Besides the CFD models, the semi-empirical models of the working process are widely used in studies concerning the working cycle of a CI engine [5, 7]. These models are applied mostly in case when the time of model answer is a crucial factor. However, models belonging to this group usually have many simplifying assumptions mostly based on formulae and relations obtained from experimental measurements [6]. This led to the use of some additional model parameters whose values have to be appropriately chosen in order to ensure an acceptable accuracy of the model. As was shown in papers [1, 9], a solution to the problem of estimating the model parameters (called the calibration or identification) can be obtained by means of optimization methods. In this case the model parameters can be the independent variables of the dynamic optimization task. This approach requires the experimental set of data in order to identify the values of model parameters.

In the paper the identification based on genetic algorithms is used for solving the dynamic optimisation task. Genetic algorithms, unlike many other optimization methods, enable us to avoid the local minimum and find the solution which is close to the optimum and ensure the minimal difference between the measured and calculated pressure courses in the cylinder [9]. The discrete identification is performed for a wide range of engine operating conditions, and in the next step in order to generalize the results of identification an approximation task was formulated and solved by using an artificial neural network. As was shown in paper [3], neural networks after training can better generalize than functions with form taken a priori. However, the type of the neural

networks, their architecture and training process determine the accuracy of prediction which has to be evaluated before implementation of the networks in the model of the working cycle.

#### 2. The model of working process

In the paper a semi-empirical single-zone model of the working cycle of the CI engine formulated by authors in earlier work [10] is considered. The model enables calculation of the pressure, mass and temperature courses in the cylinder for a given vector of independent input parameters (the engine control parameters) using some additional quantities given as a vector of model parameters. The model can be written symbolically as:

$$M_i[\mathbf{X}, \mathbf{E}, \mathbf{G}, p, m, T] = 0 \text{ for } i = 1, ..., l,$$
 (1)

where:

 $M_i$  - differential operator or function,

*l* - number of equations,

 $\mathbf{X} = [n, B_0, \varphi_w, X_{EGR}]^T$  - vector of engine control parameters,

- *n* crankshaft rotational speed,
- $B_0$  injected fuel mass,
- $\varphi_w$  injection advance angle,

 $X_{FCR}$  - degree of exhaust gas recirculation,

 $\mathbf{E} = \left[ \mu_d, \mu_w, \Delta \varphi_s, \varphi_z, \beta, e_1 \dots e_8 \right]^T - \text{vector of model parameters,}$ 

 $\mu_d$  ,  $\mu_{\scriptscriptstyle W}$  - inlet and exhaust valve discharge coefficient respectively,

 $\Delta \varphi_s$  - total combustion duration,

 $\varphi_z$  - start of combustion,

$$G = [p_d, T_d, p_w, T_w, \lambda]^T - \text{vector of auxiliary parameters,}$$
  

$$p_d, T_d, p_w, T_w - \text{pressure and temperature in the intake (d) and exhaust (w)}$$
  
manifold respectively,

 $\lambda$  - relative air/fuel ratio.

The components of the vector of model parameters appear in the following equations (subscript c is used for parameters in the cylinder, d in the intake and w in the exhaust manifold):

- flow by inlet/outlet valves:

$$\frac{dm_{d(w)}}{d\varphi} = \frac{\pi n}{30} \cdot \mu_{d(w)} \cdot A_{d(w)} \cdot p_{d(c)} \cdot \sqrt{\frac{2}{R_{d(c)} \cdot T_{d(c)}} \cdot \frac{k_{d(w)}}{k_{d(w)} - 1} \left(\beta_{d(w)}^{\frac{2}{k_{d(w)}}} - \beta_{d(w)}^{\frac{k_{d(w)} + 1}{k_{d(w)}}}\right)}, \quad (2.1)$$

where:

 $\beta_d = p_c p_d^{-1} \le 1, \ k_d = c_{pd} c_{vd}^{-1}, \ \beta_w = p_w p_c^{-1} \le 1, \ k_w = c_{pc} c_{vc}^{-1},$  $\varphi$  - crank angle,

- the Watson function:

$$y = \beta \left[ 1 - \left( 1 - \left( \frac{\varphi - \varphi_z}{\Delta \varphi_s} \right)^{e_1 \tau} \right)^{e_2} \right] + \left( 1 - \beta \right) \left\{ 1 - \exp \left[ e_3 \lambda \left( \frac{\varphi - \varphi_z}{\Delta \varphi_s} \right)^{e_4} \right] \right\},$$
(2.2)

- the Hochenberg formula for calculation of the heat transfer coefficient:

$$h_{c} = e_{5} \cdot V_{c}^{e_{6}} \cdot p_{c}^{e_{7}} \cdot T_{c}^{e_{8}} \cdot (\overline{s} + 1, 4)^{0,8}, \qquad (2.3)$$

where:

 $V_c$  - cylinder volume,

 $\overline{s}$  - mean piston speed.

Equations (2.1-2.3) together with the following equations:

$$\frac{dm_c}{d\varphi} = \frac{dm_d}{d\varphi} - \frac{dm_w}{d\varphi} + \frac{dm_f}{d\varphi}, \qquad (3.1)$$

$$B_0 W \frac{dy}{d\varphi} + \frac{30}{\pi n} h_c A_h (T_s - T_c) + c_{pd} T_d \frac{dm_d}{d\varphi} = c_{vc} T_c \frac{dm_c}{d\varphi} + c_{vc} m_c \frac{dT_c}{d\varphi} + p_c \frac{dV_c}{d\varphi} + c_{pw} T_w \frac{dm_w}{d\varphi}, \quad (3.2)$$

where:

 $A_h$  - heat transfer area,

 $c_p, c_y$  - specific heat of the medium at constant pressure and constant volume respectively,

 $m_f$  - mass of fuel,

 $T_s$  - wall temperature,

- *W* fuel caloric value,
- *y* fuel mass burning rate,

which describe the mass and energy conservation laws respectively, form a set of equations describing the single-zone model of the working cycle.

Application of a semi-empirical model is possible if values of model parameters are known. If the values of the model parameters are not appropriate, the output from the computational model differs significantly from reality. In order to obtain an acceptable accuracy of the model prediction for the main parameters describing the working cycle, identification should be performed. The identification is the process of searching for those values of model parameters (vector E) which ensure good compatibility between the real and calculated pressure courses in the cylinder.

#### 3. The problem of the discrete identification

The model presented in earlier section has 13 unknown model parameters which could be estimated on the basis of measurements carried out for a given set of engine operating conditions. In measurements pressure courses in the cylinder and auxiliary parameters such as: temperature and pressure in the engine equipment system, relative air/fuel ratio are recorded for a wide range of engine torque, load and possible recirculation degrees and injection advanced advance angles, i.e. for different sets of input parameters given as vector X [10]. In the next step the vectors of model parameters  $E^{(j)}$  are estimated for each pair of vectors  $X^{(j)}$ ,  $G^{(j)}$  where j = 1,...,400 is a number of measurements point. This stage is called the discrete identification. The values of the computational model parameters which ensure the minimal difference between the measured and calculated pressure courses in the cylinder are obtained as the results.

The discrete identification process of the vector of model parameters E for a given vector of input parameters X and auxiliary parameters G can be written as a dynamic optimisation task of the functional:

$$\Omega(\mathbf{X}, \mathbf{E}, \mathbf{G}) = c_1 \int_{0}^{4\pi} \left[ p_E(\varphi) - p_F(\varphi) \right]^2 d\varphi + c_2 \left( \max p_E(\varphi) - \max p_F(\varphi) \right)^2 \to \min$$
(4)

where:

 $p_E$  - pressure course obtained on the basis of the computational model,

 $p_F$  - smoothed course from experimental measurements,

 $c_1, c_2$  - weight coefficients.

In order to solve the optimization task in the form (4), the objective function has to be integrated at every step. In papers [4, 10] it was shown that the genetic algorithms can be used to solve the task (4) rewritten in the form of the function of adaptation as follows:

$$\Phi(\mathbf{X}, \mathbf{E}, \mathbf{G}) = \frac{1}{\Omega(\mathbf{X}, \mathbf{E}, \mathbf{G})} \to \max.$$
(5)

The real-value representation of genes in the chromosome is used, i.e. the chromosome has the form of vector [10]:

$$\mathbf{Z} = \begin{bmatrix} z_1, \dots, z_{13} \end{bmatrix}^T, \tag{6}$$

where  $z_1 = \mu_d, z_2 = \mu_w, z_3 = \Delta \varphi_s, z_4 = \varphi_z, z_5 = \beta, z_{5+i} = e_{5+i}$  for i = 1...8.

Next, the genetic operators of the arithmetical crossover and non-uniform mutation have been applied [8]. As the results of using the genetic algorithm, for given from measurements pressure courses and vectors  $\mathbf{X}^{(j)}$ ,  $\mathbf{G}^{(j)}$ , the vectors of model parameters  $\mathbf{E}^{(j)}$  are estimated.

The accuracy of the identification process can be evaluated by comparison of measured and calculated values of main parameters which characterize the working cycle such as: the mean indicated pressure  $p_i$ , thermal efficiency  $\eta_c$ , the mean indicated pressure in working part of the cycle  $p_{i(r)}$ , maximal pressure of the cycle  $p_{max}$ , the mass of the medium in the cylinder  $m_c$  and the crankshaft angle for which the maximal pressure  $\varphi_{p_{max}}$  occurred. In Tab. 1 are presented the values of error of modelling for the main parameters of the working cycle. The error is expressed as an average relative percentage error:

$$\varepsilon = \sum_{j=1}^{400} \frac{\left|o_{j} - m_{j}\right|}{o_{j}} 100\%, \qquad (7)$$

where  $o_j, m_j$  are values observed in measurement and predicted from the model respectively for the j-th engine conditions.

|  | Parameter of the working cycle |            |               |       |                         |  |  |
|--|--------------------------------|------------|---------------|-------|-------------------------|--|--|
|  | $p_i^{},\eta_c^{}$             | $p_{i(r)}$ | $p_{\rm max}$ | $m_c$ | $arphi_{p_{	ext{max}}}$ |  |  |
| Average error of modelling $\varepsilon$ [%] | 5.02                           | 2.74       | 2.7           | 2.18  | 0.73                    |  |  |

Tab. 1. The error of modelling

Analysis of the data presented in Tab. 1 lead us to conclusions of satisfactory results of modelling. It means that the pressure courses and the main parameters of the working cycle are computed with very good accuracy after using the values of model parameters obtained from the identification. Moreover, the maximal errors do not exceed in any case 5.1%.

#### 4. The generalization

The discrete identification presented in earlier chapter enables calculation of the working cycle only for a limited number of discrete engine operating conditions given by known vectors  $\mathbf{X}^{(j)}$ ,  $\mathbf{G}^{(j)}$  and using vector  $\mathbf{E}^{(j)}$  estimated for those conditions. In order to use the model for modelling the working process in the cylinder for any technically possible operating conditions, the following relationships have to be proposed:

$$\mathbf{E} = f_E(\mathbf{X}),\tag{8}$$

and

$$\mathbf{G} = f_G(\mathbf{X}). \tag{9}$$

The problem of generalization of the results of discrete identification can be treated as an approximation task of discrete vectors  $E^{(j)}$ ,  $G^{(j)}$  for j = 1,...,400. Solution of the approximation task yields the unknown function  $f_E(X)$  and  $f_G(X)$ . The task can be solved in many ways, e.g. by using the least square method if the form of the function is given a priori or with artificial neural networks otherwise. However, results of some comparisons made for similar problems of approximation presented in papers [2, 3] showed that using artificial neural networks can be a better choice.

Preliminarily two types of neural networks were tested: multi layer network and radial network [11] with different activation functions. After some numerical experiments we decided to use two feed-forward multilayer artificial neural networks: A and B. The input signals are the components of the vector  $\boldsymbol{X} = [n, B_0, \varphi_w, X_{EGR}]^T$ , the output signals are as follows:

- network *A* has output signals *Out*<sub>1</sub>:

$$Out_1 = \left[ p_d, T_d, p_w, T_w, \lambda, \mu_d, \mu_w, \Delta \varphi_s, \varphi_z \right]^T,$$
(10)

- network *B* has output signals *Out*<sub>2</sub>:

$$Out_2 = \left[\beta, e_1 \dots e_4\right]^T.$$
<sup>(11)</sup>

One can notice that the network A uses for approximation all components of the vector of auxiliary parameters G and four components of the vector of model parameters E. The network B uses for approximation only five components of the vector of model parameters E which all are the parameters of the Watson formula. It means that parameters used for calculation the heat transfer coefficient  $e_5...e_8$  are not approximated. Numerical experiments show that using the average values of those parameters obtained in discrete identification does not strongly influence the accuracy of the model.

For both neural networks we used the activation function in the form of a hyperbolic tangent:

$$f(\mathbf{w}^{T}\mathbf{I}) = \left[1 + e^{-\mathbf{w}^{T}\mathbf{I}}\right]^{-1},$$
(12)

where:

w – vector of weights on neurons' inputs,

*I* – vector of neurons' input signals.

In order to find the optimal architecture of the networks we start with one hidden layer with 9 neurons. Neural networks were trained separately using supervised momentum algorithm based on the back-propagation method in which the weights were adjusted in each training step by using the formula [11, 12]:

$$\mathbf{w}^{(n+1)} = \mathbf{w}^{(n)} - \boldsymbol{\xi}^{(n)} \nabla \delta(\mathbf{w}^{(n)}) + \alpha \left(\mathbf{w}^{(n)} - \mathbf{w}^{(n-1)}\right), \tag{13}$$

where:

 $\xi$  - learning rate,

 $\nabla \delta(w)$  - gradient of the network output error,

 $\alpha$  - moment coefficient set equal to 0.9.

The training procedure was continued until the average relative percentage error  $\varepsilon$  between the value given in verification set and the value calculated by the network for each component of the

output signals became less than 10%. If the error for any component of the output signals was still more than 10%, the network architecture was changed by adding a new neuron on the hidden layer and the training procedure was repeated. In Tab. 2 are presented the final architectures of both networks and values of the average relative percentage error  $\varepsilon$  for each component of the output signals.

Tab. 2. The final architecture of considered neural networks A and B and values of the average relative percentage error  $\varepsilon$  for each component of the output signals

|                             | <u>Network A</u>   |       |       |         |             |                  |           |                |         |                  |             |  |
|-----------------------------|--|-------|-------|---------|-------------|------------------|-----------|----------------|---------|------------------|-------------|--|
| Architecture                | The hidden layer   |       |       |         |             | The output layer |           |                |         |                  |             |  |
| The number of neurons       | 15   |       |       |         |             | 9                |           |                |         |                  |             |  |
|                             | Evaluation of the approximation quality for a verification set |       |       |         |             |                  |           |                |         |                  |             |  |
| Output signal               | $p_d$  | $T_d$ | $p_w$ | $T_w$   |             | λ                | $\mu_{a}$ | d              | $\mu_w$ | $\Delta arphi_s$ | $\varphi_z$ |  |
| <i>ε</i> [%]                | 2.226  | 1.961 | 2.076 | 1.30    | 04 2.       | 453              | 2.51      | 10             | 8.827   | 9.023            | 0.452       |  |
| Median of $\varepsilon$ [%] | 2.011  | 1.643 | 2.141 | 0.94    | 8 1.        | 818              | 1.73      | 36             | 8.475   | 8.403            | 0.286       |  |
|                             | <u>Network B</u>   |       |       |         |             |                  |           |                |         |                  |             |  |
| Architecture                | The hidden layer The output layer                              |       |       |         |             |                  |           |                |         |                  |             |  |
| The number of neurons       | 31 5   |       |       |         |             |                  |           |                |         |                  |             |  |
|                             | Evaluation of the approximation quality for a verification set |       |       |         |             |                  |           |                |         |                  |             |  |
| Output signal               | β  |       | $e_1$ |         |             | $e_2$            |           | e <sub>3</sub> |         |                  | $e_4$       |  |
| <i>ε</i> [%]                | 8.00   | 51    | 8.944 | 944 8.4 |             | 8.453            |           | 6.125          |         | 5                | 5.432       |  |
| Median of $\varepsilon$ [%] | 7.90   | 65    | 7.066 | 5       | 4.925 5.367 |                  | 3         | 3.024          |         |                  |             |  |

Analysis of values of the average relative percentage error  $\varepsilon$  for each component of the output signals presented in Tab. 2 leads to the following conclusions:

- among all output signals the smallest value of the average error was achieved for  $\varphi_z$ ,
- the components of the auxiliary vector are predicted with very good accuracy, for them the average output error does not exceed 2.5%,
- generally bigger values of error was achieved for the network *B* which was used for approximation of the model parameters used in Watson formula.

## 5. Conclusion

The identification procedure presented is a main step of formulating the semi-empirical model of the working cycle which consists of two stages. The first stage of the identification procedure ensures that the values of the model parameters are estimated properly. The second one enables estimation of those values for any conditions of engine work by solving the appropriate approximation task. The overall accuracy of the model predictions is determined by accuracy achieved in both stages. In order to verify the accuracy of the model, some additional comparisons should be done. For comparison experimental measurements are used for which values of control parameters (input signals) were different from those taken for teaching the neural networks. Then, all model and auxiliary parameters were calculated according to the approximation obtained by the neural networks. Finally, the model was used to calculate the pressure courses and the main parameters of the working cycle. A comparison of some pressure courses  $p(\varphi)$  calculated by the model and those from the measurements are presented in Fig. 1.



Fig. 1. Comparison of measured and calculated pressure courses of the medium in the cylinder

In Table 3 are presented values of error of modelling for the main parameters of the working cycle for cases presented in Fig. 1.

|                                | Error of modelling [%]                   |   |  |   |  |  |  |  |  |
|--------------------------------|--|---|--|---|--|--|--|--|--|
| Parameter of the working cycle | Case A: 1500 rpm<br>0.5 M <sub>max</sub> | Case B: 2500 rpm<br>0.75 M <sub>max</sub> | Case C: 3500 rpm<br>0.9 M <sub>max</sub> | Case D: 4000 rpm<br>0.25 M <sub>max</sub> |  |  |  |  |  |
| $p_i, \eta_c$                  | 0.72                                     | 5,08                                      | 0.63                                     | 1.7                                       |  |  |  |  |  |
| $p_{i(r)}$                     | 4.83                                     | 1.36                                      | 0.69                                     | 1.67                                      |  |  |  |  |  |
| $p_{\rm max}$                  | 3.95                                     | 5.28                                      | 0.4                                      | 2.58                                      |  |  |  |  |  |
| m <sub>c</sub>                 | 1.28                                     | 0.81                                      | 1.93                                     | 0.01                                      |  |  |  |  |  |
| $\varphi_{p_{\max}}$           | 2.01                                     | 0.66                                      | 0.54                                     | 0.14                                      |  |  |  |  |  |

Tab. 3. The errors of modelling

The results presented in Tab. 3 show that the used method of identification ensures an acceptable correspondence of calculation results and measurements. Thus, the semi-empirical model presented can be used in studies concerning modelling of the working cycle for any engine operating conditions.

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