# CAPACITY AND FRICTION FORCE IN SLIDE JOURNAL BEARING GAP BY LAMINAR UNSTEADY LUBRICATION

**Paweł Krasowski**

*Gdynia Maritime University*
*Morska Street 81-87, 81-225 Gdynia, Poland*
*tel.:+48 58 6901331, fax:+48 58 6901399*
*e-mail: pawkras@am.gdynia.pl*

***Abstract***

*This paper shows results of numerical solutions modified Reynolds equations for laminar unsteady oil flow in slide journal bearing gap. Laminar unsteady oil flow is performed during periodic and unperiodic perturbations of bearing load or is caused by the changes of gap height in time. During modelling crossbar bearing operations in combustion engines, bearing movement perturbations from engine vertical vibrations causes velocity flow perturbations of lubricating oil on the bearing race and on the bearing slider in the circumferential direction. Above perturbations occur mostly during the starting and stopping of machine. This solution example applies to isothermal bearing model with infinity length. Lubricating oil used in this model has Newtonian properties and constant dynamic viscosity. Results are presented in the dimensionless hydrodynamic pressure and tangential tension distribution diagrams. Diagrams also presents capacity and friction force change during the time of velocity perturbations. Received solutions were compared with the solution received by the stationary lubrication flow in the slide journal bearing, which were made with the same parameter assumption by constant dynamic oil viscosity. Isothermal bearing model is similar to friction node model by steady-state heat load conditions. Described effect can be used as on example of modelling the bearing friction node operations in reciprocating movement during exploitation of engines and machines.*

***Keywords:*** *laminar unsteady lubrication, journal bearing, pressure, capacity and friction forces*

## 1. Introduction

This article refer to the unsteady, laminar flows issue, in which modified Reynolds number $Re^*$ is [5, 6] smaller or equal to 2. Laminar, unsteady oil flow is performed during periodic and unperiodic perturbations of bearing load or is caused by the changes of gap height in the time. Above perturbations occur mostly during the starting and stopping of machine. Lubricated oil disturbance velocity the pin and on the bearing shell was also consider in the article. Reynolds equation system describing Newtonian oil flow in the gap of transversal slide bearing was discussed in the articles [4, 5]. Velocity perturbations of oil lubricated flow on the pin can be caused by torsion pin vibrations during the rotary movement of the shaft. Perturbations are proportional to torsion vibration amplitude, frequent constraint and to pin radius of the shaft. Oil velocity perturbations on the shell surface can be caused by rotary vibration of the shell together with bearing casing. This movement can be consider as kinematical constraint for whole bearing friction node. Isothermal bearing model can be approximate to bearing operation in friction node under steady-state thermal load conditions for example bearing in generating set on ship.

## 2. Hydrodynamic pressure and capacity forces

The unsteady, laminar and isotherm flow Newtonian oil in journal bearing gap is described for modified Reynolds equation [1, 2] from Newtonian oil with constant and variable dynamic viscosity depended for pressure. In considered model we assume small unsteady disturbances and in order to maintain the laminar flow, oil velocity $V_i^*$ and pressure $p_1^*$ are total of dependent quantities $\tilde{V}_i$ ; $\tilde{p}_1$ and independent quantities $V_i$; $p_1$ from time [4, 6] according to equation (1).

$$V_i^* = V_i + \tilde{V}_i \ , \qquad p_1^* = p_1 + \tilde{p}_1 \ , \ \ i = 1,2,3 . \tag{1}$$

Unsteady components of dimensionless oil velocity and pressure are [4] in following form of infinite series:

$$\tilde{V}_i(\varphi, r_1, z_1, t_1) = \sum_{k=1}^{\infty} V_i^{(k)}(\varphi, r_1, z_1) exp(jk\omega_0 t_0 t_1), \qquad \text{i=1,2,3,}$$

$$\tilde{p}_1(\varphi, r_1, z_1, t_1) = \sum_{k=1}^{\infty} p_1^{(k)}(\varphi, r_1, z_1) exp(jk\omega_0 t_0 t_1),$$

where:

$\omega_0$ - angular velocity perturbations in unsteady flow,

$j=\sqrt{-1}$ - imaginary unit.

Components of oil velocity $V_\varphi$ ,$V_r$ ,$V_z$ in cylindrical co-ordinates r,φ,z have presented as $V_1$,$V_2$, $V_3$ in dimensionless [1] form:

$$V_\varphi = UV_1 , \qquad V_r = \psi UV_2 , \qquad V_z = \frac{U}{L_1} V_3 , \tag{3}$$

where:

U - peripheral journal velocity U=ωR,

ω - angular journal velocity,

R - radius of the journal,

ψ - dimensionless radial clearance ($10^{-4} \le \psi \le 10^{-3}$),

2b - length of bearing,

$L_1$ - dimensionless bearing length,

ε - radial clearance:

$$\psi = \frac{\varepsilon}{R} \ ; \qquad L_1 = \frac{b}{R} , \tag{4}$$

Putting following quantities [1, 5]: dimensionless values density $\rho_1$, hydrodynamic pressure $p_1^*$, time $t_1$, longitudinal gap height $h_1$, radial co-ordinate $r_1$ and co-ordinate $z_1$.

$$\begin{aligned} &\rho = \rho_0 \, \rho_1 , \quad p = p_0 \, p_1 , \quad t = t_0 \, t_1 , \\ &z = b \, z_1 , \quad r = R(1 + \psi r_1), \quad h = \varepsilon h_1 . \end{aligned} \tag{5}$$

Rule of putting dimensionless velocity and pressure quantities in unsteady and steady part of the flow stays similar. Following symbols with bottom zero index signify density $\rho_0$, dynamic viscosity $\eta_0$ , pressure $p_0$ and time $t_0$ describe characteristic dimension values assigned to adequate quantities. Reynolds number Re , modified Reynolds number $Re^*$ are in form [1]:

$$p_0 = \frac{U\eta_0}{\psi^2 R}, \qquad Re = \frac{U \rho_0 \psi R}{\eta_0}, \qquad Re^* = \psi \, Re . \tag{6}$$

The equation solution Reynolds equation [2, 3] with disturbances of peripheral velocity $V_{10}$ on the journal and $V_{1h}$ on the sleeve for bearing with infinity length determines unsteady dimensionless hydrodynamic pressure function $\tilde{p}_1$ in following form:

$$\tilde{p}_1 = \frac{1}{2} \rho_1 \, Re^* \, n \gamma_V (V_{10} + V_{1h}) \left( \varphi - h_{1e}^3 \int_0^\varphi \frac{d\varphi}{h_1^3} \right) \sum_{k=1}^{\infty} A_{(k)} - \gamma_V \, p_{10} (V_{10} - V_{1h}) \sum_{k=1}^{\infty} B_{(k)} , \tag{7}$$

where:
$h_1$ - height of gap,
$h_{1e}$ - height of gap by film ended $\varphi = \varphi_e$,
$\lambda$ - eccentricity ratio.

$$h_1(\varphi) = 1 + \lambda \cos\varphi, \qquad h_{1e} = h_1(\varphi_e). \tag{8}$$

Quantity $\gamma_V$ are factor of scale for velocity perturbations, dependent for acceptant of term series (2). Pressure $p_{10}$ is located in the oil gap by steady flow and by constant oil dynamic viscosity.

Sum for series $\sum_{k=1}^{\infty} A_k$ and $\sum_{k=1}^{\infty} B_k$ in right side of Reynolds solution equation (7) are results from conservation of the momentum solutions and were define in work [1, 2].

$$\sum_{k=1}^{\infty} A_{(k)} = \sum_{k=1}^{\infty} \frac{\sin(k\omega_0 t_0 t_1)}{k} = \begin{cases} \frac{\pi - \omega_0 t_0 t_1}{2} & 0 < t_1 < 1, \\ 0 & t_1 = 0, 1, \end{cases}$$
$$\sum_{k=1}^{\infty} B_{(k)} = \sum_{k=1}^{\infty} \frac{\cos(k\omega_0 t_0 t_1)}{k^2} = \frac{1}{4}\left[\left(\pi - \omega_0 t_0 t_1\right)^2 - \frac{\pi^2}{3}\right] \quad 0 \le t_1 \le 1. \tag{9}$$

In presented calculation way an expression value is assumed $n\rho_1 Re^* = 12$, what is approximately equivalent to force over first frequency torsion vibrations force of six cylinder engine shaft. Examples apply to bearing with constant dependent eccentricity $\lambda = 0{,}6$. In case where oil velocity perturbations are caused by forced vibrations of engine then the number n in equation (7) define multiplication of perturbation frequency $\omega_0$ to angular velocity of engine crankshaft $\omega$. Multiplication factor n is equal to number of cylinder c in two-stroke engine (s=2) or in four-stroke engine (s=4) to number of cylinders c/2:

$$n = \frac{\omega_0}{\omega} = \begin{cases} c, & s = 2, \\ \frac{c}{2}, & s = 4. \end{cases} \tag{10}$$

We analyst cylindrical bearing infinite length with circumferential velocity perturbations on the journal $V_{10}$ and on the sleeve $V_{1h}$. Circumferential velocity perturbations are caused by torsional vibrations shaft (on the journal) or circumferential displacement frame bearing (on the sleeve). In the further numerical analysis relation time $t_0$ was taken into account as a propagation period of axial velocity perturbation of lubricating oil.

Four velocities perturbations will analysed in this article. Pressure perturbation $\tilde{p}_1$ course in point $\varphi = 150°$ presented Fig. 1 by following circumferential velocity perturbations:

1. velocity perturbations on the journal $V_{10}$= 0.05 and on the sleeve $V_{1h}$=0,
2. velocity perturbations on the journal $V_{10}$= 0.05 and on the sleeve $V_{1h}$= 0.025,
3. velocity perturbations on the journal $V_{10}$= 0.05 and on the sleeve $V_{1h}$= 0.05,
4. velocity perturbations on the journal $V_{10}$= 0.05 and on the sleeve $V_{1h}$= -0.05.

Capacity force W for cylindrical slide journal bearing has following components $W_x$ and $W_y$ to be determined [2, 6] in the local co-ordinate systems in Fig. 2. Capacity force W is resultant force from concurrent pressure forces configuration.
Thus dimensionless components $W_{1x}$ and $W_{1y}$ of capacity forces $W_1$ are as follows [2]:

$$W_{1x} = \frac{W_x}{W_0} = -\int_0^{\varphi_k} p_1 \cos\varphi d\varphi, \qquad W_{1y} = \frac{W_y}{W_0} = -\int_0^{\varphi_k} p_1 \sin\varphi d\varphi, \tag{11}$$
$$W_1 = \sqrt{W_{1x}^2 + W_{1y}^2} = \frac{W}{W_0},$$

where $W_0$ is characteristic value of capacity force $W_0 \equiv 2Rbp_0$.

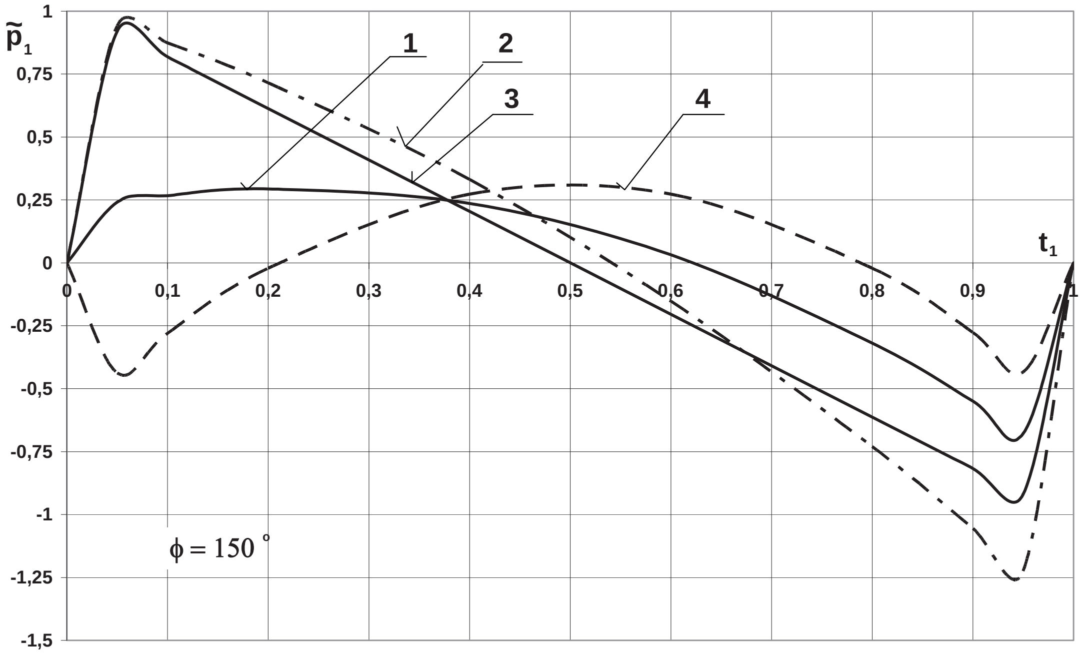


*Fig. 1. Pressure distributions $\tilde{p}_1$ in place φ=150° in the time $t_1$ by velocity perturbations: 1) $V_{10}$=0.05, $V_{1h}$=0, 2) $V_{10}$=0.05, $V_{1h}$=0.025, 3) $V_{10}$=0.05, $V_{1h}$=0.05, 4) $V_{10}$=0.05, $V_{1h}$=-0.05*

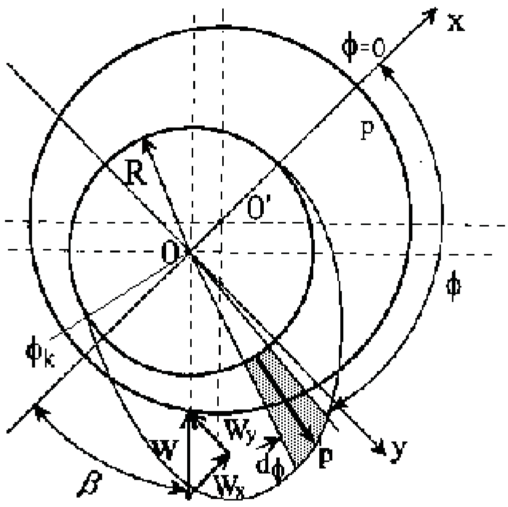


*Fig. 2. Capacity force W and components $W_x$ and $W_y$ in the local co-ordinate system*

Hydrodynamic capacity force change caused by the pressure perturbation is calculate from:

$$\tilde{W}_1 = W_1^* - W_1 \tag{12}$$

Pressure in the bearing during the perturbation is a total of stationary flow pressure and perturbation pressure according to (1). According to mentioned equation [2] if we provide stationary flow pressure $p_1$ we will obtain capacity force $W_1$. Figure 3 also presents change capacity calculation results by four oil viscosity perturbations. Capacity force in stationary flow is marked by horizontal lines with dots. Hydrodynamic capacity force $\tilde{W}_1$ changes periodically with a period equal to perturbation velocity. In case of velocity perturbation in the bearing pin, increase of capacity force above the stationary condition value last no longer than half of the perturbation period and the increase of capacity force is bigger than the decrease in the remaining time. When perturbation of velocity on the bearing pin is in the same direction as a peripheral velocity of the pin it causes then bigger increase of capacity force than decrease. It is opposite in case of oil peripheral velocity

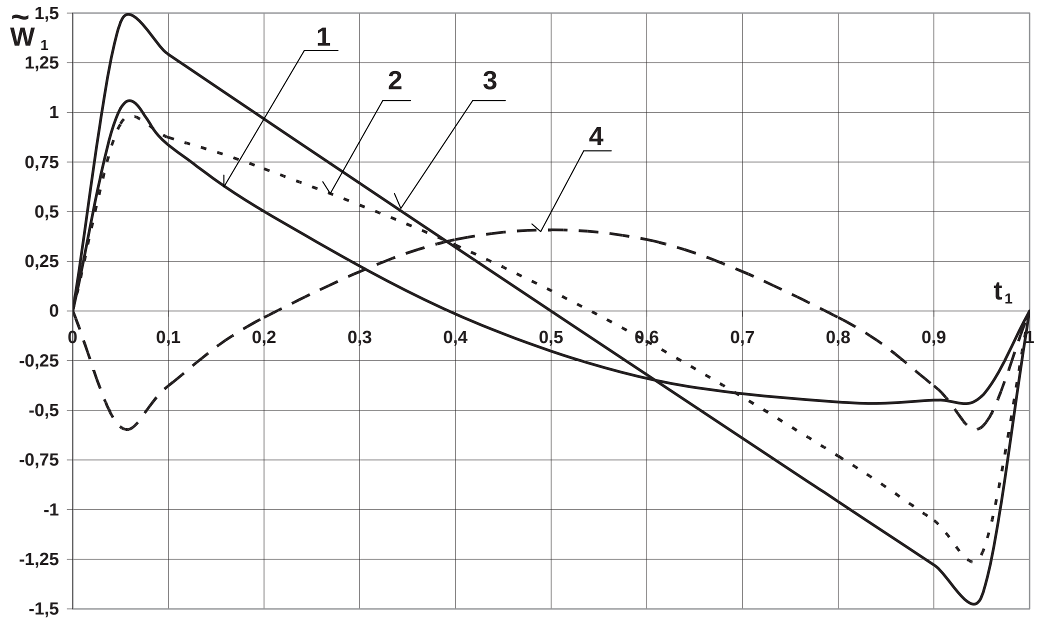


*Fig. 3. Change the dimensionless capacity forces $\widetilde{W}_1$ in the time $t_1$ by velocity perturbations: $V_{10}$=0.05, $V_{1h}$=0, 2) $V_{10}$=0.05, $V_{1h}$=0.025, 3) $V_{10}$=0.05, $V_{1h}$=0.05, 4) $V_{10}$=0.05, $V_{1h}$=-0.05*

perturbation on the shell surface, but his diagrams are not presented in his article. The case 2 effects are shown on the diagram 4. Capacity force course in time is not symmetrical for different perturbation of velocity quantities on the pin and on the shell.

## 3. Elementary friction force and friction force in bearing gap

Elementary friction forces τ and this change $\tilde{\tau}$ for dimension an dimensionless form $\tilde{\tau}_1$, we calculated as tangential stress (pressure) by Newtonian oil in formula:

$$\tau = \eta \frac{\partial V_\varphi}{\partial r}\bigg|_{r=h}, \quad \tau_1 = \eta_1 \frac{\partial V_1}{\partial r_1}\bigg|_{r_1=h_1}, \quad \tilde{\tau}_1 = \eta_1 \frac{\partial \tilde{V}_1}{\partial r_1}\bigg|_{r_1=h_1}, \tag{13}$$

$$\tau = p_0\psi\tau_1 = \tau_0\tau_1, \qquad \tau_0 = \psi p_0 .$$

Velocity circumferential perturbation (7) [3] are in formula:

$$\begin{aligned}\tilde{V}_1 = {} & \frac{6\eta_1}{\rho_1 Re^* n}\gamma_V(V_{10}-V_{1h})\frac{h_1-h_{1e}}{h_1^3}\sum_{k=1}^{\infty}\frac{P(k)}{k^3}cos(2k\pi t_1)+\gamma_V V_{10}\sum_{k=1}^{\infty}\frac{A(k)}{k^2}cos(2k\pi t_1) \\ & -\frac{\gamma_V}{2}(V_{10}+V_{1h})\frac{h_1^3-h_{1e}^3}{h_1^3}\sum_{k=1}^{\infty}\frac{P(k)}{k^2}sin(2k\pi t_1)+\gamma_V V_{1h}\sum_{k=1}^{\infty}\frac{B(k)}{k^2}cos(2k\pi t_1).\end{aligned} \tag{14}$$

Components of series P(k), A(k) i B(k) from formula (14) are in following forms [3]:

$$\begin{aligned} P(k) &= \frac{sin(sh_1X_k)sinh[(1-s)h_1X_k]+sin[(1-s)h_1X_k]\,sinh(sh_1X_k)}{cosh(sh_1X_k)+cos(sh_1X_k)}, \\ A(k) &= \frac{-cosh(sh_1X_k)cos[(2-s)h_1X_k]+cosh[(2-s)h_1X_k]\,cos(sh_1X_k)}{cosh(2h_1X_k)-cos(2h_1X_k)}, \\ B(k) &= \frac{cosh[(1+s)h_1X_k]\,cos[(1-s)h_1X_k]-cosh[(1-s)h_1X_k]\,cos[(1+s)h_1X_k]}{cosh(2h_1X_k)-cos(2h_1X_k)}, \end{aligned} \tag{15}$$

$$s = \frac{r_1}{h_1}, \qquad \gamma_V = \frac{6}{\pi^2}, \qquad X_k = \sqrt{k\frac{\rho_1}{2\eta_1}Re^* n}, \qquad \text{k=1,2,3…}$$

Additional symbol *s* marks dimensionless parameter height of gap ($0 \le s \le 1$). In numerical calculation example oil with constant density was assume, what is equivalent to quantity $\rho_1$.

$$\frac{\partial \tilde{V}_1}{\partial r_1}\Big|_{r_1=h_1} = \frac{6\eta_1}{\rho_1 Re^* n}\gamma_V(V_{10}-V_{1h})\frac{h_1-h_{1e}}{h_1^3}\sum_{k=1}^{\infty}\frac{P'(k)}{k^3}cos(2k\pi t_1)+\gamma_V V_{10}\sum_{k=1}^{\infty}\frac{A'(k)}{k^2}cos(2k\pi t_1)+ \\ -\frac{\gamma_V}{2}(V_{10}+V_{1h})\frac{h_1^3-h_{1e}^3}{h_1^3}\sum_{k=1}^{\infty}\frac{P'(k)}{k^2}sin(2k\pi t_1)+\gamma_V V_{1h}\sum_{k=1}^{\infty}\frac{B'(k)}{k^2}cos(2k\pi t_1), \tag{16}$$

where:

$$P'(k) = \frac{dP(k)}{dr_1}\Big|_{r_1=h_1} = -X_k\frac{sin(h_1X_k)+sinh(h_1X_k)}{cosh(h_1X_k)+cos(h_1X_k)},$$

$$A'(k) = \frac{dA(k)}{dr_1}\Big|_{r_1=h_1} = -2X_k\frac{sinh(h_1X_k)cos(h_1X_k)+cosh(rh_1X_k)sin(h_1X_k)}{cosh(2h_1X_k)-cos(2h_1X_k)}, \tag{17}$$

$$B'(k) = \frac{dB(k)}{dr_1}\Big|_{r_1=h_1} = X_k\frac{sinh(2h_1X_k)+sin(2h_1X_k)}{cosh(2h_1X_k)-cos(2h_1X_k)}.$$

The results change tangential pressure $\tilde{\tau}_1$ distribution in the time $t_1$ show Fig. 4 in place of coordinate φ=150°. Change elementary friction forces (tangential pressure) $\tilde{\tau}_1$ perturbation are values different sign from initial perturbations $V_{10}$, $V_{1h}$ and absolutely lesser.

Friction forces T and this change $\tilde{T}$ for dimension an dimensionless form $\tilde{T}_1$, we calcullated as sum tangential stress (pressure) in circumference co-ordinate bearing by formula:

$$\tilde{T} = \int_0^{\varphi_e} Rb\eta\frac{\partial \tilde{V}_\varphi}{\partial r}\Big|_{r=h} d\varphi = T_0\tilde{T}_1, \tag{18}$$

$$\tilde{T}_1 = \int_0^{\varphi_e}\tilde{\tau}_1(\varphi)d\varphi,$$

where $T_0$ is characteristic value of friction force $T_0 = W_0\psi$ .

In Fig. 5 presented change the dimensionless friction forces $\tilde{T}_1$ in the time $t_1$ by four of velocity perturbation.

## 4. Conclusions

Presented Reynolds equation solution for unsteady laminar Newtonian flow of lubricated oil enables initial opinion to hydrodynamic pressure, elementary friction pressure distribution and capacity, friction forces as a basic slide bearing operating parameter. Unsteady velocity perturbation on the journal and sleeve effect on capacity and friction forces in lubricated gap. Capacity and friction forces variation in bearing have periodical character equal to periodical velocity perturbation time and this variations value and character depend on type of perturbation. Author is aware of simplifications that were assumed in presented model which apply to Newtonian oil and to isothermal bearing model. Despite that presented calculation example apply to bearing with infinity length, obtained conclusions can be useful to elementary friction pressure and friction forces distribution by laminar, unsteady lubrication of cylindrical slide bearing with finite length.

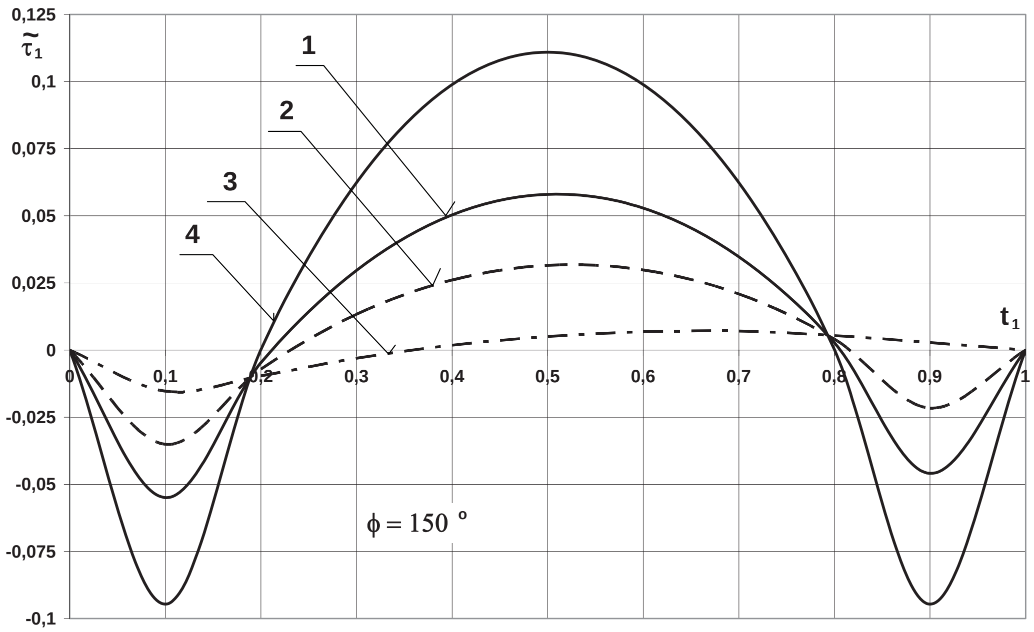


*Fig. 4. Tangential pressure distributions $\tilde{\tau}_1$ in place $\varphi$=150° in the time $t_1$ by velocity perturbations: 1) $V_{10}$=0.05, $V_{1h}$=0, 2) $V_{10}$=0.05, $V_{1h}$=0.025, 3) $V_{10}$=0.05, $V_{1h}$=0.05, 4) $V_{10}$=0.05, $V_{1h}$=-0.05*

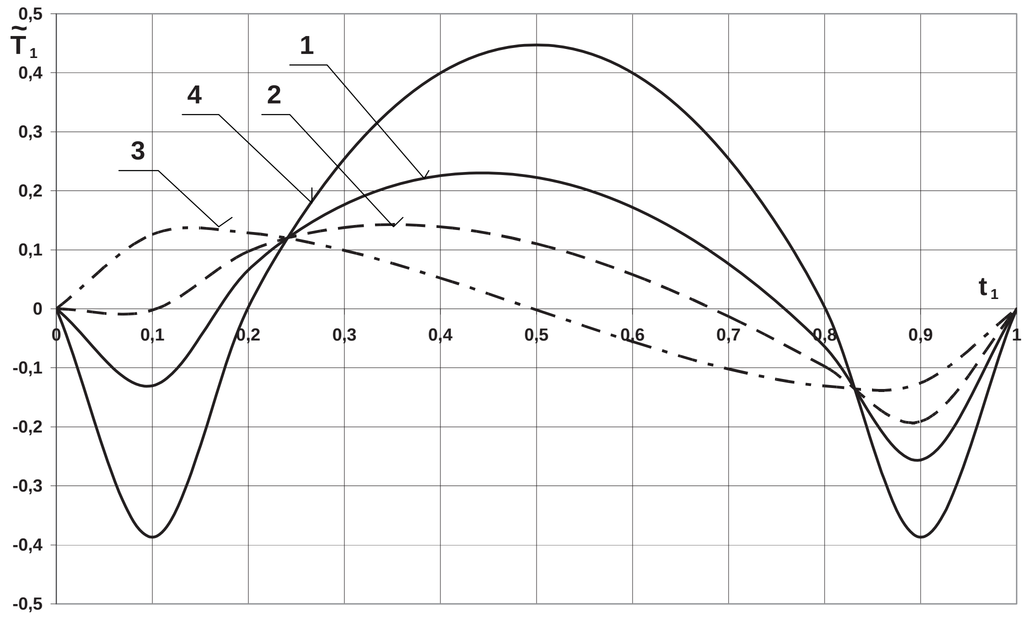


*Fig. 5. Change the dimensionless friction forces $\tilde{T}_1$ in the time $t_1$ by velocity perturbations: 1) $V_{10}$=0.05, $V_{1h}$=0, 2) $V_{10}$=0.05, $V_{1h}$=0.025, 3) $V_{10}$=0.05, $V_{1h}$=0.05, 4) $V_{10}$=0.05, $V_{1h}$=-0.05*

## References

[1] Krasowski, P., *Modelling of laminar unsteady and unsymetrical oil flow in slide journal bearing gap*, Tribologia 5, s. 1425-1436, 2002.

[2] Krasowski, P., *Pressure and capacity forces in slide journal plane bearing by laminar unsteady lubrication*, Journal of Polish CIMAC, Diagnosis, Reliability and Safety, Vol. 3, No. 2, pp. 91-98, Gdansk 2008.

[3] Krasowski, P., *Pressure and velocity distribution in slide journal bearing by laminar unsteady lubrication*, Journal of KONES Powertrain and Transport, Vol. 16, No. 2, pp. 239-246, Warsaw 2009.

[4] Teipel, I., Waterstraat, A., *The Reynolds equation for lubrication under unsteady conditions*, Proceedings the IX Canadian Congress of Applied Mechanics, pp. 497-502, University of Saskatchewan, 1983.
[5] Wierzcholski, K., *Mathematical methods in hydrodynamic theory of lubrication*, Technical University Press, Szczecin 1995.
[6] Wierzcholski, K., *Teoria niekonwencjonalnego smarowania łożysk ślizgowych*, Technical University Press, Szczecin 1995.