

## NUMBER OF MEASUREMENT CYCLES AND ACCURACY OF ENGINE INDICATION PROCESS

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### **Abstract**

*The results of measurements of instantaneous pressure in the cylinder are most often inputs to further calculations of the characteristic parameters of the working cycle of the engine. Many repetitions of measurements always feature a degree of unrepeatability in registered cycles, making determination of the so called representative round difficult. The paper considers the problem of estimating the minimal number of measurement cycles for the registration of pressure in order to obtain a representative course of pressure in the cylinder of an compression ignition engine. Maximum pressure was adopted as an indicator of cycle repeatability. After excluding systematic errors, repetition of maximum pressures recorded in subsequent measurement cycles were analyzed for fixed load and rotation speed of the engine, defining the coefficient of variation CV. Subsequently, for selected states of the engine the representativeness of the average maximum pressure depending on the number of measurement cycles averaged was examined. Full evaluation of the accuracy of the indication process in relation to the number of averaged measurement cycles requires also determination of the so called expanded uncertainty of the measurement's result. Further analysis including determination of the expanded uncertainty of the measurement result revealed that pressure course averaging over 20 consecutive cycles of measurement is sufficient to obtain a representative cycle.*

**Keywords:** *CI engine, measurement, pressure, statistical analysis*

### **1. Introduction**

In the paper are presented the results of statistical analysis of measurements of pressure course in a cylinder of an automatic ignition engine for personal car. Research was conducted on four-cylinder engine with a capacity of 1.3 dm<sup>3</sup> with direct fuel injection and four valves per cylinder, supercharged with a turbo-compressor system and equipped with an electronically controlled common rail injection system. During the tests for registration and storage of highly variable courses a SMETEC COMBI device along with a PC combined with PUMA OPEN 1.3 main control and acquisition programme was used. To register the course of pressure in the combustion

chamber of the engine a KISTLER 6058A piezoquartz transducer was used which does not require cooling. Basic parameters of the transducer are given in Tab. 1. The transducer was installed in the combustion chamber via an adapter in the shape and dimensions of a glow plug. An AVL 365X converter was used to measure the angular position of the engine's crankshaft.

Measurements of pressure courses in the cylinder were carried out for the following twenty steady states of the engine:

- engine speed: 1500 rpm, 2500 rpm, 3500 rpm, 4000 rpm,
- engine load:  $0.25 T_{\max}$ ,  $0.5 T_{\max}$ ,  $0.75 T_{\max}$ ,  $0.9 T_{\max}$ ,  $T_{\max}$  for each speed.

For each steady state of the engine 50 consecutive measurements of pressure courses in the cylinder in subsequent cycles were performed, where one cycle comprised two rotations of the crankshaft ( $720^\circ$  of crankshaft rotation). Measurement resolution was  $0.5^\circ$  of crankshaft rotation.

## 2. Repeatability of measurement of pressure in engine's cylinder

Many repetitions of measurements always feature a degree of unrepeatability in registered cycles, making determination of the so called representative round difficult [4]. One method of assessing repeatability of engine operating cycles, other than the analysis of the mean indicated pressure, is the analysis of the maximum pressure in the cylinder [2]. Both the mean indicated pressure and maximum pressure can be treated as indicators of cycle repeatability. This paper uses the maximum pressure values for the analysis of the number of measurement cycles which after averaging yield a representative cycle. To achieve this, a number of measurements of pressures for several loads and engine rotation speeds were carried out. In assessing the results of measurements the following was performed:

- check whether the distribution of the maximum recorded pressures is a normal distribution for fixed load and rotation speed of the engine,
- calculation of the rate of repetition of the registered value of the maximum pressure in the cylinder, depending on the given load and rotation speed of the engine,
- estimation of the minimal number of measurement cycles whose averaging ensures representativeness of the cycle for the fixed load and rotation speed of the engine.

Assuming that during the tests no so called gross or systematic errors were committed, the maximum pressure variability in subsequent operation cycles is mainly due to the impact of factors which cannot be unambiguously accounted for and constitutes a so called statistical (random) error. Each measurement of maximum pressure is therefore treated as a random variable. Consequently, all results of measurements are a sample of the population for which the average value can be adopted as the best estimator of the measured quantity. To perform statistical analyses an a priori level of significance  $\alpha = 0.05$  and of confidence  $p = 95\%$  were adopted adequately, assuming repeatable test conditions for subsequent repetitions. Checking whether the distribution of the maximum recorded pressure is a normal distribution and calculation of the degree of repetition of the maximum pressure value in the cylinder as well as estimation of the minimal number of measurement cycles were carried out respectively for 5, 10, 20, 30, 40 or 50 consecutive measurement cycles recorded for a fixed load and rotation speed of the engine.

## 3. Verification of conformity of the distribution of registered maximum pressure values with normal distribution

Many repetition of measurement of maximum pressure in the cylinder for fixed and unchanging load and rotation speed of the engine results in a series of values forming a distribution. This distribution should be a normal distribution obtained as a result of addition of a large number of independent factors. Deviations from the normal distribution show that in the course of the tests systematic errors were made.

Histograms may be used for the preliminary assessment of the character of the obtained distributions of the maximum pressure values. Fig. 1 shows histograms of the distribution of maximum pressure values registered over 50 consecutive cycles for rotation speed 2500 rpm and respectively different engine loads.

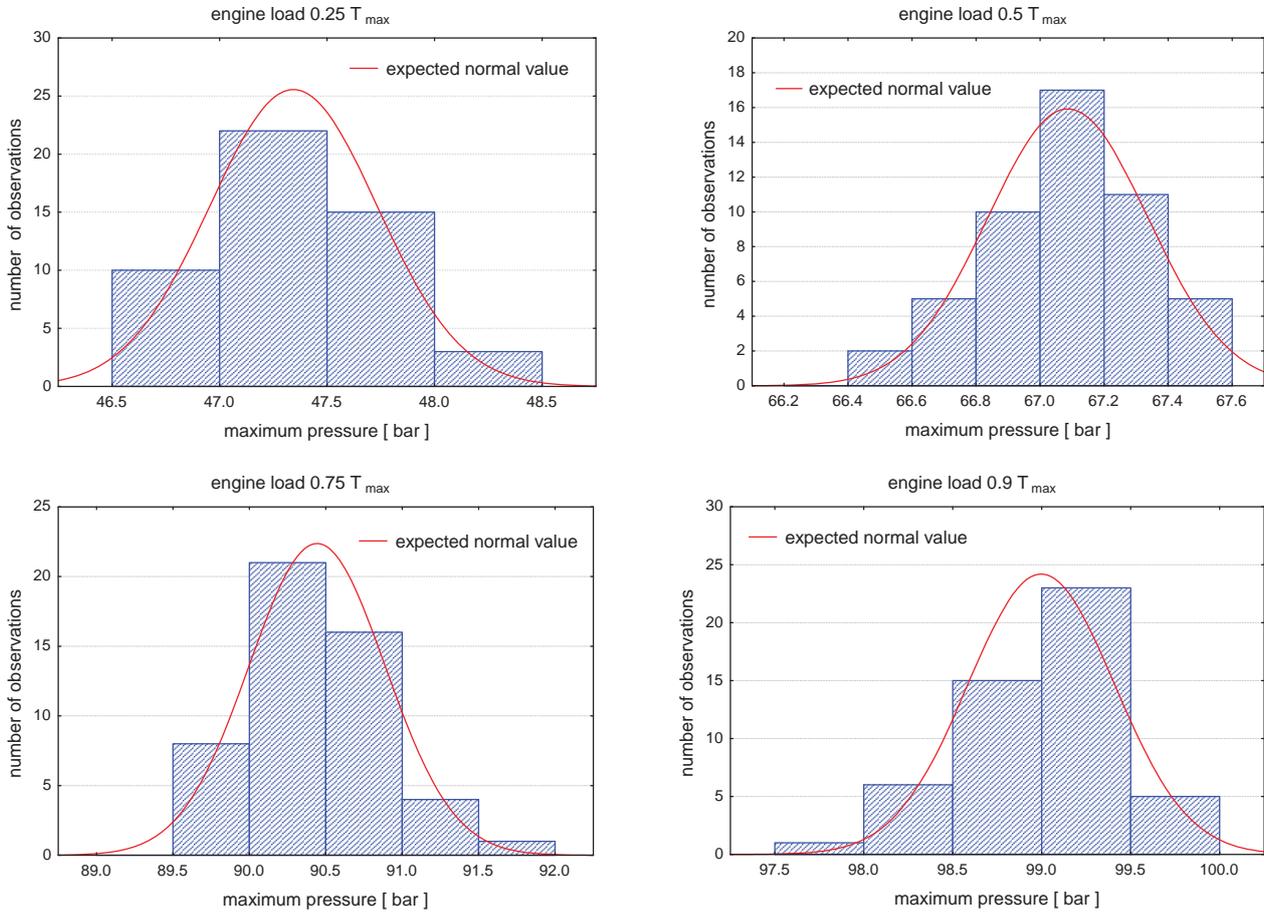


Fig. 1. Histograms of the distribution of maximum pressure values registered over 50 consecutive cycles for rotation speed 2500 rpm and respectively different engine loads

Proper evaluation of the obtained distributions requires the use of appropriate tests such as the Shapiro-Wilk test giving the probability that the sample comes from a normally distributed population [3].

#### 4. Analysis of repeatability of registered maximum pressure in the cylinder depending on load and rotation speed of the engine

For the assessment of repeatability of recorded maximum pressure values in subsequent measurement cycles and for a fixed load and rotation speed of the engine the *CV* factor was used, henceforth referred to as the coefficient of variation. This factor is the relative value of the standard deviation expressed as a percentage, thus:

$$CV = \frac{SD}{\bar{P}_{\max}} \cdot 100\% , \quad (1)$$

where:

*SD* - standard deviation,

$\bar{P}_{\max}$  - arithmetic mean of maximum pressure calculated over *n* measurements.

The standard deviation  $SD$  for  $n$  measurements being calculated by the formula:

$$SD = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (p_{\max,i} - \bar{p}_{\max})^2}, \quad (2)$$

where:

$\bar{p}_{\max,i}$  - maximum pressure registered in a single subsequent cycle.

The coefficient of variation determined in this study for  $n = 50$  results of measurements carried out in each of the considered points of the state of engine operation. Dependency of the calculated CV coefficient on the load and rotation speed are shown in Fig. 2.

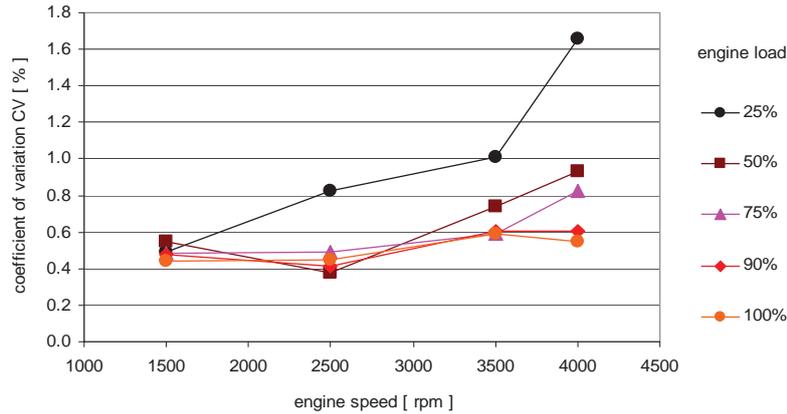


Fig. 2. The CV coefficient of variation for 50 pressure measurements performed for different engine operation states

The analysis of the calculated values of the CV coefficient indicates that in the case of the engine used in the tests the lowest repeatability of the registered values of maximum pressure in the cylinder applies in those states of engine operation where the engine load is low or moderate (up to  $0.5 T_{\max}$ ) and at the same time the rotation speed is high, i.e. 3500 rpm and 4000 rpm.

It is also easy to notice that for those states of engine operation where the loads are high ( $\geq 0.9 T_{\max}$ ) the CV coefficient did not exceed 0.6%, nor did it feature as apparent growing tendency along with the increase of the rotation speed as it did for lower loads. For such loads of the engine a relatively small variability of the registered maximum pressure values regardless of engine speed should therefore be expected.

## 5. Estimation of the minimal number of measurement cycles required to obtain a representative cycle

Registration even of a large number of measurement cycles of pressure courses in the cylinder of an engine is hardly a lengthy process. Higher investment of time and work is required to merely prepare the engine and the test bench. Nevertheless, when numerous measurement cycles are to be registered, the size of the set of data collected can be a problem. It is therefore vital to estimate the minimal number of measurement cycles, averaging over which ensures representativeness of the cycle for a given load and rotation speed of the engine.

In the present chapter a procedure is presented enabling the estimation of the minimal number of measurement cycles. Among the states of engine operation analyzed the three characterized by highest values of the coefficient of variation CV in 50 cycles averaged and three other for which the coefficient of variation CV reached the lowest values among the analyzed operation states were selected for the estimation of the minimal number of measurement cycles. Tab. 1 summarizes those states of engine operation for which the estimation of the minimal number of measurement cycles averaged to obtain a representative cycle was performed.

Tab. 1. Operation states for which the minimal number of measurement cycles required to obtain a representative cycle was estimated

Operation state	Rotation speed [rpm]	Load	Calculated CV ( $n = 50$ ) [%]
A1	4000	0.25 $T_{\max}$	1.66
A2	3500	0.25 $T_{\max}$	1.01
A3	4000	0.5 $T_{\max}$	0.94
A4	1500	$T_{\max}$	0.44
A5	2500	$T_{\max}$	0.44
A6	1500	0.9 $T_{\max}$	0.48

For the courses of pressure registered in consecutive 5, 10, 20, 30, 40 and 50 measurement cycles the highest and lowest value of maximum pressure respectively was determined and calculations were performed of the mean value and standard deviation value and standard error of the determination of the mean maximum pressure by the relationship:

$$SE = \frac{SD}{\sqrt{n}} . \quad (3)$$

Additionally the lowest and highest value of mean maximum pressure was determined, fixing the confidence interval  $\langle -95\%, 95\% \rangle$  for the mean value of maximum pressure, satisfying the relationship:

$$\bar{p}_{\max} \pm \frac{u_{\alpha} \cdot SD}{\sqrt{n}} , \quad (4)$$

where:

$u_{\alpha}$  - value of the statistic for the normal distribution ( $u_{\alpha} = 1.96$  for significance level  $\alpha = 0.95$ ), in case of small cardinality of the sample ( $n < 30$ )  $u_{\alpha}$  is replaced by the statistic  $t_{\alpha, n-1}$  taken from Student's t-distribution depending on the significance level  $\alpha$  and the cardinality of the sample  $n$ .

Results of calculations for the considered states of engine operation are shown in Fig. 3.

Analysis of the required number of measurement cycles necessary for obtaining a representative mean value of the maximum pressure requires adopting an acceptable value of standard error. It may be assumed that an acceptable value of standard error is equal to the value of maximal error of the measurement instrument used. According to the characteristics of the converter shown in Tab. 1 the acceptable value of the standard error was each time set as 0.4% of the measured value.

On the ground of the results placed in Tab. 1 the following can be proposed:

- in the case of tests covering states of engine operation characterized by high, close to maximum, loads of the engine ( $\geq 0.9 T_{\max}$ ) and at the same time low rotation speeds (1500 and 2500 rpm) it is sufficient to average over 5 measurement cycles to achieve the minimum, acceptable value of the standard error,
- in other cases, i.e. for the measurements performed for states of engine operation characterized by low or moderate load of the engine (up to 0.5  $T_{\max}$ ) and at the same time higher rotation speeds (3500 and 4000 rpm), achieving the acceptable value of the standard error requires carrying out and averaging of at least 10 measurement cycles.

Furthermore, as can be seen in Fig. 3, and also as should be expected, increasing the number of measurement cycles leads to decreasing the value of the standard error. Analyzing the variation of the calculated mean value of the maximum pressure with increasing the number of measurement cycles allows in the analyzed states of engine operation to observe a pattern of decreasing differences between the calculated mean values on averaging over  $n = 20$  and more measurement cycles. In addition to that, in most of the analyzed states of engine operation the mean value of the maximum pressure calculated for  $n = 20$  measurement cycles falls within the confidence interval fixed for the mean value determined using the maximum number of measurement cycles ( $n = 20$ ).

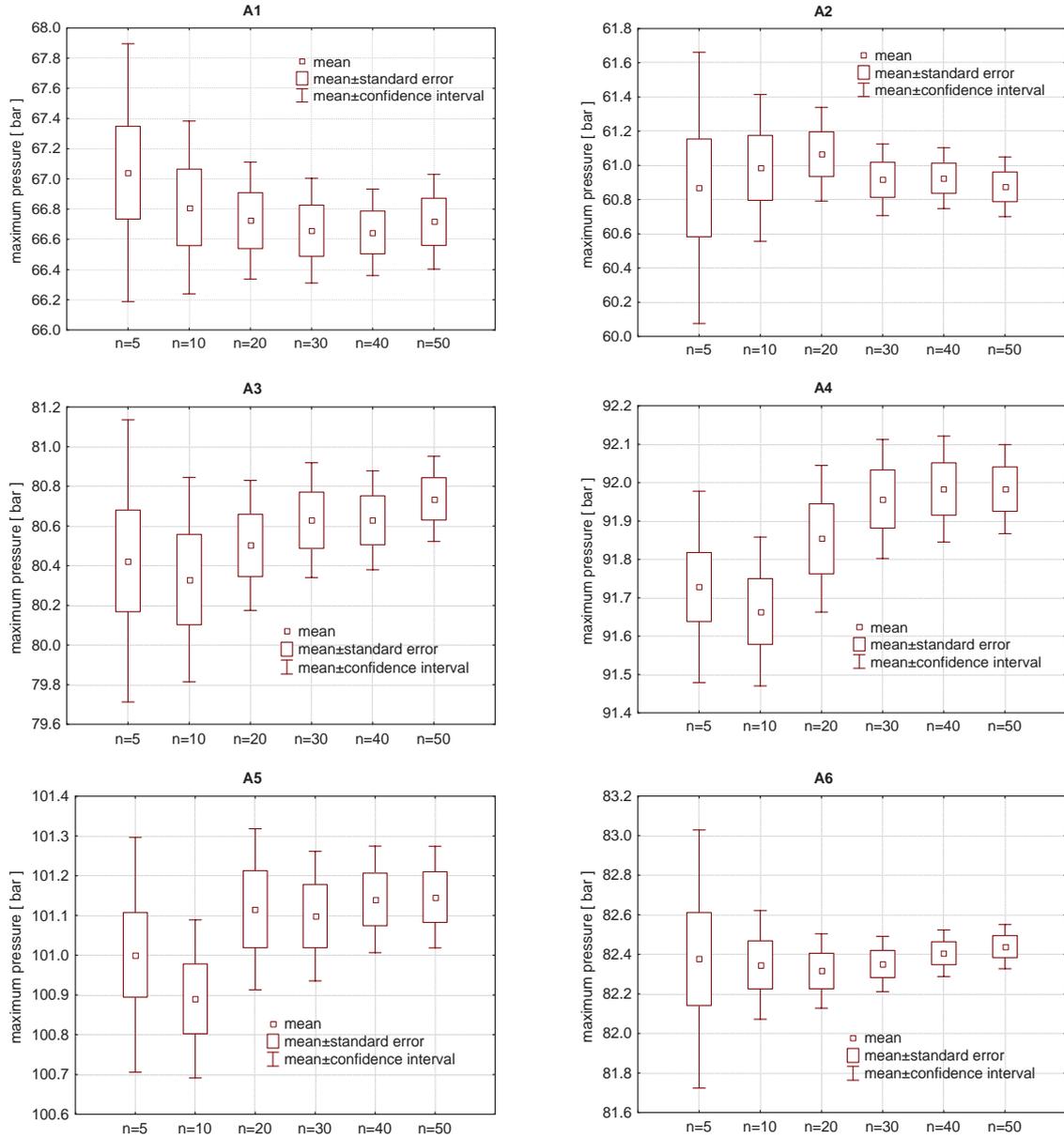


Fig. 3. Representativeness of the mean maximum pressure depending on the number of averaged measurement cycles in the states of engine operation analyzed

## 6. The uncertainty of the measurement

Full evaluation of the accuracy of the indication process in relation to the number of averaged measurement cycles requires also determination of the so called expanded uncertainty of the measurement's result. The methodology of determining the uncertainty of a measurement firstly requires calculating the composite standard uncertainty [1]:

$$u_c = \sqrt{u_1^2 + u_2^2}, \quad (5)$$

where:

$u_1$  - standard uncertainty due to dispersion of the results determined by statistical methods (Type-A uncertainty),

$u_2$  - another kind of uncertainty (Type-B uncertainty), in the paper the uncertainty due to the converter's error was adopted assuming a uniform readings probability distribution and ignoring other components of uncertainty able to influence the result of a measurement, e.g. the

uncertainty of load parameters and of the rotation speed given by the engine test bench. In cases considered in the paper we may write:

$$u_1 = SE, \tag{6}$$

$$u_2 = \frac{0.004 \bar{p}_{\max}}{\sqrt{3}}. \tag{7}$$

The expanded uncertainty is next calculated as the product:

$$U = k u_c, \tag{8}$$

where:

$k$  - expansion coefficient - is taken from a table based on Student's t-distribution for confidence level  $p = 95\%$ .

The calculated values of expanded uncertainty of the result of measurement for analyzed states of engine operation are shown in Fig. 4 in relation to the performed number of measurement cycles. The analysis of the results of calculation of expanded uncertainty of the measurement allows to propose the following patterns:

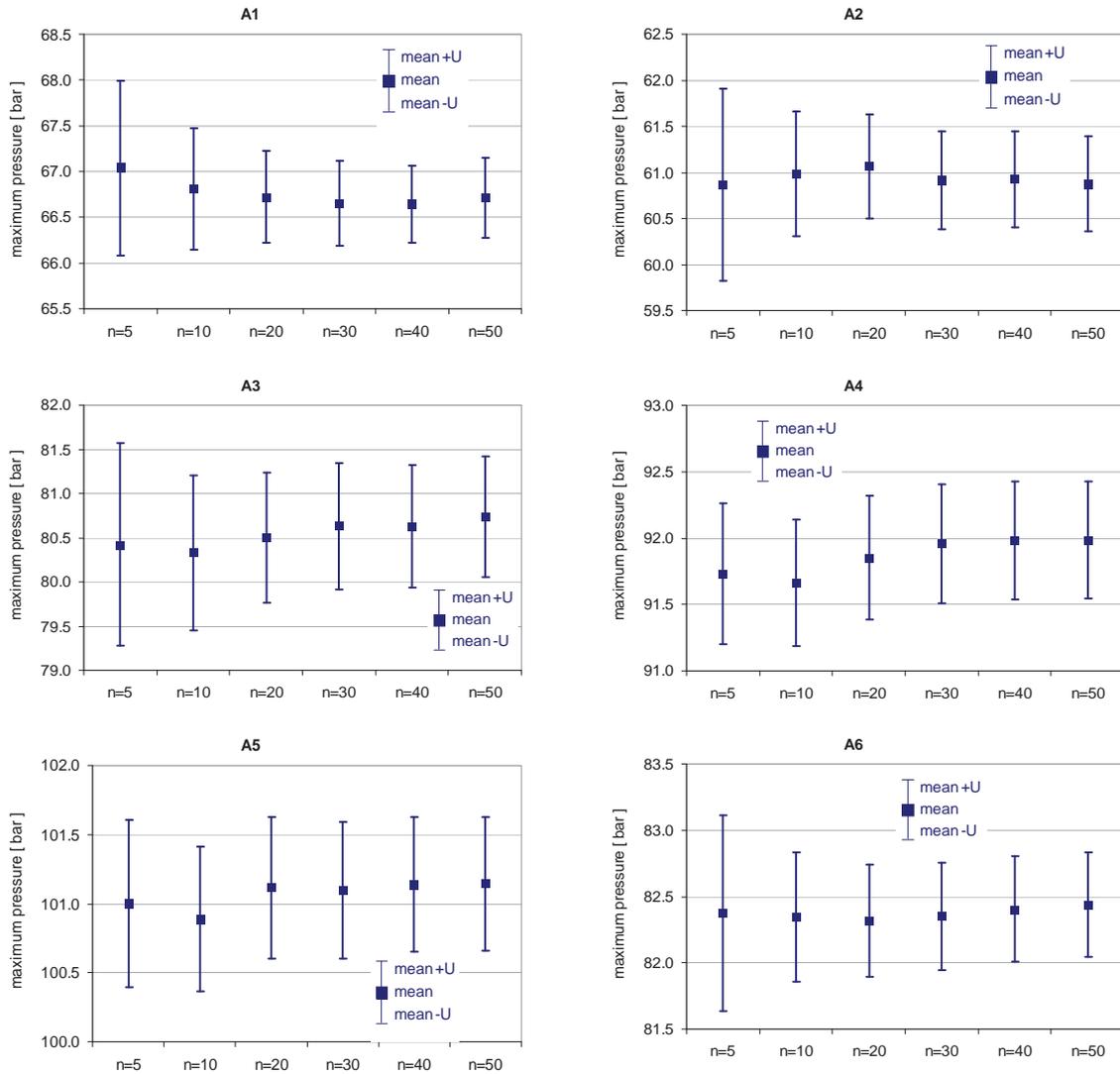


Fig. 4. Values of expanded uncertainty of the result of measurement in analyzed states of engine operation depending on the number of measurement cycles averaged

- in all analyzed states of engine operation the determined values of measurement uncertainty ought to be construed as low – for its values do not exceed 1.5% of the estimate of the measured quantity in case of using 5 and 10 measurement cycles, whereas for 20 and more cycles they do not exceed 1.0%,
- in states of engine operation characterized by low and moderate load of the engine (up to  $0.5 T_{\max}$ ) and at the same time higher rotation speeds (respectively 3500 and 4000 rpm) the influence on higher values of the expanded uncertainty for low numbers of cycles ( $n = 5$ ,  $n = 10$ ) is due to higher values of the uncertainty coming from statistical errors (type-A uncertainty) – for example the dependency between the  $u_1$  and  $u_2$  uncertainties for the A1 state of engine operation is charted in Fig. 5,
- in states of engine operation characterized by high load of the engine ( $\geq 0.9 T_{\max}$ ) and at the same time low rotation speeds (1500 and 2500 rpm) the calculated expanded uncertainty is less sensitive to the number of measurements; this is caused by a greater role of the uncertainty due to the accuracy of the measurement instrument used.

The analyzes performed within the study for selected twenty states of engine operation characterized by different load and rotation speed indicate that averaging courses of pressure over 20 consecutive measurement cycles is sufficient to obtain a representative cycle.

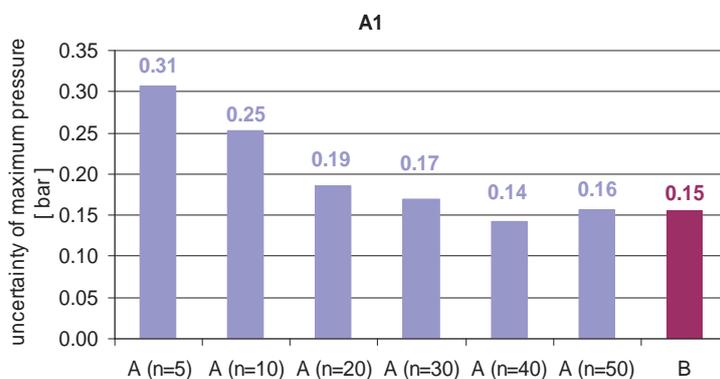


Fig. 5. Comparison of the uncertainties  $u_1$  and  $u_2$  for the A1 state of engine operation

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